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NONLINEAR AEROELASTIC ANALYSIS AND CONTROL OF AIRFOIL SECTION WITH CONTROL SURFACE FREEPLAY

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Abstract. *Control surface freeplay is an important structural nonlinearity in aeroelastic systems. In this paper, the effect of the control surface freeplay nonlinearity on the dynamic responses of an airfoil section is studied, and an active vibration suppression system with compensator is designed. Wagner function and Duhamel formulation are used to model the aerodynamic loads based on Theodorsen theory. The vibration characteristics of the nonlinear systems with different freeplay gaps are analyzed, and compared to the linear system without freeplay. Bifurcation and limit cycle oscillations occur due to the control surface freeplay, and the nonlinear systems are also divergent above the linear flutter speed. The influence of the freeplay nonlinearity on the closed-loop system is studied with a controller designed using LQR and over-damping phenomenon is found. The response speed of the closed-loop system for vibration suppression is improved effectively with the compensator.*
Keywords: Airfoil sections, Control surface freeplay, Limit cycle oscillations, Compensator

1. **Introduction.** Nonlinear aeroelastic systems exhibit a variety of phenomena such as limit cycle oscillation and chaotic vibration which are different from those of linear systems and may lead to structural problems and material fatigue [1,2]. Therefore, analysis and vibration suppression of the nonlinear aeroelastic systems are very important. There are various nonlinearities in aeroelastic systems and a great deal of research activities have been devoted to them, especially continuous polynomial nonlinearities [3,4]. However, for the discontinuous nonlinearities caused by freeplay, further studies are needed.

The control surface freeplay is an important structural nonlinearity in aeroelastic systems. Vasconcellos et al. [6] assessed different representations for freeplay. Conner et al. [7] investigated the effect of freeplay on an open-loop system response in numerical and experimental approach, and the result showed the limit cycle oscillation was observed when the free stream velocity was far lower than linear flutter speed. Bae et al. [5] demonstrated that limit cycle oscillations have amplitudes significantly greater than the size of the freeplay gap. Tang et al. [8] studied limit cycle behavior using harmonic balance calculations and found that the bifurcation velocities are independent of the freeplay range. Zhao's study [9] showed that the jump phenomenon appears in the amplitudes of limit cycle oscillations with the increase of air speed. Active vibration suppression of the nonlinear aeroelastic systems has caused widespread concern [10-12,14]. However, in the process of control system design, the two DOFs' systems which just consist of plunge and pitch motions were adopted in structural model and quasi steady assumption was always used in aerodynamic model [3,12]. Hence, the inertial coupling of control surface motion and other two motions, and unsteady aerodynamic effect would not be reflected. In this study, the three DOFs' structural dynamics equations including the control surface motion and unsteady time-domain aerodynamic loads are used to model the aeroelastic system. Based on the precise representation, the effect of control surface freeplay on both open-loop and closed-loop aeroelastic system of an airfoil section is analyzed and a novel active vibration control law with compensator for this kind of nonlinear aeroelastic system is presented.

The organization of the paper is as follows. Section 2 presents nonlinear aeroelastic model, including structural dynamics equations and unsteady areodynamic loads. Stability and dynamic characteristics analysis are conducted in Section 3. Active control law with compensator is designed in Section 4. Section 5 draws the conclusions.

2. Aeroelastic Model. The two-dimensional airfoil section with a trailing-edge control surface is shown in Figure 1. The plunge pitch and control surface motions are denoted by h, α and β, respectively. The semi-chord length of airfoil section is b. The distance between the elastic axis and midchord is represented by a and the distance from midchord to the hinge line of control surface is denoted by c. K_h and K_α are used to represent the plunge and pitch stiffness and K_β represents the stiffness of the control surface hinge.

The control surface freeplay nonlinearity produces a piecewise linear change in the structural stiffness of hinge. The relationship between the restoring moment of the control surface and its motion is illustrated in Figure 2. δ is the size of the freeplay gap and $\tilde{M}(\beta)$ represents the restoring moment. Thus, when the rotation angle of the control surface β is in the freeplay gap, the stiffness of the control surface hinge is zero; and when β is beyond the gap, the stiffness is a constant value.

