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IMPROVED CPHD FILTER BASED ON WEIGHT OPTIMIZATION

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Abstract. *Multitarget tracking (MTT) is a popular topic in various surveillance systems. To deal with imprecise estimations of the existing cardinalized probability hypothesis density (CPHD) filter, an improved filter is presented in this paper. First the limitations of the standard CPHD filter are discussed. Afterwards employing the novel lemmas on weight optimization, we design the proposed CPHD filter. Simulation results have been carried out to confirm the validity of the proposed CPHD filter.*
Keywords: Multitarget tracking, Weight optimization, Probability hypothesis density, Particle

1. **Introduction.** Multitarget tracking (MTT) is to jointly estimate the number of targets and their state from noise-corrupted measurements [1]. By modelling targets and measurements as the random finite set (RFS), the MTT is rigorously formulated in the Bayesian filtering framework. However, the Bayes recursion is computationally intractable owing to complex multi-integration [2]. To alleviate computational intractability, Mahler proposed the famous cardinalized probability hypothesis density (CPHD) recursion in 2006.

Recently many articles with respect to the CPHD filter have been reported in important academic periodicals. Based on the Gaussian mixture (GM) solution, a CPHD filter was developed for linear Gaussian assumption in [3]. Then a CPHD filter in [4] was presented to deal with nonlinear and non-Gaussian problem by the sequential Monte Carlo (SMC) method. In [5] an extension of the CPHD filter was presented to distinguish the survival and newborn targets. However, the mentioned methods have unstable target number estimations. Therefore, [6] depicted a new CPHD filter, where the time-updated cardinality distribution was exact and the estimated number of targets was rectified. Lately an improved GM implementation of the CPHD filter in [7] was proposed to minimize the effect of estimation errors. Nevertheless both [6] and [7] are still restricted to linear Gaussian assumption, which lack universality in actual applications to a certain extent.

In this paper, an improved CPHD filter for nonlinear and non-Gaussian system is presented. We propose two lemmas on weight optimization to overcome unstable number estimations by balancing the number of undetected targets and that of false alarms. The rest of this note is organized as follows. In Section 2, the standard CPHD filter is discussed. Section 3 presents the principle and SMC implementation of the proposed CPHD filter. In Section 4, the numerical simulation evaluates tracking performance of the proposed filter. Section 5 draws conclusions by providing the future work.

2. **The Standard CPHD Filter.** The SMC implementation of the CPHD filter is to propagate a particle set representing the posterior probability hypothesis density (PHD) and the posterior cardinality distribution. It contains four steps: initialization, time-update, measurement-update and state estimation.

Step 1. Initialization. Assume χ is the particle number per target, N_0 is the expected number of targets, $\delta(\cdot)$ denotes the Dirac Delta function, the initial PHD $D_0(x)$ and the initial cardinality distribution $\rho_0(n)$ are approximated by the particles $x_0^{(i)}$ and weights $w_0^{(i)} = \chi^{-1}$

$$D_0(x) = \sum_{i=1}^{L_0} w_0^{(i)} \delta\left(x - x_0^{(i)}\right) \tag{1}$$

$$\rho_0(n) = C_{L_0}^n \chi^{-n} \left(1 - \chi^{-1}\right)^{L_0 - n} \tag{2}$$

where $L_0 = \chi N_0$ is the total number of particles and $C_{L_0}^n$ is the binomial coefficient.

Step 2. Time update. The PHD $D_{k-1}(x)$ at scan $k-1$ can be approximated by the particle system $\left\{w_{k-1}^{(i)}, x_{k-1}^{(i)}\right\}_{i=1}^{L_{k-1}}$

$$D_{k-1}(x) = \sum_{i=1}^{L_{k-1}} w_{k-1}^{(i)} \delta\left(x - x_{k-1}^{(i)}\right) \tag{3}$$

where L_{k-1} is the number of particles, and the sum of weights $N_{k-1} = \sum_{i=1}^{L_{k-1}} w_{k-1}^{(i)}$ equals the expected number of targets.

