# MAP PROJECTION POSITIONING METHOD IN RANGING-MODE OF AUTOMATIC IDENTIFICATION SYSTEM 

Yi Jiang ${ }^{1,2}$, Qing Hu ${ }^{1}$, Dongkai Yang ${ }^{2}$ and Kai Zheng ${ }^{1}$<br>${ }^{1}$ Information Science and Technology College Dalian Maritime University<br>No. 1, Linghai Road, Dalian 116026, P. R. China<br>j_y@dlmu.edu.cn<br>${ }^{2}$ School of Electronic and Information Engineering Beihang University<br>No. 37, Xueyuan Road, Haidian District, Beijing 100191, P. R. China<br>Received October 2015; accepted January 2016


#### Abstract

This paper investigates a positioning method based on the map projection in the ranging-mode ( $R$-mode) of Automatic Identification System (AIS). The geographic location of the vessel is calculated into a flat two-dimensional plane using the direct and inverse transformations based on the map projection. Furthermore, the degree zone conversion algorithm is also proposed when the vessel and the AIS shore stations are not in the same degree zone. Finally, the positioning method is verified by the simulation results using information provided by the real AIS shore stations.


Keywords: Positioning method, Gauss-Kruger projection, Degree zone, Ranging-mode of automatic identification system

1. Introduction. Marine navigation is highly dependent on Global Navigation Satellite System (GNSS) for position, navigation and timing (PNT) information [1]. The awareness of the vulnerability of GNSS to both intentional and non-intentional interference has been raised $[2,3]$. Temporary interruptions of GNSS are not expected to be frequent, but would have a high impact on users [4]. A new ranging-mode (R-mode) of Automatic Identification System (AIS) is proposed by the International Maritime Organization (IMO) as an alternative system for complementing the existing GNSS for the robust PNT information. Moreover, members of IMO are encouraged to investigate and develop the positioning method using the R-mode of AIS [5]. The location of the vessel usually refers to the geographical coordinates (latitude and longitude). It is the two-dimensional positioning. Some studies have focused on the position solution algorithm, such as Fang's closed-form algorithm, Friedlander localization algorithm, a least square algorithm, Chan's algorithm. Fang has studied an exact solution when the number of measurements is equal to the number of unknowns [6]. However, the redundant measurements cannot be utilized to improve its solution accuracy. Although the closed-form solutions with redundant measurements were considered by Friedlander [7], their estimators are not optimum. The least square algorithm can also make use of redundant measurements [8,9]. However, its performance depends highly on an initial estimation of the iterative process. Chan's algorithm has a small computational and high accuracy in the Gaussian noise environment [10]. However, it performs degradation under the NLOS. However, the above position solution algorithms are all discussed in a Cartesian plane coordinate system. They cannot be used to solve geographical coordinates directly.

A positioning method based on the map projection is presented in this paper using in two-dimensional position of R-mode AIS. The location of the vessel is indicated by geographical coordinates. It can also be extended to any other ground-based navigation
system easily. Furthermore, the degree zone conversion algorithm is also proposed when the vessel and AIS shore stations span multiple degree zones. This paper is organized as follows. The principle of Gauss-Kruger projection is given in Section 2. Section 3 presents the map projection positioning method in R-mode of AIS and gives the detailed processing flow. The proposed positioning method is verified by simulation using information of the real AIS shore stations in Section 4. Some concluding remarks are given in Section 5.
2. Gauss-Kruger Projection. In the real world, the location of the vessel is described by the geodetic coordinates, namely the latitude and longitude $(\lambda, \varphi)$ on earth surface. In the Cartesian plane, positions can be described using plane rectangular coordinates ( $x, y$ ). A map projection is a rigorous mathematical means of translating a particular region of three dimensional surfaces into a flat two-dimensional plane. A fundamental projection classification is based on the type of projection surface onto which the globe is conceptually projected. Therefore, there are a number of map projections. The projection selected for the proposed positioning method is Gauss-Kruger projection. It is an internationally popular map projection which has been widely used in the maritime field.

The Gauss-Kruger projection is a transverse cylindrical projection. The projection principle is keeping the projected central meridian as a straight line of the same length and the equator as a straight line. Project the two parts divided by the central meridian within certain scope of longitude degree to the cylinder surface by conformal projection. Upon cutting and unfolding the imaginary cylinder, the required flat projection map is produced, which is the Gauss-Kruger projection plane. The illustration is shown in Figure 1.