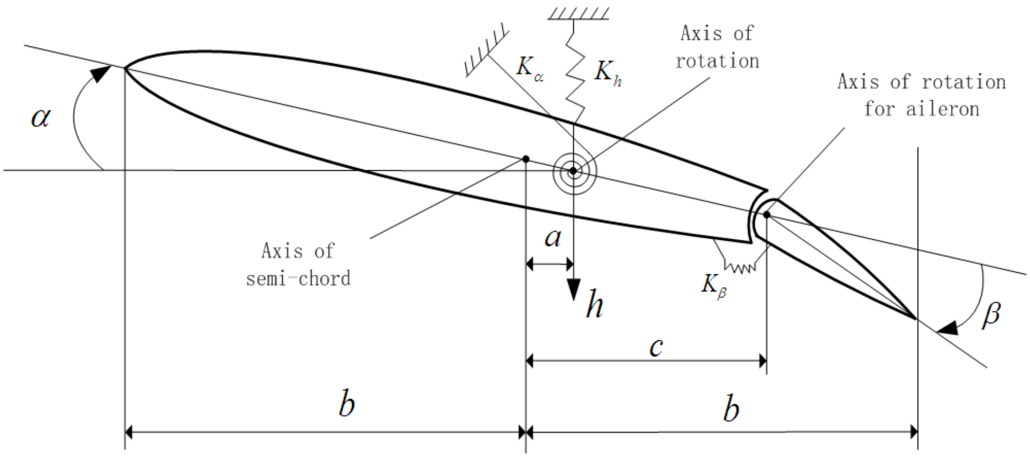


FIGURE 1. Airfoil section with control surface

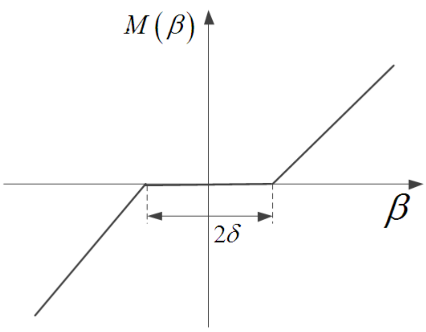


FIGURE 2. Control surface freeplay

2.1. **Structural dynamics equations.** Using Lagrange's equation, the equations of motion of the two-dimensional airfoil section with three DOFs are written as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{Q} \tag{1}$$

where $\mathbf{q} = \begin{bmatrix} h & \alpha & \beta \end{bmatrix}^{\mathrm{T}}$ denotes the displacements of the three DOFs' motion. $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass matrix, damping matrix and stiffness matrix of the aeroelastic system, respectively. It should be noted that $\mathbf{K}$ is related to the control surface motion β. $\mathbf{Q}$ is the aerodynamic loads.

2.2. **Areodynamic loads.** The unsteady aerodynamic force and moment in incompressible flow are given using Theodorsen approach [13]. The original aerodynamic loads are obtained in frequency expressions assuming the airfoil section is in the state of harmonic vibration. To simulate the arbitrary motions of the system, the time-domain unsteady aerodynamic loads are needed. Based on the Wagner function, the loads associated with Theodorsen's function can be replaced by Duhamel formulation in the time domain

$$L_c = f(t)\,\varphi_w(0) + \int_0^t f(\sigma)\,\frac{d\varphi_w(t-\sigma)}{d\sigma}d\sigma \tag{2}$$

where $f(t)$ is the function of displacements and velocities of three DOFs motions. And φ_w is the approximated Wagner function

$$\varphi_w(t) = 1 - A_1 e^{-b_1 \frac{V}{b} t} - A_2 e^{-b_2 \frac{V}{b} t} \tag{3}$$

where $A_1 = 0.165$, $A_2 = 0.335$, $b_1 = 0.0455$ and $b_2 = 0.3$. For the integral items in Equation (2), it is hard to solve directly. Two augmented variables will be introduced as

$$v_j(t) = \int_0^t f(\sigma) e^{-b_j \frac{V}{b}(t-\sigma)} d\sigma \tag{4}$$

with $j = 1, 2$. The time derivative of the introduced variables leads to

$$\frac{dv_j(t)}{dt} = f(t) - b_j \frac{V}{b} v_j(t) \tag{5}$$

In matrix representation

$$\dot{\mathbf{v}} = \mathbf{E}_1\mathbf{q} + \mathbf{E}_2\dot{\mathbf{q}} + \mathbf{F}\mathbf{v} \tag{6}$$