The time-updated PHD $D_{k|k-1}(x)$ and its cardinality distribution $\rho_{k|k-1}(n)$ are

$$D_{k|k-1}(x) = \sum_{i=1}^{L_{k-1}} w_{S,k|k-1}^{(i)} \delta\left(x - x_{S,k|k-1}^{(i)}\right) + \sum_{i=1}^{L_{B,k}} w_{B,k}^{(i)} \delta\left(x - x_{B,k}^{(i)}\right) \tag{4}$$

$$\begin{aligned}&\rho_{k|k-1}(n)\\ &= \sum_{j=0}^{n} \rho_{B,k}(n-i) \sum_{l=i}^{\infty} C_i^l \left\langle p_{S,k}^{(1:L_{k-1})}, w_{k-1}\right\rangle^i \left\langle 1 - p_{S,k}^{(1:L_{k-1})}, w_{k-1}\right\rangle^{l-i} \rho_{k-1}(l) / \langle 1, w_{k-1}\rangle^i\end{aligned} \tag{5}$$

where $\rho_{B,k}(\cdot)$ is the cardinality distribution of newborn targets, $L_{B,k}$ is the number of newborn-target particles, $\langle\cdot,\cdot\rangle$ denotes the inner product. Assume $p_{S,k}\left(x_{k-1}^{(1)}\right)$ is the probability of survival targets, $[\cdot]^T$ denotes the transposition matrix, and we have

$$p_{S,k}^{(1:L_{k-1})} = \left[p_{S,k}\left(x_{k-1}^{(1)}\right), \cdots, p_{S,k}\left(x_{k-1}^{(L_{k-1})}\right)\right]^T \tag{6}$$

$$w_{k-1} = \left[w_{k-1}^{(1)}, \cdots, w_{k-1}^{(L_{k-1})}\right]^T \tag{7}$$

Let $q_k(\cdot|x_{k-1}, Z_k)$ and $b_k\left(x_{B,k}^{(i)}\middle|, Z_k\right)$ be the proposal density of survival targets and newborn targets; the survival-target particles and newborn-target particles in (4) are

$$x_{S,k|k-1}^{(i)} \sim q_k\left(\cdot\middle|x_{k-1}^{(i)}, Z_k\right), \quad i = 1, \cdots, L_{k-1} \tag{8}$$

$$x_{B,k}^{(i)} \sim b_k(\cdot|, Z_k), \quad i = 1, \cdots, L_{B,k} \tag{9}$$

Considering the Markov transition probability of survival targets $f_{k|k-1}(x|\cdot)$ and the PHD of newborn targets $\gamma_k(\cdot)$, we have the time-updated weights

$$w_{k|k-1}^{(i)} = \begin{cases} w_{S,k|k-1}^{(i)} = p_{S,k}\left(x_{k-1}^{(i)}\right) f_{k|k-1}\left(x_{S,k|k-1}^{(i)}\middle|x_{k-1}^{(i)}\right) w_{k-1}^{(i)} \Big/ q_k\left(x_{S,k|k-1}^{(i)}\middle|x_{k-1}^{(i)}, Z_k\right), & i=1,\cdots,L_{k-1} \\ w_{B,k}^{(i)} = \gamma_k\left(x_{B,k}^{(i)}\right) \Big/ L_{B,k} b_k\left(x_{B,k}^{(i)}\middle|, Z_k\right), & i=1,\cdots,L_{B,k} \end{cases} \tag{10}$$

where $w_{S,k|k-1}^{(i)}$ and $w_{B,k}^{(i)}$ are the weights for survival particles and newborn particles.

Step 3. Measurement update. The measurement-updated PHD $D_k(x)$ and its cardinality distribution $\rho_k(n)$ at scan k are

$$D_k(x) = \sum_{i=1}^{L_{k-1}+L_{B,k}} w_k^{(i)} \delta\left(x - x_k^{(i)}\right) \tag{11}$$

$$\rho_k(n) = \Upsilon_k^0\left[w_{k|k-1}; Z_k\right](n)\rho_{k|k-1}(n) \Big/ \left\langle \Upsilon_k^0\left[w_{k|k-1}; Z_k\right](n), \rho_{k|k-1}(n)\right\rangle \tag{12}$$

