ICIC Express Letters ICIC International ©2016 ISSN 1881-803X
Volume **10**, Number **4**, April **2016** pp. **915–922**

MULTI-OBJECTIVE OPTIMAL TRACKING PERFORMANCE OF NETWORKED SYSTEMS WITH COMMUNICATION CONSTRAINTS

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Received October 2015; accepted January 2016

Abstract. *The multi-objective optimal tracking performance of single-input single-output (SISO) networked systems with the packet dropouts and channel noise is studied in this paper. The tracking performance is measured by the energy of the error signal between the output of the plant and the reference signal. The obtained result shows that the optimal tracking performance is dependent on the non-minimum phase zeros, unstable poles of a given plant, as well as the packet dropout probability and the power spectral density of additive white Gaussian noise (AWGN) of communication channel. A typical example is given to illustrate the theoretical result.*
Keywords: Optimal tracking performance, Packet dropout probability, Non-minimum phase zeros, Multi-objective optimization

1. **Introduction.** Control systems in which communication takes place over nontransparent communication links are called networked systems. In recent years, the application of networked systems has shown great growth. Due to their advantages, such as reduced cost, low weight, high resource utilization and simple installation, networked systems have been widely applied in many areas, for example, military, the robot control, automated highway systems. While networked systems have received increasing research attention, they also brought about new challenges due to the limitation of the network resource. The network-induced delay and packet dropout are always inevitable as the sampling data is transmitted through the network, and they cause the deterioration and instability of system performance, so they have attracted much research interest.

The optimal H_2 performance can be found in [1], and there were many analyses about the tracking control problems [2]. The tracking performance of multi-input multi-output (MIMO) linear time-invariant systems was studied in [2]. It showed that its unstable poles and non-minimum phase (NMP) zeros of a given plant as well as the additive white Gaussian noise impose unavoidable limitations on achievable performance. It also pointed out that two-parameter controller structure can improve the tracking performance, however, the optimal tracking performance has not considered the packet dropout in [2]. In recent years, the topic of optimal tracking performance has been extended to networked systems [3]. The optimal tracking performance problem for MIMO networked systems with communication constraints was studied in paper [3]. The optimal tracking problem for SISO networked systems over a communication channel with packet dropouts was studied in [4]. However, in [3], in order to attain the minimal tracking error, the control input and channel input of networked systems often required to have an infinite energy in the optimal tracking problem. This requirement cannot be met in general in practice. Thus, the control input and channel input energy of networked systems should be considered in the performance index to address this issue. In this paper, we study the multi-objective optimization performance problem in terms of the tracking error energy with considering the channel input energy and control input energy, and meanwhile we consider communication channel of the packet dropouts and channel noise, which are more realistic models of communication link than those in [3].

In this paper, we study the multi-objective optimization performance problem of SISO networked systems with packet dropout and channel noise under the channel input energy and control input energy constraints. Using the square of the 2-norm to define the multi-object optimization performance index and the explicit expression of multi-objective optimization performance is obtained by applying the spectral factorization and two-parameter compensator scheme. The obtained results show the optimal tracking performance of networked systems which was determined by plant internal structure and networked parameters, no matter what compensator is adopted, which will be guidance for the design of networked systems.

This paper is organized as follows. Section 2 introduces the problem formulation. The multi-objective optimization performance of networked systems with considering packet dropout and channel noise is studied in Section 3. A typical example is given to illustrate the results in Section 4. The paper conclusions and future research directions are presented in Section 5.

2. **Problem Statement.** For any complex number z, we denote its complex conjugate by $\bar{z}$. For any signal $x(t)$, we denote its Laplace transform by X. The expectation operator is defined as $E\{\cdot\}$. Let the open right-half plane be denoted by $\mathbb{C}_+ := \{s : \mathrm{Re}(s) > 0\}$, the open left-half plane by $\mathbb{C}_- := \{s : \mathrm{Re}(s) < 0\}$. $\mathcal{L}_2$ is the standard frequency domain Lebesgue space. $\mathcal{H}_2$ and $\mathcal{H}_2^{\perp}$ are subspaces of containing functions that are analytic in $\mathbb{C}_+$ and $\mathbb{C}_-$, respectively. Moreover, let $\|\cdot\|$ denote the Euclidean vector norm and the norm $\|\cdot\|_2$ in the space $\mathcal{L}_2$. The space $\mathcal{L}_2$ is the Hilbert space with inner product $\langle f, g\rangle := \frac{1}{2\pi}\int_{-\infty}^{\infty}(f^H(jw)g(jw))dw$. For any f, $g \in \mathcal{L}_2$, they are orthogonal if $\langle f, g\rangle = 0$. It is well known that $\mathcal{L}_2$ can be decomposed into two orthogonal subspaces $\mathcal{H}_2$ and $\mathcal{H}_2^{\perp}$. Finally, $\mathbb{R}\mathcal{H}_\infty$ denotes the set of all stable, proper, rational transfer function.