where $\mathbf{E}_1$, $\mathbf{E}_2$ and $\mathbf{F}$ represent the influence matrices. Then, the aerodynamic loads $\mathbf{Q}$ in Equation (1) can be written as

$$\mathbf{Q} = \mathbf{M}_{\mathrm{nc}}\ddot{\mathbf{q}} + \mathbf{C}_{\mathrm{nc}}\dot{\mathbf{q}} + \mathbf{K}_{\mathrm{nc}}\mathbf{q} + \mathbf{D}\mathbf{v} \tag{7}$$

where $\mathbf{M}_{\mathrm{nc}}$, $\mathbf{C}_{\mathrm{nc}}$ and $\mathbf{K}_{\mathrm{nc}}$ are used to represent the influence matrices of plunge, pitch and control surface motions to aerodynamic loads, respectively. $\mathbf{D}$ is the influence matrix of aerodynamic variables, and that is the characterization of unsteady effect.

2.3. **State space representation.** The nonlinear aeroelastic system can be obtained in second order ordinary differential equation representations with the interconnection of the structure dynamics equations and unsteady aerodynamic loads. Substitute Equation (7) into Equation (2)

$$(\mathbf{M} - \mathbf{M}_{\mathrm{nc}})\,\ddot{\mathbf{q}} + (\mathbf{C} - \mathbf{C}_{\mathrm{nc}})\,\dot{\mathbf{q}} + (\mathbf{K} - \mathbf{K}_{\mathrm{nc}})\,\mathbf{q} = \mathbf{D}\mathbf{v} \tag{8}$$

Let $\mathbf{M}_{\mathrm{tot}} = \mathbf{M} - \mathbf{M}_{\mathrm{nc}}$, $\mathbf{C}_{\mathrm{tot}} = \mathbf{C} - \mathbf{C}_{\mathrm{nc}}$ and $\mathbf{K}_{\mathrm{tot}} = \mathbf{K} - \mathbf{K}_{\mathrm{nc}}$. In order to analyze the stability of the system and design the active control law, the state space representation is needed. The aeroelastic model above can be written in state space form

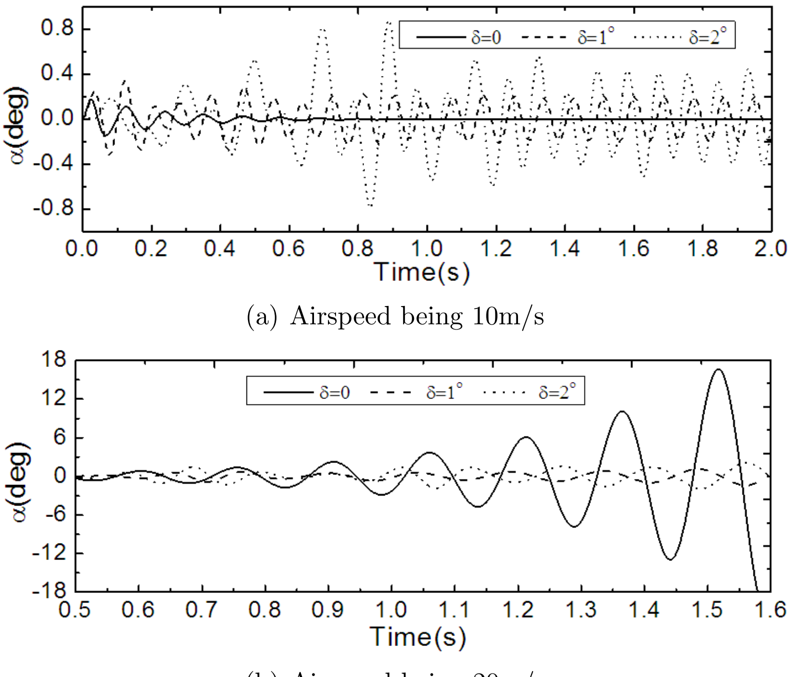
$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x})\,\mathbf{x} + \mathbf{B}u \tag{9}$$

with $\mathbf{x} = \left\{ \begin{array}{ccc} \mathbf{q}^{\mathrm{T}} & \dot{\mathbf{q}}^{\mathrm{T}} & \mathbf{v}^{\mathrm{T}} \end{array} \right\}^{\mathrm{T}}$, and u is the scalar control input. The matrix $\mathbf{A}$ and $\mathbf{B}$ are