Subsequently the measurement-updated equation for weights $w_k^{(i)}$ can be written as

$$\begin{aligned} w_k^{(i)} = & \underbrace{\left(1 - p_{D,k}\left(x_k^{(i)}\right)\right) \frac{\left\langle \Upsilon_k^1\left[w_{k|k-1}; Z_k\right](n), \rho_{k|k-1}(n)\right\rangle}{\left\langle \Upsilon_k^0\left[w_{k|k-1}; Z_k\right](n), \rho_{k|k-1}(n)\right\rangle} w_{k|k-1}^{(i)}}_{\text{undetected component}} \\ & + \underbrace{\sum_{z \in Z_k} g_k\left(z|x_k^{(i)}\right) p_{D,k}\left(x_k^{(i)}\right) \frac{\langle 1, C_k \rangle}{C_k(z)} \frac{\left\langle \Upsilon_k^1\left[w_{k|k-1}; Z_k - \{z\}\right](n), \rho_{k|k-1}(n)\right\rangle}{\left\langle \Upsilon_k^0\left[w_{k|k-1}; Z_k\right](n), \rho_{k|k-1}(n)\right\rangle} w_{k|k-1}^{(i)}}_{\text{detected component}} \end{aligned} \tag{13}$$

where $C_k(\cdot)$ is the PHD of clutters, $p_{D,k}(\cdot)$ is the detection probability of sensors. Define the cardinality distribution of clutters $p_{C,k}(\cdot)$ and the elementary symmetric function $e_j(\cdot)$; the related parameters in (12) and (13) are given by

$$\begin{aligned} & \Upsilon_k^n\left[w_{k|k-1}; Z_k\right](n) \\ & = \sum_{i=0}^{\min(|Z_k|,n)} \left(\begin{array}{l} (|Z_k| - i)! p_{C,k}(|Z_k| - i) P_{j+u}^n \\ \cdot \left\langle 1 - p_{D,k}^{(1:L_{k-1})}, w_{k|k-1} \right\rangle^{n-i-u} e_i\left(\Lambda_k\left(w_{k|k-1}, Z_k\right)\right) \end{array} \right) \Bigg/ \left\langle 1, w_{k|k-1} \right\rangle^n \end{aligned} \tag{14}$$

$$\begin{aligned} & \Lambda_k\left(w_{k|k-1}, Z_k\right) \\ & = \left\{ \left\langle w_{k|k-1}, \langle 1, C_k \rangle \left[g_k\left(z|x_k^{(1)}\right) p_{D,k} x_k^{(1)}, \cdots, g_k\left(z|x_k^{(L_{k|k-1})}\right) p_{D,k} x_k^{(L_{k|k-1})} \right]^T \Big/ C_k(z) \right\rangle : z \in Z_k \right\} \end{aligned} \tag{15}$$

$$p_{D,k}^{(1:L_{k-1})} = \left[p_{D,k}\left(x_{k-1}^{(1)}\right), \cdots, p_{D,k}\left(x_{k-1}^{(L_{k-1})}\right) \right]^T \tag{16}$$

$$w_{k|k-1} = \left[w_{k|k-1}^{(1)}, \cdots, w_{k|k-1}^{(L_{k-1})} \right]^T \tag{17}$$

Then the expected number of targets is

$$\tilde{N}_k = \sum_{i=1}^{L_{k-1}+L_{B,k}} w_k^{(i)} \tag{18}$$

We resample $L_k = \chi \tilde{N}_k$ particles from $\left\{ w_k^{(i)}, x_k^{(i)} \right\}_{i=1}^{L_{k-1}+L_{B,k}}$, and obtain $\left\{ w_k^{(i)}, x_k^{(i)} \right\}_{i=1}^{L_k}$.

