ICIC Express Letters
Volume **10**, Number **5**, May **2016**

pp. 1001–1007

DESIGN OF CORNER-CUBE RETROREFLECTOR ARRAYS FOR LASER RANGING USING GENETIC ALGORITHM OPTIMIZATION

Chengyi Li[1], Chunhui Wang[2] and Zhonghe Jin[2]

[1]Academy of Information Science and Electronics Engineering
[2]School of Aeronautics and Astronautics
Zhejiang University
No. 866, Yuhangtang Road, Hangzhou 310058, P. R. China
lichengyi.tanran@163.com

Received November 2015; accepted February 2016

Abstract. *Corner-cube retroreflector (CCR) arrays with both enough effective reflection area and small installation area are important for microsatellites, which have limited sizes. However, the design of such arrays is challenging due to the inherently large number of degrees of freedom involved. To solve this problem, in this work, a genetic algorithm (GA) optimization is applied to the design of CCR arrays for laser ranging for the first time. This algorithm compares a desired effective reflection area with that of trial CCR arrays, and decides whether to introduce a penalty factor to the fitness function or not. The GA uses the information of fitness function to rank and select trial arrays, and creates new arrays at each subsequent generation via GA operators, i.e., selection, crossover, and mutation. Finally, an array of 49 CCRs is presented, and results of both GA optimization and traditional try and error method are shown to illustrate the efficiency of the algorithm. The GA optimization saves 15.8% installation area and spends only 1/288 working time compared with that optimized by traditional try and error method.*
Keywords: CCR arrays, Microsatellites, Laser ranging, Genetic algorithm (GA), Optimization

1. **Introduction.** More and more systems employing laser ranging for precise orbit determination, earth orientation and rotation parameters have been proposed since late 1964 [1-5]. This is due to its higher range accuracy comparing with microwave techniques [6]. CCR arrays are essential for laser ranging [7,8]. During the operation, CCR arrays provide reflected rays directly back toward the source when illuminated with a collimated beam of laser from the ground stations. The received reflection lasers can be used to determine the satellite range. The accuracy of laser ranging is mainly associated with the effective reflecting area afforded by CCR arrays [9].

The traditional method to design CCR arrays is trail and error [7,8]. Traditional satellites usually have installation areas bigger than several square meters. They have enough areas to install CCR arrays [3,4,7]. However, with the development of microsatellites, the design of CCR arrays becomes challenging. Since microsatellites usually have installation areas about only several percent square meters, to design CCR arrays with as small as possible installation areas is necessary [10]. However, it becomes a hard mission by the traditional try and error method when complex coverage areas are investigated and a large number of degrees of freedom are involved. Therefore, more advanced design and optimization techniques are needed.

GA is efficient for the problem. GA is a technique based on the biological laws of Darwinian evolution [11]. GA has been widely used in electromagnetics due to its ability to optimize in complex multimodal search spaces [11-14]. In this work, GA is employed to optimize the CCR arrays for the first time. It is capable to converge to the desired solution in a quite large search space of about 8×10^6 possible solutions in this paper.

2. **Model for CCR Arrays.** The general expression of the reflecting area of a single CCR can be expressed as [9]

$$\eta = \frac{2}{\pi}\left[\arcsin\left(\sqrt{1-2\tan^2 i}\right) - \sqrt{2}\tan i\sqrt{1-2\tan^2 i}\right]\cos i_0, \tag{1}$$

where η denotes the relative effective reflecting area normalized by the aperture area of a single CCR, and i_0 is the input angle of the laser on CCR. And $i = \arcsin(\sin i_0/n)$, where n denotes the index of refraction relative to air of the CCR's glass. The input angle of laser refers to the angle between laser and CCR's normal. It is obvious that η is associated with i_0.