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ -\mathbf{M}_{\mathrm{tot}}^{-1}\mathbf{K}_{\mathrm{tot}} & -\mathbf{M}_{\mathrm{tot}}^{-1}\mathbf{C}_{\mathrm{tot}} & \mathbf{M}_{\mathrm{tot}}^{-1}\mathbf{D} \\ \mathbf{E}_1 & \mathbf{E}_2 & \mathbf{F} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}_{\mathrm{tot}}^{-1}\mathbf{G} \\ \mathbf{0} \end{bmatrix} \tag{10}$$

where $\mathbf{G} = \left[\begin{array}{ccc} 0 & 0 & 1 \end{array} \right]^{\mathrm{T}}$. Note that the matrix $\mathbf{A}$ is dependent on the states of the system (the motion of control surface), and that makes Equation (9) a nonlinear aeroelastic system.

3. **Stability and Dynamic Characteristics.** If the control surface freeplay is not considered, i.e., $\delta = 0°$, Equation (9) will be a linear aeroelastic system. The linear flutter speed $V_F = 18.2$m/s of an airfoil section can be obtained based on the stability theory of linear systems. While considering the freeplay, the system will present complicated nonlinear characteristics. We can judge that Hopf bifurcation phenomenon will occur when the airspeed reaches $V_B = 4.5$m/s which is far lower than V_F. V_B can be obtained by analyzing eigenvalues of the Jaccobi matrix of the system. Figure 3 gives the time histories of pitch motion of the airfoil section with different freeplay gaps under the same initial conditions. The airspeeds are chosen as 10m/s (between bifurcation speed V_B and linear flutter speed V_F) and 20m/s (a little higher than V_F).



(a) Airspeed being 10m/s

(b) Airspeed being 20m/s

FIGURE 3. Time histories of pitch motion

Figure 3(a) shows that stable limit cycle oscillations occur in the aeroelastic system with freeplay nonlinearity. The amplitude of limit cycle oscillation increases with freeplay gap δ. Figure 4 represents the phase diagram of the pitch motion under the condition that airspeed V is 10m/s and freeplay gap δ is 2°. Figure 5 gives the variation of maximum amplitude of pitch motion versus airspeed. We can see that, when the airspeed is higher than linear flutter speed V_F, both the linear system and the nonlinear system will diverge. The dynamic responses of pitch motion in this situation are shown in Figure 3(b).