Figure 1. Illustration of Gauss-Kruger projection

In Gauss-Kruger projection, the globe is divided into zones according to certain longitude value scope. On the one hand, the length distortion should be controlled within the measurement error. On the other hand, the number of the zones should be kept the minimum to decrease the calculation complexity. As a result, the earth is divided into equal size of melon-shaped zones from the prime meridian. Usually, there are two kinds of dividing zones: the six degree zone and the three degree zone. The six degree zone divides from the west to the east, starting at the prime meridian with 6 degree intervals. The number of the divided zones is $1,2, \ldots, 60$. The three degree zone is carried on the basis of the six degree zone. With the same central meridian and sub zone's central meridian, it starts at the 1.5 degree meridian. The number of the divided zones is $1,2, \ldots, 120$.

The direct projection is given point $\mathrm{P}=(\lambda, \varphi)$ to get projected point $\mathrm{P}^{\prime}=(x, y)$, according to

$$
\left\{\begin{align*}
x= & X+\frac{1}{2} N \sin \varphi \cos \varphi \lambda^{\prime 2}+\frac{1}{24} N \sin \varphi \cos ^{3} \varphi\left(5-t^{2}+9 \eta^{2}+4 \eta^{4}\right) \lambda^{\prime 4}  \tag{1}\\
& +\frac{1}{720} N \sin \varphi \cos ^{5} \varphi\left(61-58 t^{2}+t^{4}\right) \lambda^{\prime 6} \\
y= & N \cos \varphi \lambda^{\prime}+\frac{1}{6} N \cos ^{3} \varphi\left(1-t^{2}+\eta^{2}\right) \lambda^{\prime 3} \\
& +\frac{1}{120} N \cos ^{5} \varphi\left(5-18 t^{2}+t^{4}+14 \eta^{2}-58 \eta^{2} t^{2}\right) \lambda^{\prime 5}
\end{align*}\right.
$$

where $N$ is the radius of curvature in prime vertical; $\lambda^{\prime}$ is the longitude difference from the central meridian, given by

$$
\begin{equation*}
\lambda^{\prime}=\lambda-\lambda_{0} \tag{2}
\end{equation*}
$$

where $\lambda_{0}$ is the longitude of the central meridian. $X$ is the meridional arc length from the equator obtained by

$$
\begin{equation*}
X=6367452.1328 \varphi-\left[32144.5189-\left(135.3646-0.7034 \cos ^{2} \varphi\right) \cos ^{2} \varphi\right] \sin \varphi \cos \varphi \tag{3}
\end{equation*}
$$

And other parameters can be calculated by the following formulas:

$$
\begin{gather*}
t=\tan \varphi  \tag{4}\\
\eta^{2}=e_{2}^{2} \cos ^{2} \varphi \tag{5}
\end{gather*}
$$

where $e_{2}$ is the second eccentricity of the earth.
On the contrary, the transformation of the plane rectangular coordinates into the geodetic coordinates is known as the inverse map projection. Given the position of $\mathrm{P}^{\prime}(x, y)$ in a two-dimensional plane found the geographic coordinates $(\lambda, \varphi)$ of point P . It can be calculated according to the following equations:

$$
\left\{\begin{array}{l}
\lambda=\left(1-\left(d_{3}-d_{5} d_{1}^{2}\right) d_{1}^{2}\right) d_{1}+\lambda_{0}  \tag{6}\\
\varphi=\varphi_{1}-\left(1-\left(d_{4}-0.147 d_{1}^{2}\right) d_{1}^{2}\right) d_{1}^{2} d_{2}
\end{array}\right.
$$

where

$$
\begin{align*}
& \beta=x / 6367452.1328  \tag{7}\\
& \varphi_{1}=\beta+\left(502289760+\left(2936975+23830 \cos ^{2} \beta\right) \cos ^{2} \beta\right) \times 10^{-11} \sin \beta \cos \beta  \tag{8}\\
& d_{1}=\frac{y}{C\left(1+e_{2}^{2} \cos ^{2} \varphi_{1}\right)^{-\frac{1}{2}} \cos \varphi_{1}}  \tag{9}\\
& d_{2}=\left(0.5+0.00336975 \cos ^{2} \varphi_{1}\right) \sin \varphi_{1} \cos \varphi_{1}  \tag{10}\\
& d_{3}=0.3333333-\left(0.1666667-0.0011232 \cos ^{2} \varphi_{1}\right) \cos ^{2} \varphi_{1}  \tag{11}\\
& d_{4}=0.25+\left(0.161612+0.005617 \cos ^{2} \varphi_{1}\right) \sin \varphi_{1} \cos ^{2} \varphi_{1}  \tag{12}\\
& d_{5}=0.2-\left(0.16667-0.00878 \cos ^{2} \varphi_{1}\right) \cos ^{2} \varphi_{1}  \tag{13}\\
& C \text { is the earth's polar radius }
\end{align*}
$$