Step 4. State estimation. We estimate the number of targets using maximum a posteriori method

$$\hat{N}_k = \arg\max{}_n\, \rho_k(n) \tag{19}$$

where $\hat{N}_k$ means the selected posterior PHDs $m_k^{(i_1)}, \cdots, m_k^{\left(i_{\hat{N}_k}\right)}$ corresponding to the highest weights $w_k^{(i_1)}, \cdots, w_k^{\left(i_{\hat{N}_k}\right)}$. Then the estimated state of targets can be written as

$$\hat{X}_k = \left\{ m_k^{(i_1)}, \cdots, m_k^{\left(i_{\hat{N}_k}\right)} \right\} \tag{20}$$

Remark 2.1. *The random false alarms are approximated by newborn particles. Once a clutter appears around the true track, the target will be interfered. All newborn particles for newborn targets and false alarms would lead to overestimated number. In general, there is* $1 - p_{D,k}\left(x_{k|k-1}^{(i)}\right) > 0$ *when* $p_{D,k}\left(x_{k|k-1}^{(i)}\right) < 1$*. The PHD would be discarded if the sensor cannot collect target-originated measurement and only the undetected component is in (13). The detected component does not concentrate on the true target but moves proportionally to other targets. If a clutter-originated measurement near the track is similar to the value of newborn particles, the increasing weight would lead to unreliable number estimates. Some false alarms are mistaken for true targets when the variance of clutters is small. Although the occurrence probability is small, the standard CPHD filter still employs many newborn particles to look for targets, which causes inaccurate number estimates.*

3. The Proposed CPHD Filter.

To pave the way towards the proposed CPHD filter, we present two lemmas on weights optimization.

Lemma 3.1. *Assume* μ *is the weight threshold, the optimized weights* $w_{\mathrm{m},k}^{(i)}$ *are*

$$w_{\mathrm{m},k}^{(i)} = \begin{cases} w_{S',k}^{(i)} = \left(1 + \sum\limits_{i=1}^{L_{B,k}} \left(w_{B,k}^{(i)} - w_{B',k}^{(i)}\right) \Big/ \sum\limits_{i=1}^{L_{k-1}} w_{S,k}^{(i)}\right) w_{S,k}^{(i)}, & i = 1, \cdots, L_{k-1} \\ w_{B',k}^{(i)} = \begin{cases} \mu, & w_{B,k}^{(i)} \geq \mu \\ w_{B,k}^{(i)}, & w_{B,k}^{(i)} < \mu \end{cases}, & i = 1, \cdots, L_{B,k} \end{cases} \tag{21}$$

where $w_{S',k}^{(i)}$ *and* $w_{B',k}^{(i)}$ *are optimized weights for survival particles and newborn particles.*

Proof: We assign excess weights to survival particles when $w_{B,k}^{(i)} \geq \mu$. Then the sum of $w_{\mathrm{m},k}^{(i)}$ is

$$\begin{aligned} \sum_{i=1}^{L_{k-1}+L_{B,k}} w_{\mathrm{m},k}^{(i)} &= \sum_{i=1}^{L_{k-1}} w_{S',k}^{(i)} + \sum_{i=1}^{L_{B,k}} w_{B',k}^{(i)} \\ &= \sum_{i=1}^{L_{k-1}} w_{S,k}^{(i)} + \sum_{i=1}^{L_{k-1}} \left(\sum_{i=1}^{L_{B,k}} \left(w_{B,k}^{(i)} - w_{B',k}^{(i)}\right) \Big/ \sum_{i=1}^{L_{k-1}} w_{S,k}^{(i)} \right) w_{S,k}^{(i)} + \sum_{i=1}^{L_{B,k}} w_{B',k}^{(i)} \\ &= \sum_{i=1}^{L_{k-1}} w_{S,k}^{(i)} + \sum_{i=1}^{L_{B,k}} w_{B,k}^{(i)} = \sum_{i=1}^{L_{k-1}+L_{B,k}} w_k^{(i)} \end{aligned} \tag{22}$$

In (22), we note that the sum of weights keeps unchanged after weight optimization.

Lemma 3.2. *If* i' *survival particles have optimized weights* $w_{S',k}^{(i')} \geq 1$ *and* i'' *survival particles have optimized weights* $w_{S',k}^{(i'')} < 1$*, the re-optimized weights of survival particles are*

$$w_{S'',k}^{(i)} = \begin{cases} 1, & w_{S',k}^{(i')} \geq 1 \\ \left(1 + \sum\limits_{i'} \left(w_{S',k}^{(i')} - 1\right) \Big/ \sum\limits_{i''} w_{S',k}^{(i'')}\right) w_{S',k}^{(i'')}, & w_{S',k}^{(i'')} < 1 \end{cases} \tag{23}$$