We consider SISO networked systems with packet dropouts and channel noise of the communication channel by two-parameter compensator as depicted in Figure 1, where the problem is to obtain the multi-objective tracking performance of networked systems. In this setup, G represents the plant model, and $K = [K_1\ K_2]$ represents the two degree compensator, whose transfer function are denoted as $G(s)$, $\begin{bmatrix} K_1(s) & K_2(s) \end{bmatrix}$, respectively. The communication channel is characterized by two components: the parameter d_r and AWGN n. The parameter d_r represents whether or not a packet is dropped:

$$d_r = \begin{cases} 0 & \text{if the systems output is not successfully transmitted to the controller} \\ 1 & \text{if the systems output is successfully transmitted to the controller} \end{cases}$$

where the stochastic variable $d_r \in R$ is a Bernoulli distributed white sequence with $Prob\{d_r = 0\} = q$, $Prob\{d_r = 1\} = 1 - q$, where q is the packet dropout probability.

The signal r, y, $\hat{y}$ and u represent, respectively, the reference input, the system output, channel input and the system input, whose transfer functions are R, Y, $\tilde{Y}$ and U. The

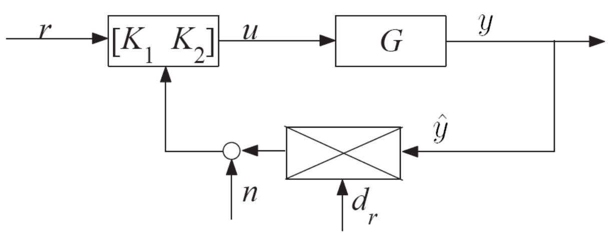

Figure 1. Networked system with packet dropout and channel noise

reference signal r is a Brownian motion, and $E\{|r(t)|\} = 0$, $E\{|r(t)|^2\} = \sigma_1^2$. Channel noise signal n is a zero mean Gaussian white noise, $E\{|n(t)|^2\} = \sigma_2^2$. And r, n, d_r are uncorrelated with each other in this paper.

For the reference signal r, $\tilde{E} = R - Y$ is defined as the tracking error. According to Figure 1, we can obtain $Y = GRK_1 + G(Yd_r + n)K_2$, $Y = GU$, $Y = \hat{Y}$.

According to the calculation methods of [3], a direct calculation is given as

$$S(e^{j\omega}) = \left(1 - \frac{G(e^{j\omega})K_1(e^{j\omega})}{1 - (1-q)\,G(e^{j\omega})K_2(e^{j\omega})}\right) S_{re} - \frac{(e^{j\omega})G(e^{j\omega})K_2(e^{j\omega})}{1 - (1-q)\,G(e^{j\omega})K_2(e^{j\omega})} S_{ne}.$$

S_{re} is the frequency characteristics from r to e and S_{ne} is the frequency characteristics from n to e. r, n, d_r are uncorrelated with each other in this paper, and we can obtain

$$E\left\{\left\|\tilde{E}\right\|_2^2\right\} = \left\|1 - \frac{GK_1}{1-(1-q)GK_2}\right\|_2^2 \sigma_1^2 + \left\|\frac{GK_2}{1-(1-q)GK_2}\right\|_2^2 \sigma_2^2.$$