Here we take the Cartesian coordinate system fixed on satellite as the reference coordinate system. As shown in Figure 1, $\mathrm{O}(0,0,0)$ is the coordinate origin, and $\mathrm{OX}(1,0,0)$, $\mathrm{OY}(0,1,0)$, $\mathrm{OZ}(0,0,1)$ are unit orientation vectors of XYZ coordinate axes. The origin is the center of the satellite. The X-axis directs toward the satellite's velocity vector. The Z-axis originates from the satellite center and directs toward the earth's center. The Y-axis explements the right-handed coordinate system. Vector ON is the unit orientation vector of installed CCR, which makes an angle α to Z, and an angle β between its project on XY plane and X. Vector OP is the unit vector points toward the instantaneous input laser, which makes an angle θ to Z, and an angle φ between its project on XY plane and X.

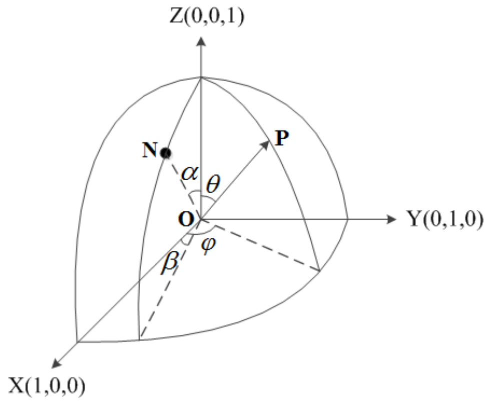


FIGURE 1. Coordinate system fixed on satellite

To take full advantage of a CCR array, it generally consists of several well designed CCRs, with different installation angles and different aperture areas. The effective reflecting area afforded by a CCR array is given by

$$A(\theta,\varphi) = \sum_{n=1}^{N} S_n \eta_n(\alpha,\beta,\theta,\varphi), \tag{2}$$

where $A(\theta,\varphi)$ is the total effective reflecting area afforded by a CCR array on the sample point (θ,φ), S_n is the aperture area of the nth CCR, η_n $(\alpha,\beta,\theta,\varphi)$ is the relative effective reflecting area of the nth CCR with the installation angle (α,β).

3. **GA Optimization of CCR Arrays.** Since parameters of this problem are continuous, we choose to use a real encoded chromosome in order to avoid the quantization errors

of the binary encoding. The population is given by (3)

$$X^n = \begin{bmatrix} X_{1,\alpha}^{n,1} & X_{1,\beta}^{n,1} & X_{1,S}^{n,1} & X_{2,\alpha}^{n,1} & X_{2,\beta}^{n,1} & X_{2,S}^{n,1} & \cdots & X_{var,\alpha}^{n,1} & X_{var,\beta}^{n,1} & X_{var,S}^{n,1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_{1,\alpha}^{n,pop} & X_{1,\beta}^{n,pop} & X_{1,S}^{n,pop} & X_{2,\alpha}^{n,pop} & X_{2,\beta}^{n,pop} & X_{2,S}^{n,pop} & \cdots & X_{var,\alpha}^{n,pop} & X_{var,\beta}^{n,pop} & X_{var,S}^{n,pop} \end{bmatrix} \tag{3}$$

where each line represents an individual at generation n, pop is the population size and var is the number of CCRs composing the CCR array. The number of optimization variables is up to treble var. Each line in X^n is a chromosome with treble var genes, which represents the installation angles and aperture areas for CCRs. For example, $X_{1,\alpha}^{1,1}$ is the α angle of the 1st individual's 1st CCR at generation 1, $X_{1,\beta}^{1,1}$ is the β angle of the 1st individual's 1st CCR at generation 1, $X_{1,S}^{1,1}$ is the aperture area of the 1st individual's 1st CCR at generation 1.