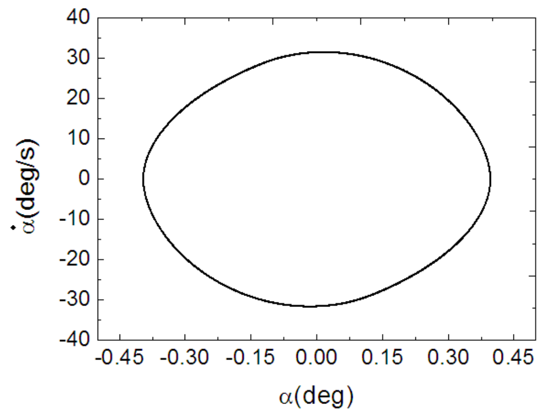


FIGURE 4. The phase diagram of pitch motion

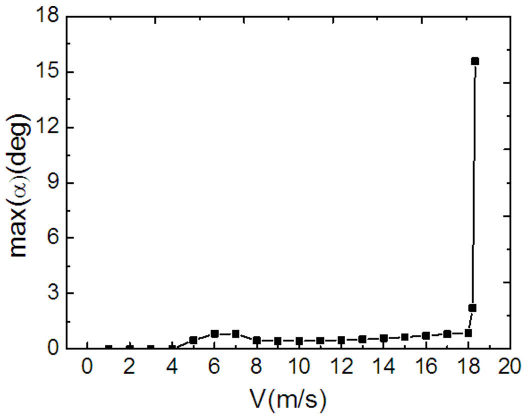


FIGURE 5. The maximum amplitude of pitch motion

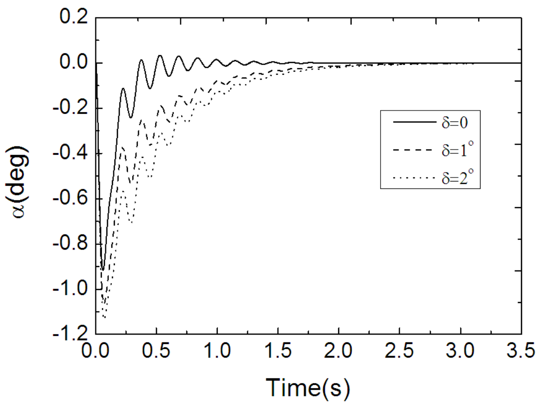


FIGURE 6. Time histories of pitch motion with controller

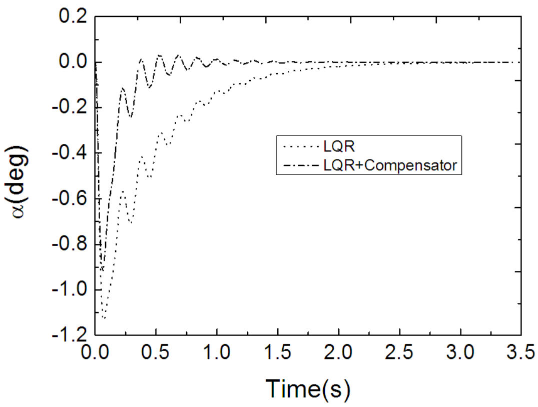


FIGURE 7. Time histories of pitch motion with controller and compensator

4. **Control Design.** The vibration of the airfoil section may reduce the flight envelope or cause structure fatigue problems even the damage of aircrafts. It is necessary to stabilize aeroelastic systems and suppress the vibration. In this paper, LQR method is used to get the control law, and $\mathbf{K}_c$ denotes the obtained control gain matrix. The control performance is shown in Figure 6. In these simulations, the airspeed is chosen as $V = 20$m/s.

Obviously, the systems can converge with the controller. However, the response time will increase due to the existence of the freeplay and it will be longer with a larger value of δ. The control surface freeplay impairs the performance of controller and the pitch motion behaves like a response of an over-damping system. The same problem was also found in [10].

From the above analysis, in order to obtain ideal vibration suppression performance, the impact of the freeplay should be considered during the design of controller. Rewrite the control surface moment rotation relationship as

$$\tilde{M}(\beta) = K_\beta \beta + K_\beta \beta L_1(\beta) - K_\beta \delta L_2(\beta) \tag{11}$$

and

$$\begin{aligned} L_1(\beta) &= \tfrac{1}{2}\left[sign(\beta-\delta) - sign(\beta+\delta)\right] \\ L_2(\beta) &= \tfrac{1}{2}\left[sign(\beta-\delta) + sign(\beta+\delta)\right] \end{aligned} \tag{12}$$

where *sign* is a symbolic function. When β is in the freeplay gap, $L_1 = -1$ and $L_2 = 0$. When β is beyond the freeplay gap, $L_1 = 0$ and $L_2 = \pm 1$. Obviously, it can be seen that the difference of freeplay nonlinear system and the linear system is reflected in last two terms of the right-hand side of Equation (11). The effect of freeplay can be compensated with added input u_c and the closed-loop system will become

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\left(-\mathbf{K}_c\mathbf{x} + u_c\right) \tag{13}$$

where $u_c = K_\beta \beta L_1(\beta) - K_\beta \delta L_2(\beta)$.

In the case of $V = 20$m/s and $\delta = 2°$, the simulation results are shown in Figure 7.

It is indicated that the response time of system is obviously reduced under the effect of the compensator. Over-damping phenomenon disappears and the requirement to vibration suppression can be met for the airfoil section.

5. **Conclusions.** The effect of the control surface freeplay on the aeroelastic system of an airfoil section has been studied in this paper. The three DOFs' structural dynamics equations are used. The time-domain unsteady aerodynamic loads for arbitrary motions of airfoil section are obtained by Wagner function and Duhamel integration. The results show that bifurcation and limit cycle oscillation occur in the nonlinear aeroelastic system under a certain airspeed which is far lower than linear flutter speed, and the system is also divergent above this flutter speed. To suppress the vibration of the airfoil section, active controller is designed. The time consumed in stabilizing the system increases due to the control surface freeplay, and has been effectively reduced by the introduction of compensator in controller.

The future work will consider full three-dimensional wing model with high-aspect ratio and a more practical controller will be designed with incomplete and noisy measurements.

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