Similarly, we can obtain

$$E\left\{\|Y\|_2^2\right\} = \left\|\frac{GK_1}{1-(1-q)GK_2}\right\|_2^2 \sigma_1^2 + \left\|\frac{GK_2}{1-(1-q)GK_2}\right\|_2^2 \sigma_2^2,$$

$$E\left\{\|U\|_2^2\right\} = \left\|\frac{K_1}{1-(1-q)GK_2}\right\|_2^2 \sigma_1^2 + \left\|\frac{K_2}{1-(1-q)GK_2}\right\|_2^2 \sigma_2^2.$$

3. **Multi-object Tracking Performance with Channel Noise.** In this paper, we want to obtain the optimal tracking error subject to the channel input energy and control input energy constraints. We denote the multi-objective performance as

$$J = (1 - \varepsilon_1 - \varepsilon_2)\, E\left\{\left\|\tilde{E}\right\|_2^2\right\} + \varepsilon_1 \left(E\left\{\|U\|_2^2\right\}\right) - \Gamma + \varepsilon_2 \left(E\left\{\left\|\tilde{Y}\right\|_2^2\right\} - \Upsilon\right) \tag{1}$$

where $0 \le \varepsilon_1 + \varepsilon_2 \le 1$, which represents the trade-off among the optimal tracking error, the channel input energy and control input energy. Γ is the allowed control input energy, and Υ is the allowed channel input energy.

The multi-objective tracking performance is measured by the possible minimal tracking error achievable by all possible linear stabilizing controllers (denoted by K), determined as

$$J^* = \inf_{K \in \mathcal{K}} J \tag{2}$$

Therefore,

$$\begin{aligned} J = {} & (1 - \varepsilon_1 - \varepsilon_2) \left(\left\|1 - \frac{GK_1}{1+(1-q)GK_2}\right\|_2^2 \sigma_1^2 + \left\|\frac{GK_2}{1+(1-q)GK_2}\right\|_2^2 \sigma_2^2 \right) \\ & + \varepsilon_2 \left(\left\|\frac{GK_1}{1+(1-q)GK_2}\right\|_2^2 \sigma_1^2 + \left\|\frac{GK_2}{1+(1-q)GK_2}\right\|_2^2 \sigma_2^2 \right) - \varepsilon_2 \Upsilon \\ & + \varepsilon_1 \left(\left\|\frac{K_1}{1+(1-q)GK_2}\right\|_2^2 \sigma_1^2 + \left\|\frac{K_2}{1+(1-q)GK_2}\right\|_2^2 \sigma_2^2 \right) - \varepsilon_1 \Gamma \end{aligned} \tag{3}$$

For the rational transfer function $(1-q)G$, let its coprime factorization be given by $(1-q) = NM^{-1}$, where $N, M \in \mathbb{R}\mathcal{H}_\infty$, and satisfy the Bezout identity $MX - NY = 1$, where $X, Y \in \mathbb{R}\mathcal{H}_\infty$.

It is well known that every stabilizing compensator K can be characterized by Youla parameterization [5]

$$\mathcal{K} := \left\{K : K = \begin{bmatrix} K_1 & K_2 \end{bmatrix} = (X - DN)^{-1} \begin{bmatrix} Q & Y - DM \end{bmatrix}, Q \in \mathbb{R}\mathcal{H}_\infty, D \in \mathbb{R}\mathcal{H}_\infty\right\}.$$

It is also well known that a non-minimum phase transfer function could factorize a minimum phase part and an all pass factor $N = L_z N_m, M = B_p M_m$, where L_z and B_p are the all pass factors, and N_m and M_m are the minimum phase parts. L_z includes all the right half plane zeros of the plant $z_i \in \mathbb{C}_+$, $i = 1, \cdots, n$, and B_p includes all the right half plane poles of the plant $p_j \in \mathbb{C}_+$, $j = 1, \cdots, m$. We consider coprime factorization of $L_z(s)$ and $B_p(s)$ respectively as $L_z(s) = \prod_{i=1}^{n} \frac{s-z_i}{s+\bar{z}_i}$, $B_p(s) = \prod_{j=1}^{m} \frac{s-p_j}{s+\bar{p}_j}$.