In Gauss-Kruger projection, there is no angle distortion, little length and square dimension distortion. Except that the central meridian and the equator are straight lines, other longitude lines are symmetric arcs to the central meridian. Moreover, the central meridian is the same as it is. The distortion increases from the central meridian toward the borders. As we move away from the central meridian, the projected features will suffer from distortion. The farther we are from the central meridian, the greater the distortion is. Since it is highly precise, low distortion, and easy to calculate, it is commonly applied to large scale maps. Additionally, the coordinate system is the same for all projected zones. So we just need to work out data in one zone which could be used in all other zones. It can offer the precise measurement of distance. In conclusion, Gauss-Kruger projection is popular in large scale mapping, such as coastal navigation.
3. Map Projection Positioning Method in R-mode of AIS. Map projection positioning method in AIS is based on the theory of map projection. It is an extension of GNSS positioning method. Firstly, the geographical coordinates of the shore stations, which are the reference nodes in the R-mode of AIS, are converted to the plane rectangular coordinates according to the direct projection in Equation (1). Then vessel's position can be solved using the least square algorithm or Chan's algorithm in the two-dimensional Cartesian plane. The position $(x, y)$ of the vessel in the plane rectangular coordinates is obtained. Finally, the geographical coordinates $(\lambda, \varphi)$ of the vessel can be calculated by the inverse projection according to Equation (6).
3.1. Degree zone conversion algorithm. The positioning method in R-mode of AIS only applies to coastal vessels, limited by the arrangement of AIS shore stations and the propagation range of AIS very high frequency (VHF) signals. Thus, the Gauss-Kruger projection suitable for accurate calculation in large scale mapping is selected. In order to minimize the length distortion there are a number of degree zones in the Gauss-Kruger projection. In each zone, the intersection of the central meridian and the equator is the coordinate origin. The coordinate calculation is the same for all the degree zones. The transmission range of VHF signals is generally $35-50$ nautical miles. In a six degree zone, a zone spans about 360 nautical miles ( 668 kilometers). Thus in practice, for one vessel, the locations of this vessel and all the AIS shore stations used as its reference nodes for position estimation are either in the same degree zone, or in the two adjacent degree zones. If the vessel and its reference nodes are all in the same degree zone, the position of the vessel can be solved directly. If the vessel and its reference nodes are in the two adjacent degree zones, the degree zone conversion algorithm is proposed. In the degree zone conversion algorithm, the coordinates of the vessel and its reference nodes are all moved into one zone to solve the position of the vessel.

Therefore, before estimating the vessel's position in the two-dimensional plane, it is needed to decide whether the vessel and its reference nodes are in the same degree zone in terms of the comparison of their central meridians. Firstly, the central meridians corresponding to the initial estimated position of the vessel and its reference nodes are calculated. If all the central meridians are same, there is no need to convert degree zones. If not, the vessel and its reference nodes are in two adjacent degree zones. Take the same central meridian corresponding to greater than or equal to two reference nodes as a benchmark. The degree zone, which the central meridian benchmark belongs to, is called the reference zone. The points which are not in the reference zone should be moving. That is to say, the maximum (or minimum) central meridian is chosen as the benchmark. The position corresponding to the minimum (or maximum) longitude should be moved into the reference zone. Keep a record of the moving longitude value. And all the rest position coordinates are moved the same radian. The flow chart is shown in Figure 2.
3.2. Map projection positioning method. The map projection positioning method in R-mode of AIS is divided into three parts denoted by the dotted box in Figure 3. First of all, the geographical coordinates of the initial vessel estimated position and its reference nodes are converted to the plane rectangular coordinates using Gauss-Kruger projection. Secondly, judgment and conversion of the degree zone span shown in Figure 2 are processed. Then the vessel's position is estimated in the two-dimensional plane. The position equation using time difference of arrival (TDOA) technology is expressed in the rectangular coordinate plane as

$$
\begin{equation*}
\Delta R_{i}=\sqrt{\left(x_{m}-x\right)^{2}+\left(y_{m}-y\right)^{2}}-\sqrt{\left(x_{i}-x\right)^{2}+\left(y_{i}-y\right)^{2}} \tag{14}
\end{equation*}
$$

where $(x, y)$ are the coordinates of the vessel; $\left(x_{i}, y_{i}\right)$ are the position coordinates of the $i$ th reference node; the subscript $m$ denotes the main reference node; $\Delta R_{i}$ denotes the distance difference between the vessel and the different reference nodes.