Proof: We re-assign excess weights to other survival particles when $w_{S',k}^{(i')} \geq 1$. Then the sum of $w_{S'',k}^{(i)}$ is

$$\sum_{i=1}^{L_{k-1}} w_{S'',k}^{(i)} = \sum_{i'} 1 + \sum_{i''} \left(1 + \sum_{i'} \left(w_{S',k}^{(i')} - 1\right) \Big/ \sum_{i''} w_{S',k}^{(i'')}\right) w_{S',k}^{(i'')}$$

$$= \sum_{i'} 1 + \sum_{i''} w_{S',k}^{(i'')} + \sum_{i'} \left(w_{S',k}^{(i')-1} \right) \tag{24}$$

$$= \sum_{i'} w_{S',k}^{(i')} + \sum_{i''} w_{S',k}^{(i'')} = \sum_{i=1}^{L_{k-1}} w_{S',k}^{(i)}$$

In (24), we also note that the sum of weights keeps unchanged after weight re-optimization.

Let round$(\cdot)$ denote integer approximation, and we rewrite (18) as

$$\tilde{N}_k = \text{round}\left(\sum_{i=1}^{L_{k-1}} w_{S'',k}^{(i)} \right) + \text{round}\left(\sum_{i=1}^{L_{B,k}} w_{B',k}^{(i)} \right) \tag{25}$$

Subsequently the process of the proposed CPHD filter is summarized as follows:

Initialization: At scan 0, the initialization operation is

For $i = 1, \cdots, N_0$

-Generate particle set $\left\{ w_0^{(i)}, x_0^{(i)} \right\}_{i=1}^{L_0}$, compute $D_0(x)$ and $\rho_0(n)$ using (1) and (2);

Iteration: At scan k, there are time-update, measurement-update and state estimation.

For $k = 1, 2, \cdots$

For $i = 1, \cdots, \tilde{N}_k$

-Generate particle set $\left\{ w_{k-1}^{(i)}, x_{k-1}^{(i)} \right\}_{i=1}^{L_{k-1}}$, compute $x_{S,k|k-1}^{(i)}$ and $x_{B,k}^{(i)}$ using (8) and (9);

-Compute $w_{k|k-1}^{(i)}$, $D_{k|k-1}(x)$ and $\rho_{k|k-1}(n)$ using (10), (4) and (5);

-Compute $w_k^{(i)}$ and $w_{\mathrm{m},k}^{(i)}$ using (13) and (21), compute $w_{S'',k}^{(i)}$ using (23) if necessary;

-Compute $D_k(x)$, $\rho_k(n)$ and $\tilde{N}_k$ using (11), (12) and (25);

-Resample L_k particles from set $\left\{ w_k^{(i)}, x_k^{(i)} \right\}_{i=1}^{L_{k-1}+L_{B,k}}$ to get a new set $\left\{ w_k^{(i)}, x_k^{(i)} \right\}_{i=1}^{L_k}$;

-Estimate $\hat{N}_k$ and $\hat{X}_k$ using (19) and (20).

4. **Experimental Results and Discussions.** The numerical study is presented to evaluate the proposed CPHD filter. During the surveillance period of 60 scans, the targets move with the constant velocity (CV) motion and the constant turn (CT) motion. In the scenario, 100 Monte Carlo runs are done to obtain simulation results, where the sampling period is 1s.

Figure 1 shows the target tracks and measurements. We note that four targets move in cluttered environment: Target 1 moves with velocity of $(-20, -5)$m/s and turn of $0.25°$/s