Therefore, we can get

$$\begin{aligned} J = {} & (1-\varepsilon_1-\varepsilon_2)\left(\left\|1-\frac{NQ}{(1-q)}\right\|_2^2 \sigma_1^2 + \left\|\frac{N(Y-DM)}{(1-q)}\right\|_2^2 \sigma_2^2\right) \\ & + \varepsilon_2\left(\left\|\frac{NQ}{(1-q)}\right\|_2^2 \sigma_1^2 + \left\|\frac{N(Y-DM)}{(1-q)}\right\|_2^2 \sigma_2^2\right) \\ & - \varepsilon_2 \Upsilon + \varepsilon_1\left(\|MQ\|_2^2 \sigma_1^2 + \|M(Y-DM)\|_2^2 \sigma_2^2\right) - \varepsilon_1 \Gamma. \end{aligned} \tag{4}$$

Then, we have

$$\begin{aligned} J^* = \inf_{D,Q\in\mathbb{R}\mathcal{H}_\infty} & \left\{\left((1-\varepsilon_1-\varepsilon_2)\left\|1-\frac{NQ}{(1-q)}\right\|_2^2 \sigma_1^2 + \varepsilon_2\left\|\frac{NQ}{(1-q)}\right\|_2^2 \sigma_1^2 + \varepsilon_1\|MQ\|_2^2 \sigma_1^2\right)\right. \\ & \left. + \left(\varepsilon_1\|M(Y-DM)\|_2^2 \sigma_2^2 + (1-\varepsilon_1)\left\|\frac{N(Y-DM)}{(1-q)}\right\|_2^2 \sigma_2^2\right)\right\} - \varepsilon_1\Gamma - \varepsilon_2\Upsilon. \end{aligned}$$

It is clear that in order to obtain J^*, Q and D must be appropriately selected.

Theorem 3.1. *For given networked systems such as Figure 1, the multi-object performance under the channel input energy and control input energy constraints can be expressed as:*

$$J^* = \sum_{j=1}^{m}\sum_{k=1}^{m} \frac{\gamma_j \gamma_j^H}{p_j+\bar{p}_k}\sigma_2^2 + A\sum_{i=1}^{n} 2\mathrm{Re}(z_i)\sigma_1^2 - \varepsilon_2\Upsilon - \varepsilon_1\Gamma + A\left\|\begin{matrix} 1-\frac{N_m\Delta_0^{-1}\Delta_0^{-H}N_m^H}{(1-q)^2} \\ \sqrt{\frac{\varepsilon_2}{1-\varepsilon_1-\varepsilon_2}}\frac{N_m\Delta_0^{-1}\Delta_0^{-H}N_m^H}{(1-q)^2} \\ \sqrt{\frac{\varepsilon_1}{1-\varepsilon_1-\varepsilon_2}}\frac{M_m\Delta_0^{-1}\Delta_0^{-H}N_m^H}{(1-q)} \end{matrix}\right\|_2^2 \sigma_1^2,$$

where $\gamma_j = -2\,\mathrm{Re}(p_j)\Lambda_o(p_j)N_m^{-1}(p_j)L_z^{-1}(p_j)\prod_{k\neq j}\frac{p_j+\bar{p}_k}{p_j-p_k}$, $A = (1-\varepsilon_1-\varepsilon_2)$.

Proof: Because the B_p and L_z are all pass factors, we can obtain

$$\begin{aligned} J^* = {} & \inf_{Q\in\mathbb{R}\mathcal{H}_\infty}\left\|\begin{bmatrix} \sqrt{1-\varepsilon_1-\varepsilon_2}\left(L_z^{-1}-\frac{N_mQ}{(1-q)}\right) \\ \sqrt{\varepsilon_2}\frac{N_mQ}{(1-q)} \\ \sqrt{\varepsilon_1}M_mQ \end{bmatrix}\right\|_2^2 \sigma_1^2 - \varepsilon_2\Upsilon \\ & + \inf_{D\in\mathbb{R}\mathcal{H}_\infty}\left\|\begin{matrix} \sqrt{\varepsilon_1}M_m(Y-DM) \\ \sqrt{1-\varepsilon_1}\frac{N_m(Y-DM)}{(1-q)} \end{matrix}\right\|_2^2 \sigma_2^2 - \varepsilon_1\Gamma. \end{aligned}$$