The GA must simultaneously optimize the total effective reflecting area and the installation area to synthesize a desired CCR array. The goal of the total effective reflecting area optimization is to provide adequate laser returning to ground stations. While, the goal of the installation area optimization is to arrive at the minimum value. Therefore, the GA can take the installation area as its fitness function, given by

$$f = \sum_{k=1}^{var} S_k. \tag{4}$$

In order to provide adequate laser return to earth-based ground stations, the GA must comply with a constraint, given by

$$A(\theta, \varphi) \geq threshold(\theta, \varphi). \tag{5}$$

threshold (θ, φ) is the minimum required total effective reflecting area for each sample point (θ, φ). Whether the sample point (θ, φ) satisfies the constraint is described as $e(\theta, \varphi)$

$$e(\theta, \varphi) = \begin{cases} 1, & A(\theta, \varphi) \geq threshold(\theta, \varphi) \\ 0, & A(\theta, \varphi) < threshold(\theta, \varphi) \end{cases}. \tag{6}$$

We introduce a penalty factor to the fitness function in (4) as constraint

$$f = \sum_{k=1}^{var} S_k + S_{\max} \iint_D e(\theta, \varphi) d\sigma, \quad (\theta, \varphi) \in D \tag{7}$$

where $S_{\max}$ is the maximum installation area among all the possible installation patterns. D is the required coverage area.

Since each inadequate $A(\theta, \varphi)$ causes a serious deterioration to the fitness function in (7), the GA tends to favor the constraint. Figure 2 shows the flowchart of the GA optimization. The specific steps are as follows.

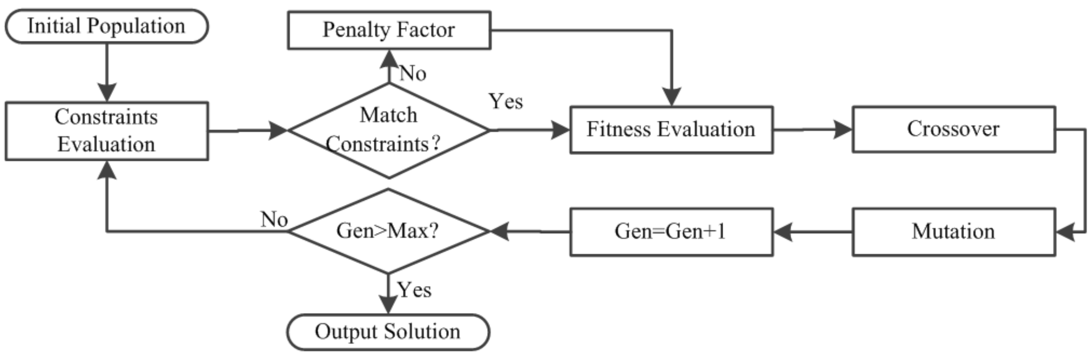


Figure 2. Flowchart of the GA optimization in this study

Step 1. Initial population. A certain number of real encoded chromosomes mentioned in expression (3) are used to generate initial population.

Step 2. Constraint evaluation. The constraint function as expression (5) is used to evaluate whether the solutions satisfy the constraint. The satisfied solutions go to step 3 directly, and the unsatisfied solutions go to step 3 with a penalty factor.

Step 3. Fitness evaluation. Fitness function as expression (7) is used to evaluate the fitness of the solutions. And the fitness of the solutions determines whether they can survive and create new populations.

Step 4. Crossover and mutation. Crossover function and mutation function are used to combine the survival chromosomes and generate new populations.

Step 5. Output. Output the solutions when an enough optimization generation is reached.

4. **Results.** In this study, a 7×7 CCR array is optimized with both the traditional try and error method and GA by the same professional designer. Five types of CCRs are used, e.g., type0 = 0cm^2, type1 = 3cm^2, type2 = 7cm^2, type3 = 11cm^2, type4 = 16cm^2, $n = 1.4$. The coverage area in this study is given by (8)

$$\begin{cases} 0^\circ \leq \theta \leq 45^\circ \\ 0^\circ \leq \varphi \leq 90^\circ \\ A(\theta,\varphi) \geq 160\text{cm}^2 \end{cases} \quad \begin{cases} 0^\circ \leq \theta \leq 30^\circ \\ 90^\circ \leq \varphi \leq 180^\circ \\ A(\theta,\varphi) \geq 80\text{cm}^2 \end{cases} . \tag{8}$$

Figure 3 shows the coverage area of the CCR array optimized with traditional try and error method. Figure 4 is a 3-D surface plot of the coverage area.