Figure 2. Flow chart of degree zone conversion


Figure 3. Flow chart of map projection positioning method

Obtain the linear position equation using Taylor-series, keeping only terms below the second order.

$$
\begin{equation*}
\Delta \bar{R}_{i}=\Delta \hat{R}_{i}+\frac{\partial \Delta R_{i}}{\partial x} \Delta x+\frac{\partial \Delta R_{i}}{\partial y} \Delta y \tag{15}
\end{equation*}
$$

where $\Delta \bar{R}_{i}$ and $\Delta \hat{R}_{i}$ denote the measured distance difference and its estimation, respectively; $(\Delta x, \Delta y)$ are the corrections to the estimated position of the vessel. Stacking all the measurements from different reference nodes, the above Equation (15) can be given in the matrix form as follows:

$$
\left[\begin{array}{c}
\delta L_{1}  \tag{16}\\
\vdots \\
\delta L_{n}
\end{array}\right]=\left[\begin{array}{cc}
\frac{\partial \Delta R_{1}}{\partial x} & \frac{\partial \Delta R_{1}}{\partial x} \\
\vdots & \vdots \\
\frac{\partial \Delta R_{n}}{\partial y} & \frac{\partial \Delta R_{n}}{\partial y}
\end{array}\right]\left[\begin{array}{c}
\Delta x \\
\Delta y
\end{array}\right]
$$

where $\delta L_{i}=\Delta \bar{R}_{i}-\Delta \hat{R}_{i}$. We can write (16) more compactly as

$$
\mathbf{L}=\mathbf{A} \Delta \mathbf{X}=\left[\begin{array}{ll}
\boldsymbol{\alpha} & \boldsymbol{\beta} \tag{17}
\end{array}\right] \Delta \mathbf{X}
$$

$\mathbf{A}$ is called positioning matrix. The elements in the matrix are expressed as

$$
\left.\begin{array}{rl}
\alpha_{i} & =\frac{\partial \Delta R_{i}}{\partial x}=\frac{x-x_{m}}{\hat{R}_{m}}-\frac{x-x_{i}}{\hat{R}_{i}}  \tag{18}\\
\beta_{i} & =\frac{\partial \Delta R_{i}}{\partial y}=\frac{y-y_{m}}{\hat{R}_{m}}-\frac{y-y_{i}}{\hat{R}_{i}}
\end{array}\right\}
$$

The least square solution for the corrections $(\Delta x, \Delta y)$ to the estimated position of the vessel can be written as

$$
\begin{equation*}
\Delta \mathbf{X}=\left(\mathbf{A}^{\mathrm{T}} \mathbf{A}\right)^{-1} \mathbf{A}^{\mathrm{T}} \mathbf{L} \tag{19}
\end{equation*}
$$

The convergence is decided by $\Delta \mathbf{X}$. The convergence condition given in the paper is $\sqrt{(\Delta x)^{2}+(\Delta y)^{2}} \leq 0.00001$ when the number of iterations is less than 10 times. Finally, the position $(x, y)$ of the vessel, which meets the above condition, is converted to geographic coordinates by the inverse Gauss-Kruger projection.
4. Simulations. To verify the proposed method, AIS shore stations named Laotieshan, Huangbaizui and Beihuangcheng in the real situation are used. The vessel is located in Yuandao. Table 1 gives information about these shore stations and the location of the vessel, including the latitude and longitude coordinates and their maritime mobile communications service identity (MMSI).

Table 1. Information of the AIS shore stations and vessel location

|  | MMSI | Latitude | Longitude |
| :---: | :---: | :---: | :---: |
| Laotieshan | 4131101 | $38^{\circ} 43.6420^{\prime} \mathrm{N}$ | $121^{\circ} 08.1330^{\prime} \mathrm{E}$ |
| Huangbaizui | 4131104 | $38^{\circ} 54.2850^{\prime} \mathrm{N}$ | $121^{\circ} 42.9500^{\prime} \mathrm{E}$ |
| Beihuangcheng | 4131504 | $38^{\circ} 23.6880^{\prime} \mathrm{N}$ | $120^{\circ} 54.6137^{\prime} \mathrm{E}$ |
| Yuandao |  | $38^{\circ} 40.1690^{\prime} \mathrm{N}$ | $122^{\circ} 10.1020^{\prime} \mathrm{E}$ |

The location distribution of the vessel and the AIS shore stations is shown in Figure 4. Figrue 4(a) is drawn according to the geographic coordinates given in Table 1. The orange box indicates the shore station. A red dot indicates the vessel. Figure 4(b) is drawn on the rectangular coordinate plane. The black circle denotes the shore station and the red star is the position solution obtained by the proposed method. It can be seen that the distribution of the AIS shore stations and the vessel in Figure 4(b) agrees with Figure 4(a).