By a simple calculation, J^* can be expressed as

$$J^* = \inf_{Q\in\mathbb{R}\mathcal{H}_\infty} \left\| \begin{bmatrix} \sqrt{1-\varepsilon_1-\varepsilon_2}\left(L_z^{-1}-1+1-\frac{N_mQ}{(1-q)}\right) \\ \sqrt{\varepsilon_2}\frac{N_mQ}{(1-q)} \\ \sqrt{\varepsilon_1}M_mQ \end{bmatrix} \right\|_2^2 \sigma_1^2 - \varepsilon_2\Upsilon$$

$$+ \inf_{D\in\mathbb{R}\mathcal{H}_\infty} \left\| \begin{bmatrix} \sqrt{\varepsilon_1}M_m \\ \frac{\sqrt{1-\varepsilon_1}N_m}{(1-q)} \end{bmatrix} (Y - DM) \right\|_2^2 \sigma_2^2 - \varepsilon_1\Gamma.$$

In order to calculate the J^*, we denote

$$J_1^* = \inf_{Q\in\mathbb{R}\mathcal{H}_\infty} \left\| \begin{bmatrix} \sqrt{1-\varepsilon_1-\varepsilon_2}\left(L_z^{-1}-1+1-\frac{N_mQ}{(1-q)}\right) \\ \sqrt{\varepsilon_2}\frac{N_mQ}{(1-q)} \\ \sqrt{\varepsilon_1}M_mQ \end{bmatrix} \right\|_2^2 \sigma_1^2 \tag{5}$$

$$J_2^* = \inf_{D\in\mathbb{R}\mathcal{H}_\infty} \left\| \begin{bmatrix} \sqrt{\varepsilon_1}M_m \\ \frac{\sqrt{1-\varepsilon_1}N_m}{(1-q)} \end{bmatrix} (Y - DM) \right\|_2^2 \sigma_2^2 \tag{6}$$

By a simple calculation, J_1^* can be expressed as

$$J_1^* = \inf_{Q\in\mathbb{R}\mathcal{H}_\infty} \left\| \begin{bmatrix} \sqrt{1-\varepsilon_1-\varepsilon_2}\,(L_z^{-1}-1) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \sqrt{1-\varepsilon_1-\varepsilon_2}\left(1-\frac{N_mQ}{(1-q)}\right) \\ \sqrt{\varepsilon_2}\frac{N_mQ}{(1-q)} \\ \sqrt{\varepsilon_1}M_mQ \end{bmatrix} \right\|_2^2 \sigma_1^2.$$

Because $\begin{bmatrix} \sqrt{1-\varepsilon_1-\varepsilon_2}(L_z^{-1}-1) \\ 0 \\ 0 \end{bmatrix}$ is in $\mathcal{H}_2^\perp$, and conversely, $\begin{bmatrix} \sqrt{1-\varepsilon_1-\varepsilon_2}\left(1-\frac{N_mQ}{(1-q)}\right) \\ \sqrt{\varepsilon_2}\frac{N_mQ}{(1-q)} \\ \sqrt{\varepsilon_1}M_mQ \end{bmatrix}$ is in $\mathcal{H}_2$, J_1^* can be expressed as

$$J_1^* = \left\| \begin{bmatrix} \sqrt{1-\varepsilon_1-\varepsilon_2}(L_z^{-1}-1) \\ 0 \\ 0 \end{bmatrix} \right\|_2^2 \sigma_1^2 + \inf_{Q\in\mathbb{R}\mathcal{H}_\infty} \left\| \begin{bmatrix} \sqrt{1-\varepsilon_1-\varepsilon_2}\left(1-\frac{N_mQ}{(1-q)}\right) \\ \sqrt{\varepsilon_2}\frac{N_mQ}{(1-q)} \\ \sqrt{\varepsilon_1}M_mQ \end{bmatrix} \right\|_2^2 \sigma_1^2.$$

In order to calculate J_1^*, define

$$J_{11}^* = \left\| \begin{bmatrix} \sqrt{1-\varepsilon_1-\varepsilon_2}(L_z^{-1}-1) \\ 0 \\ 0 \end{bmatrix} \right\|_2^2 \sigma_1^2,$$

$$J_{12}^* = \inf_{Q\in\mathbb{R}\mathcal{H}_\infty} \left\| \begin{bmatrix} \sqrt{1-\varepsilon_1-\varepsilon_2}\left(1-\frac{N_mQ}{(1-q)}\right) \\ \sqrt{\varepsilon_2}\frac{N_mQ}{(1-q)} \\ \sqrt{\varepsilon_1}M_mQ \end{bmatrix} \right\|_2^2 \sigma_1^2.$$

It follows by the same argument, and we have $J_{11}^* = (1-\varepsilon_1-\varepsilon_2)\sum_{i=1}^{n} 2\,\mathrm{Re}(z_i)\sigma_1^2$.