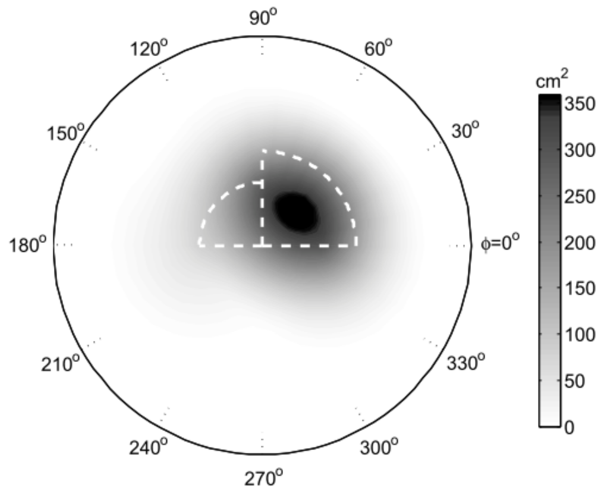


Figure 3. Coverage area of the CCR array optimized with traditional method

Although the result provides enough effective reflecting area for the required coverage area, it spends the professional designer 24 working hours to reach. The efficiency is very low. Moreover, a large portion of the effective reflecting area is wasted since it is not the requirement of constraint, which is shown in Figure 3 as dark gray and in Figure 4 as high peak.

For the GA optimization, the following parameters are used: population size = 1500, number of generations = 200, GA encoding = real, fitness scaling function = rank, replacement percentage = 0.1, selection function = Stochastic uniform, crossover function = scattered, crossover probability = 0.8, mutation rate = 0.2.

Figure 5 shows the coverage area of the CCR array optimized with GA. Figure 6 is a 3-D surface plot of the coverage area. Compared with Figure 3 and Figure 4, less dark gray and lower peak are contained in the result, which demonstrates the GA optimization is more efficient to distribute the effective reflecting area than the traditional try and