Table 2 gives the position error between the position solution of the vessel derived from the map projection positioning method and its exact location. As the simulation


Figure 4. Location distribution of AIS shore stations and vessel
Table 2. Position error

|  | Latitude | Longitude |
| :---: | :---: | :---: |
| Yuandao | $38^{\circ} 40^{\prime} 10.140000^{\prime} \mathrm{N}$ | $122^{\circ} 10^{\prime} 6.120000 \mathrm{E}$ |
| Solution | $38^{\circ} 40^{\prime} 10.1399237^{\prime \prime} \mathrm{N}$ | $121^{\circ} 10^{\prime} 6.1199999 \mathrm{E}$ |
| Error $(\mathrm{m})$ | $-2.356777 \times 10^{-3}$ | $-3.57566 \times 10^{-6}$ |

environment is ideal, there is no measurement error and other effective factors. Thus position solution has very small deviation. The error of the latitude is $\left(2 \times 10^{-8}\right)^{\circ}$, that is 0.002 meters. The error of the longitude is $\left(3 \times 10^{-11}\right)^{\circ}$, that is 0.000004 meters.
5. Conclusions. The map projection positioning method is investigated in the R-mode of AIS. The position solution of the vessel is converted to a flat two-dimensional plane, using the direct and inverse map projection based on the map projection theory. Furthermore, the degree zone conversion algorithm is also proposed when the vessel and the AIS shore stations are not in the same degree zone. Finally, the positioning method is verified by the positioning simulation in the real AIS situation. The proposed method can also be extended to any other ground based navigation system easily.

Acknowledgment. This research is partially supported by the Chinese National Science Foundation (No. 61501079 and 61231006), Foundation of Liaoning Educational Committee (No. L2015059), Liaoning Provincial Natural Science Foundation (No. 2014025002) and the Fundamental Research Funds for the Central Universities (No. 3132014329).

## REFERENCES

[1] A. Weintrit, Prioritized main potential solutions for the e-navigation concept, International Journal on Marine Navigation and Safety of Sea Transportation, vol.7, no.1, pp.27-38, 2013.
[2] IALA Recommendation R-129, GNSS Vulnerability and Mitigation Measures, 3rd Edition, 2012.
[3] K. Y. Chen, C. A. C. Heckel-Jones, N. G. Maupin, S. M. Rubin, J. M. Bogdanor, Z. Y. Guo and Y. Y. Haimes, Risk analysis of GPS-dependent critical infrastructure system of systems, Proc. of Systems and Information Engineering Design Symposium, Charlottesville, Virginia, pp.316-321, 2014.
[4] P. Papadimitratos and A. Jovanovic, Protection and fundamental vulnerability of GNSS, Proc. of IEEE International Workshop on Satellite and Space Communications, Toulouse, France, pp.167171, 2008.
[5] International Association of Lighthouse Authorities, IALA Worldwide Radio Navigation Plan, Version 2, 2012.
[6] B. T. Fang, Simple solutions for hyperbolic and related fixes, IEEE Trans. Aerospace and Electronic Systems, vol.26, pp.748-753, 1990.
[7] B. Friedlander, A passive localization algorithm and its accuracy analysis, IEEE Journal of Oceanic Engineering, vol.OE-12, no.1, pp.234-244, 1987.
[8] M. N. Alam and M. M. Haque, A least square approach for TDOA/AOA wireless location in WCDMA system, Proc. of the 11th International Conference on Computer and Information Technology, Khulna, Bangladesh, pp.686-690, 2008.
[9] M. Ye, L. Wang, Y. Jiang and S. D. Xie, Localization algorithm based on nonlinear least square principle, Journal of Computational Information Systems, vol.10, no.14, pp.6033-6039, 2014.
[10] T. C. Yang, L. Jin and J. Cheng, An improvement Chan algorithm based on TOA position, Acta Electronica Sinca, vol.37, no.4, pp.819-822, 2009.