By a simple calculation, J_{12}^* can be expressed as

$$J_{12}^* = (1-\varepsilon_1-\varepsilon_2) \inf_{Q\in\mathbb{R}\mathcal{H}_\infty} \left\| \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{-N_m}{(1-q)} \\ \sqrt{\frac{\varepsilon_2}{1-\varepsilon_1-\varepsilon_2}}\frac{N_m}{(1-q)} \\ \sqrt{\frac{\varepsilon_1}{1-\varepsilon_1-\varepsilon_2}}M_m \end{bmatrix} Q \right\|_2^2 \sigma_1^2 \tag{7}$$

We introduce an inner-outer factorization $\begin{bmatrix} \frac{-N_m}{(1-q)} \\ \sqrt{\frac{\varepsilon_2}{1-\varepsilon_1-\varepsilon_2}}\frac{N_m}{(1-q)} \\ \sqrt{\frac{\varepsilon_1}{1-\varepsilon_1-\varepsilon_2}}M_m \end{bmatrix} = \Delta_i\Delta_o$.

To find the optimal Q, introduce $\psi \triangleq \begin{bmatrix} \Delta_i^T(-s) \\ I - \Delta_i\Delta_i^T(-s) \end{bmatrix}$. Then, we have $\psi_i^T\psi_i = I$, and it follows that $J_{12}^* = (1-\varepsilon_1-\varepsilon_2) \inf_{Q\in\mathbb{R}\mathcal{H}_\infty} \left\| \psi \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{-N_m}{(1-q)} \\ \sqrt{\frac{\varepsilon_2}{1-\varepsilon_1-\varepsilon_2}}\frac{N_m}{(1-q)} \\ \sqrt{\frac{\varepsilon_1}{1-\varepsilon_1-\varepsilon_2}}M_m \end{bmatrix} \right) Q \right\|_2^2 \sigma_1^2$.

By a simple calculation, we can get

$$J_{12}^* = (1-\varepsilon_1-\varepsilon_2) \inf_{Q\in\mathbb{R}\mathcal{H}_\infty} \left\| \Delta_i^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \Delta_0 Q \right\|_2^2 \sigma_1^2 + (1-\varepsilon_1-\varepsilon_2) \left\| \left(\mathrm{I} - \Delta_i\Delta_i^T\right) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\|_2^2 \sigma_1^2.$$

According to $Q \in \mathcal{R}H_\infty$, $\left\| \Delta_i^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \Delta_0 Q \right\|_2^2$ can be made arbitrarily small by properly choosing $Q \in \mathcal{R}H_\infty$. By an simple calculation, we have

$$J_{12}^* = (1-\varepsilon_1-\varepsilon_2) \left\| \begin{matrix} 1 - \frac{N_m\Delta_0^{-1}\Delta_0^{-H}N_m^H}{(1-q)^2} \\ \sqrt{\frac{\varepsilon_2}{1-\varepsilon_1-\varepsilon_2}}\frac{N_m\Delta_0^{-1}\Delta_0^{-H}N_m^H}{(1-q)^2} \\ \sqrt{\frac{\varepsilon_1}{1-\varepsilon_1-\varepsilon_2}}\frac{M_m\Delta_0^{-1}\Delta_0^{-H}N_m^H}{(1-q)} \end{matrix} \right\|_2^2 \sigma_1^2.$$