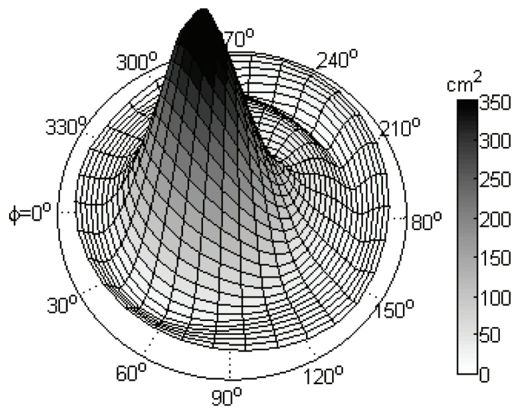


Figure 4. Surface (3-D) plot of the coverage area shown in Figure 3

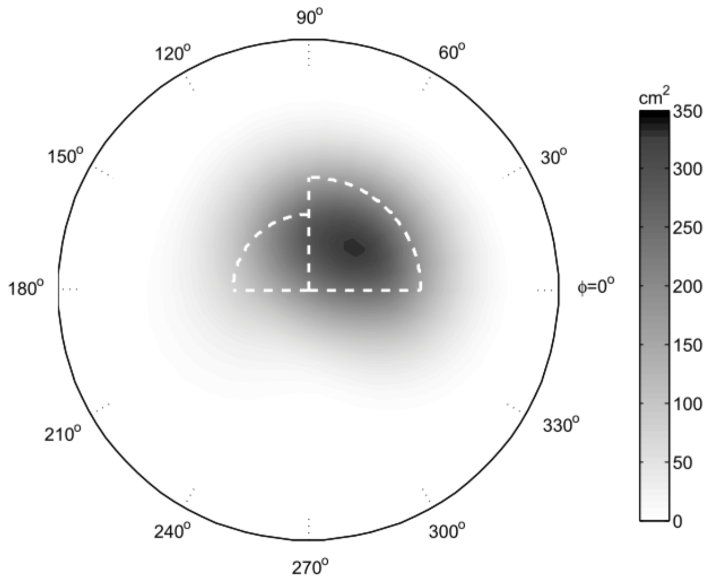


Figure 5. Coverage area of the CCR array optimized with GA

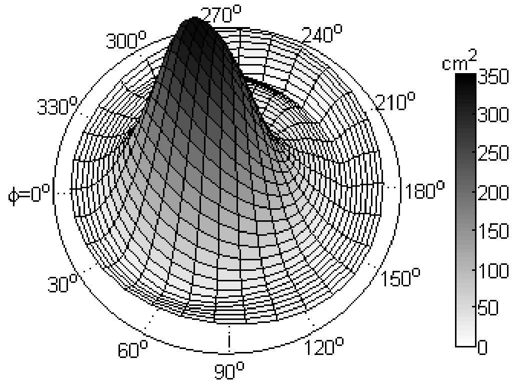
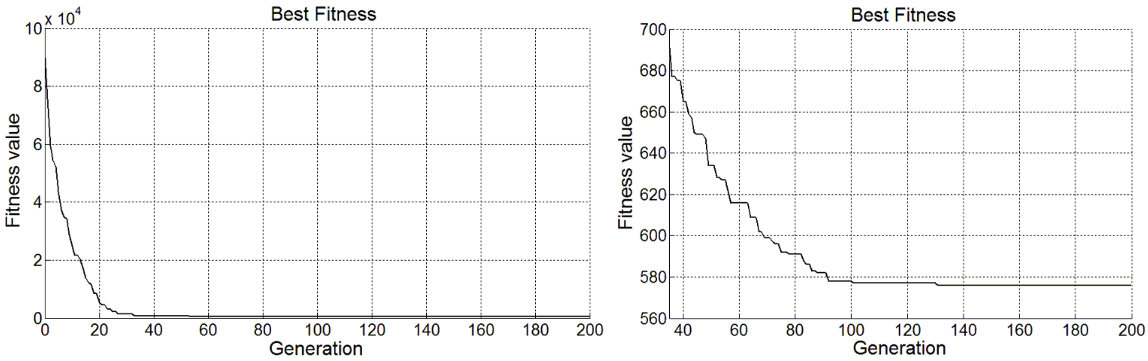


Figure 6. Surface (3-D) plot of the coverage area shown in Figure 5

error method. The result spends the same designer only 5 minutes to set parameters and computer completes the design automatically in 2 hours.

The convergence behavior of GA optimization is shown in Figure 7. Figure 7(a) shows the best fitness of generation 0-200. It begins from a quite large value, and represents the constraint has not been satisfied. Figure 7(b) shows the best fitness of generation 35-200.



(a) Generation 0-200 (b) Generation 35-200

FIGURE 7. Convergence of the GA optimization

It converges below 784 (49×16), represents the constraint has been satisfied in just 35 generations optimization, and demonstrates the efficiency of the penalty factor.

Table 1 shows the efficiencies of the two optimization methods in this paper.

TABLE 1. Efficiencies of two optimization methods

Optimization method	Total area/cm^2	Area advantage	Spent working time
Traditional method	667	0	24 hours
GA optimization	576	15.8%	5 minutes

The GA optimization saves 15.8% installation area and spends only 1/288 working time compared with traditional try and error method.