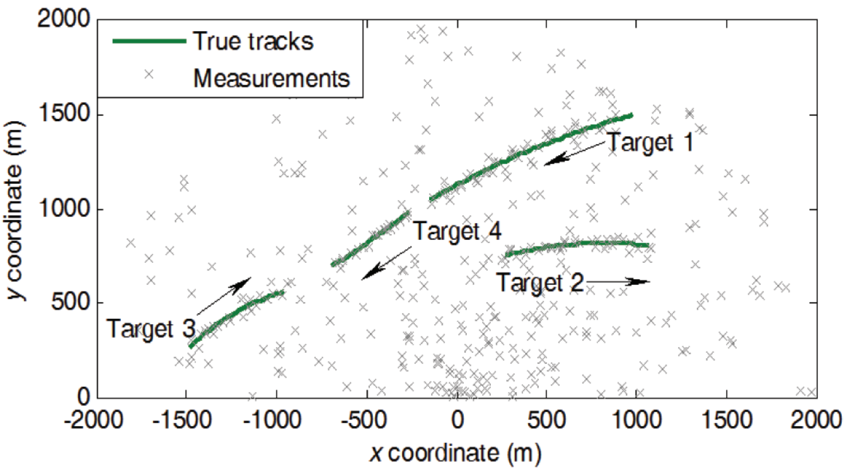

Figure 1. Target tracks and measurements

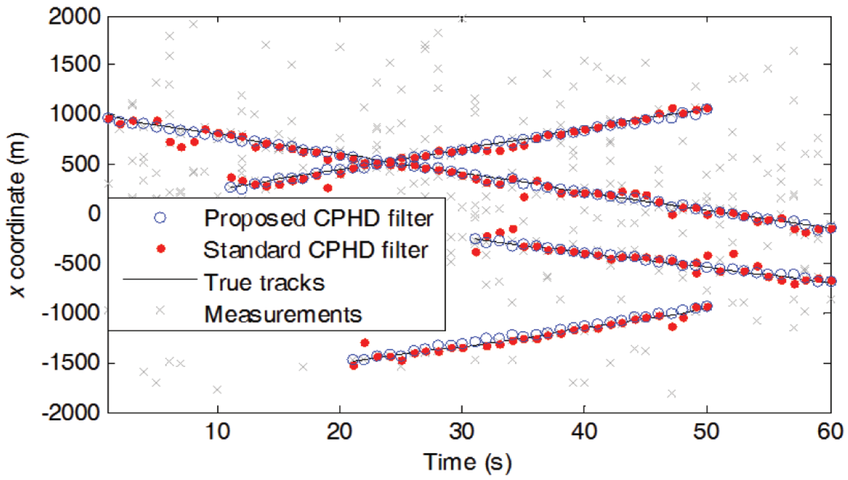

FIGURE 2. Target tracks in x coordinate

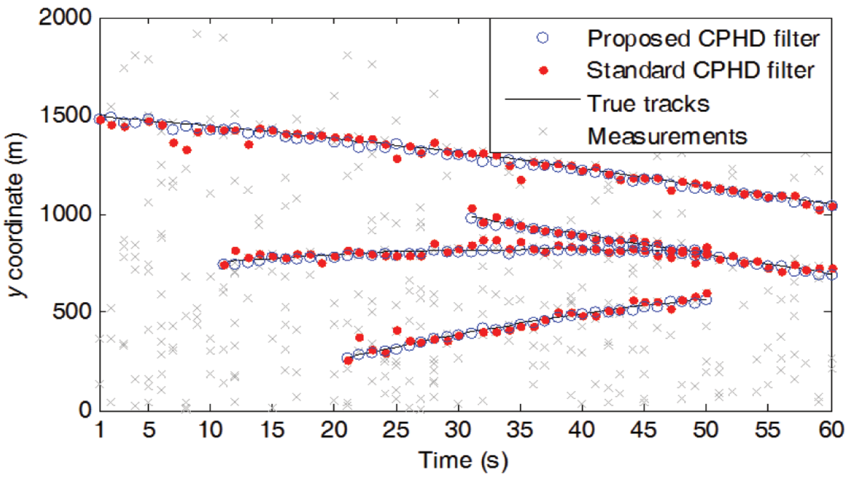

FIGURE 3. Target tracks in y coordinate

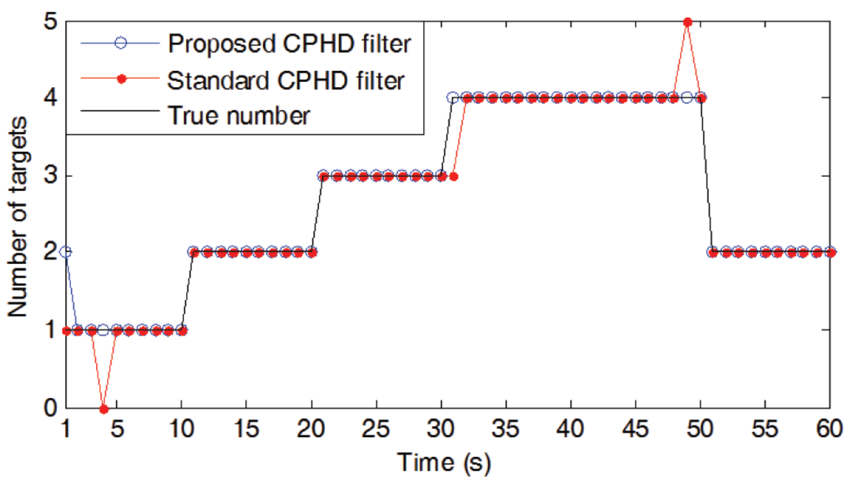

FIGURE 4. Target number estimates

from position $(1000, 1500)$m during 1∼60s. Target 2 travels from position $(250, 750)$m with velocity of $(20, 5)$m/s and turn of $-0.5°$/s during 11∼50s. Target 3 moves with velocity of $(15, 15)$m/s and turn of $-1°$/s from position $(-1500, 250)$m during 21∼50s. Target 4 keeps CV motion with velocity of $(-15, -10)$m/s from position $(-250, 1000)$m during

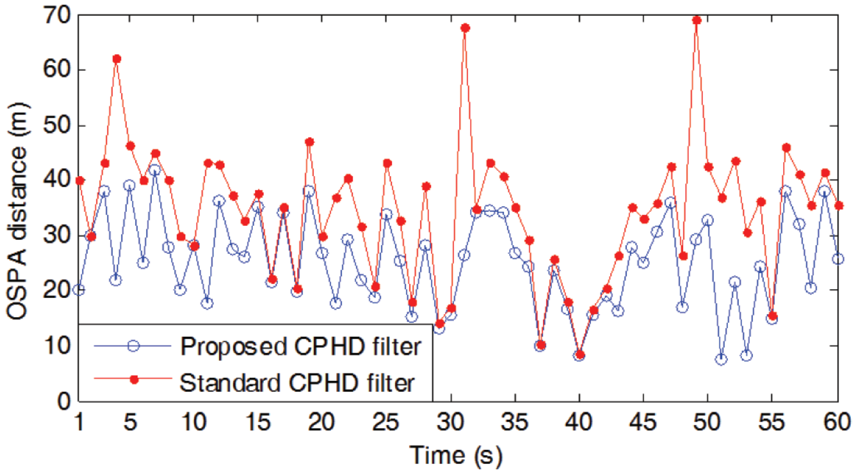

FIGURE 5. OSPA distance

31∼60s. Figures 2 and 3 demonstrate the tracks in x and y coordinates. As seen, the estimated positions of the proposed CPHD filter are near the true tracks. By comparison, the standard filter has position deviation. Figure 4 shows the estimated number of targets. We note that the standard filter has unstable number estimates. It over-estimates one target at scan 49 because of the effect from close clutters. Additionally it misses one target at scans 4 and 31 respectively due to the imperfection of sensors. On the contrary, the estimated number of targets using the proposed filter during the surveillance period coincides with the ground truth. The reason is that the excessive particle weights of newborn targets are automatically assigned to survival targets with certain percentage when the newborn targets appear. Then the survival-target particles mainly concentrate on the true target. Figure 5 plots the optimal sub pattern assignment (OSPA) distance against time. It can be observed that the tracking performance of the standard filter is worse because it exaggerates the biased position. It has three intensity peaks as a result of the miscalculated number estimates. And what is more, the maximum value of the OSPA distance 69.43m is obtained at scan 49. In contrast, it can be verified that the proposed filter has the promising performance of track continuity whether the target motion is maneuvering or not.

5. **Conclusions.** This paper introduces an improved CPHD filter for the MTT. The work proposes two lemmas on weight optimization for correcting estimated number of targets. The numerical study suggests that the proposed filter has a significant improvement in tracking performance over the standard filter. As future developments of this work, we plan to adaptively compute the weight threshold for various applications.

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