According to J_{11}^* and J_{12}^*, we can obtain J_1^*

$$J_1^* = (1-\varepsilon_1-\varepsilon_2)\sum_{i=1}^n 2\operatorname{Re}(z_i)\sigma_1^2 + (1-\varepsilon_1-\varepsilon_2) \left\| \begin{matrix} 1 - \frac{N_m\Delta_0^{-1}\Delta_0^{-H}N_m^H}{(1-q)^2} \\ \sqrt{\frac{\varepsilon_2}{1-\varepsilon_1-\varepsilon_2}}\frac{N_m\Delta_0^{-1}\Delta_0^{-H}N_m^H}{(1-q)^2} \\ \sqrt{\frac{\varepsilon_1}{1-\varepsilon_1-\varepsilon_2}}\frac{M_m\Delta_0^{-1}\Delta_0^{-H}N_m^H}{(1-q)} \end{matrix} \right\|_2^2 \sigma_1^2.$$

In order to calculate J_2^*, we consider following an inner-outer factorization $\begin{bmatrix} \sqrt{\varepsilon_1}M_m \\ \frac{\sqrt{1-\varepsilon_1}N_m}{(1-q)} \end{bmatrix}$ $= \Lambda_i\Lambda_o$ where Λ_i is an inner factor, and Λ_o is an outer factor.

Therefore, we have $J_2^* = \inf_{D\in\mathbb{R}\mathcal{H}_\infty} \|\Lambda_i\Lambda_o(Y-DM)\|_2^2\sigma_2^2$.

Because Λ_i is an inner factor, and B_p is the all pass factor, J_2^* can be rewriten $J_2^* = \inf_{D\in\mathbb{R}\mathcal{H}_\infty} \left\|\Lambda_o Y B_p^{-1} - \Lambda_o D M_m\right\|_2^2\sigma_2^2$. Because $\Lambda_o Y B_p^{-1} = \Gamma_1^\perp + \Gamma_1$, $\Gamma_1^\perp$ is in $\mathcal{H}_2^\perp$, Γ_1 is in $\mathcal{H}_2$, then $J_2^* = \inf_{D\in\mathbb{R}\mathcal{H}_\infty} \left\|\Gamma_1^\perp + \Gamma_1 - \Lambda_o D M_m\right\|_2^2\sigma_2^2$. Simultaneously, $\Gamma_1 - \Lambda_o D M_m$ is in $\mathcal{H}_2$, and it follows that $J_2^* = \left\|\Gamma_1^\perp\right\|_2^2\sigma_2^2 + \inf_{D\in\mathbb{R}\mathcal{H}_\infty}\|\Gamma_1 - \Lambda_o D M_m\|_2^2\sigma_2^2$.

We can select an appropriate Q, making $\inf_{D\in\mathbb{R}\mathcal{H}_\infty}\|\Gamma_1 - \Lambda_o D M_m\|_2^2\sigma_2^2 = 0$.

The $\mathbb{C}_+$ poles of $\Gamma^\perp$ are precisely the plant poles p_j. Therefore, we apply residue calculus to obtaining $\Gamma_1^\perp = \sum_{j=1}^m \left(\frac{\alpha_j}{s-p_j}\Lambda_o(p_j)Y(p_j)\right)$, where α_j is the residue of B_p^{-1} evaluated at $s = p_j$, so $\alpha_j = 2\operatorname{Re}(p_j)\prod_{k\neq j}\frac{p_j+\bar{p}_k}{p_j-p_k}$.

According to $MX - NY = 1$ and $M(p_j) = 0$, we can obtain $Y(p_j) = -N_m^{-1}(p_j)L_z^{-1}(p_j)$.

Define $\gamma_j = -2\,\mathrm{Re}(p_j)\Lambda_o(p_j)N_m^{-1}(p_j)L_z^{-1}(p_j)\prod\limits_{k\neq j}\frac{p_j+\bar{p}_k}{p_j-p_k}$, and it follows that

$$J_2^* = \sum_{j=1}^{m}\sum_{k=1}^{m}\left(\frac{\gamma_j\gamma_j^H}{p_j+\bar{p}_k}\right)\sigma_2^2.$$