5. **Conclusion.** In this paper, we present a technique of designing CCR arrays for laser ranging using GA optimization. The technique takes the installation area modified by a penalty factor as the fitness function for the GA. The penalty factor is defined by the constraint, which requires the CCR array to provide adequate laser return to ground stations. Considering the continuous nature of the parameters for this optimization problem, the GA chromosome employs a real parameter encoding scheme in order to avoid the quantization errors of the binary encoding. An array of 49 CCRs is presented, and results of both GA optimization and traditional try and error method are shown to illustrate the efficiency of the algorithm. Using the GA optimization proposed in this paper to design other arrays besides CCR array is the future direction of research.

REFERENCES

[1] D. Kucharski, G. Kirchner, F. Koidl et al., Attitude and spin period of space debris envisat measured by satellite laser ranging, *IEEE Trans. Geoscience and Remote Sensing*, vol.52, no.12, pp.7651-7657, 2014.

[2] D. Kucharski, H. C. Lim, G. Kirchner, T. Otsubo, G. Bianco and J. Y. Hwang, Spin axis precession of LARES measured by satellite laser ranging, *IEEE Geoscience and Remote Sensing Letters*, vol.11, no.3, pp.646-650, 2014.

[3] S. Moss and T. S. Johnson, Performance of the Nasa laser ranging system in satellite tracking, *IEEE Trans. Geoscience Electronics*, vol.9, no.1, pp.1-9, 1971.

[4] J. J. Degnan, Satellite laser ranging: Current status and future prospects, *IEEE Trans. Geoscience and Remote Sensing*, vol.23, no.4, pp.398-413, 1985.

[5] T. Otsubo, J. Amagai, H. Kunimori et al., Spin motion of the AJISAI satellite derived from spectral analysis of laser ranging data, *IEEE Trans. Geoscience and Remote Sensing*, vol.38, no.3, pp.1417-1424, 2000.

[6] S. Dell'Agnello, D. G. Currie, G. O. D. Monache et al., Next generation lunar laser ranging and its GNSS applications, *IEEE Aerospace Conference*, pp.1-9, 2010.

[7] D. G. Currie, S. Dell'Agnello and G. D. Monache, A lunar laser ranging retroreflector array for the 21st century, *Acta Astronautica*, vol.68, pp.667-680, 2011.

[8] E. P. W. Attema, The active microwave instrument on-board the ERS-1 satellite, *Proc. of the IEEE*, vol.79, no.6, pp.791-799, 1991.

[9] J. J. Degnan, Millimeter accuracy satellite laser ranging: A review, *Contributions of Space Geodesy to Geodynamics: Technology*, 1993.

[10] M. Yang, H. Wang, C. Wu et al., Space flight validation of design and engineering of the ZDPS-1A pico-satellite, *Chinese Journal of Aeronautics*, vol.25, no.5, pp.725-738, 2012.

[11] D. S. Weile, E. Michielssen, D. S. Weile et al., Genetic algorithm optimization applied to electromagnetics: A review, *IEEE Trans. Antennas and Propagation*, vol.45, no.3, pp.343-353, 1997.

[12] H. Tao, G. Liao and Y. Jiang, Space-borne antenna adaptive anti-jamming method based on gradient-genetic algorithm, *Journal of Systems Engineering & Electronics*, vol.18, no.3, pp.469-475, 2007.

[13] A. Deb, J. S. Roy and B. Gupta, Performance comparison of differential evolution, particle swarm optimization and genetic algorithm in the design of circularly polarized microstrip antennas, *IEEE Trans. Antennas and Propagation*, vol.62, no.8, pp.3920-3928, 2014.

[14] C. Cheng, K. Fallahi, H. Leung et al., A genetic algorithm-inspired UUV path planner based on dynamic programming, *IEEE Trans. Systems, Man, and Cybernetics, Part C: Applications and Reviews*, vol.42, no.6, pp.1128-1134, 2012.