According to J_1^* and J_2^*, we can obtain J^*

$$J^* = \sum_{j=1}^{m}\sum_{k=1}^{m}\left(\frac{\gamma_j\gamma_j^H}{p_j+\bar{p}_k}\right)\sigma_2^2 + A\sum_{i=1}^{n}2\,\mathrm{Re}(z_i)\sigma_1^2 - \varepsilon_2\Upsilon - \varepsilon_1\Gamma$$
$$+ A\left\|\begin{array}{c} 1-\frac{N_m\Delta_0^{-1}\Delta_0^{-H}N_m^H}{(1-q)^2} \\ \sqrt{\frac{\varepsilon_2}{1-\varepsilon_1-\varepsilon_2}}\frac{N_m\Delta_0^{-1}\Delta_0^{-H}N_m^H}{(1-q)^2} \\ \sqrt{\frac{\varepsilon_1}{1-\varepsilon_1-\varepsilon_2}}\frac{M_m\Delta_0^{-1}\Delta_0^{-H}N_m^H}{(1-q)} \end{array}\right\|_2^2\sigma_1^2.$$

where $A = (1-\varepsilon_1-\varepsilon_2)$, which completes the proof.

4. **Numerical Example.** Consider a linear time-invariant control system model described by $G(s) = \frac{s-k}{(s+1)(s-2)}$, where $k \in (1,4)$. This plant is non-minimum phase. The unstable pole is located at $p = 2$, for any $k > 0$, and it has a non-minimum phase zero at $z = k$. The simulation parameters are as follows: $N_m = \frac{s+k}{(s+2)(s+1)}$, $M_m = 1$, $q = 0.5$, $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.4$.

The multi-objective optimal performance about networked system with different non-minimum phase zeros is shown in Figure 2. The multi-objective optimal performance is obtained by applying two-parameter scheme. It can be observed from Figure 2, the packet dropout probability is higher, the performance will become worse. It can be also seen from Figure 2 that the multi-objective optimal performance has been degraded because of the packet dropouts probability of the communication channel in the feedback control system.

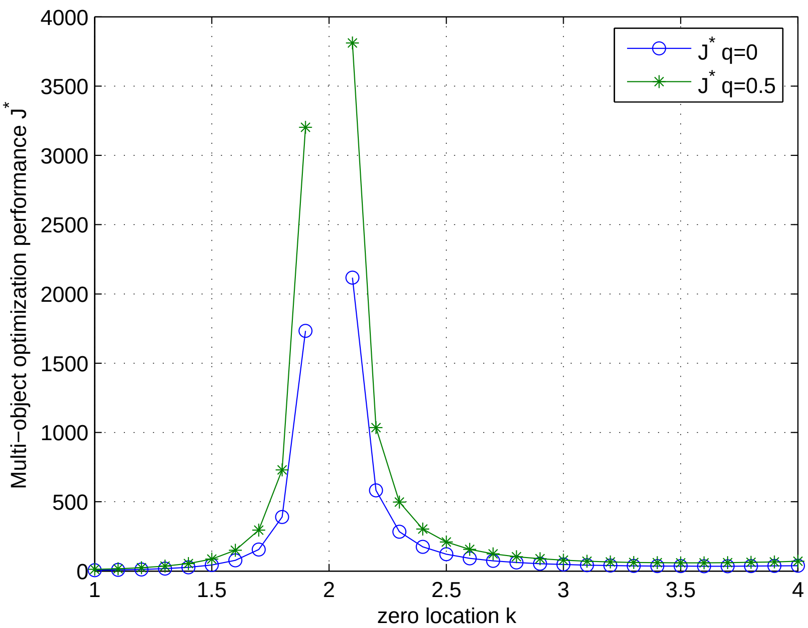

FIGURE 2. Networked system with channel noise and packet dropout

5. Conclusions. The problem of the multi-objective optimal performance with packet dropouts and channel noise under control input energy and channel input energy constraints has been studied in this paper. The network constraints under consideration are packet dropouts and channel noise. Explicit expression of the multi-objective optimal performance has been obtained by applying two parameter compensator and inner-outer factorization. It is shown that the optimal tracking performance depends on the non-minimum phase zeros and unstable poles of the given plant, as well as the reference input signal, and the packet dropouts probability and channel noise of the communication channel. The result shows how the packet dropouts probability and channel noise of communication channel may fundamentally constrain a control system's tracking capability. An example has been given to illustrate the obtained results.

Future extensions to this work will include nonlinear networked control systems, and more parameters of communication channel such as bandwidth, network-induced delay and quantization effect.

Acknowledgment. This work is partially supported by the Emphasis Foundation of Department of Education of Hubei province under Grant D20152502.

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