## GUARANTEED COST TRACKING CONTROL FOR AN OMNIDIRECTIONAL REHABILITATIVE TRAINING WALKER WITH SAFETY VELOCITY PERFORMANCE

PING SUN<sup>1</sup> AND SHUOYU WANG<sup>2</sup>

<sup>1</sup>School of Information Science and Engineering Shenyang University of Technology No. 111, Shenliao West Road, Eco. and Tech. Development Zone, Shenyang 110870, P. R. China tonglongsun@sut.edu.cn

> <sup>2</sup>Department of Intelligent Mechanical Systems Engineering Kochi University of Technology Tosayamada, Kami city, Kochi 782-8502, Japan wang.shuoyu@kochi-tech.ac.jp

Received October 2015; accepted January 2016

ABSTRACT. A new nonlinear tracking control method with safety velocity performance is proposed for an omnidirectional rehabilitative training walker. The goal of our study was to design an asymptotically stable controller with which users can complete rehabilitation training under the safety velocity. According to Lyapunov theory, the controller can be designed to maintain stability in terms of solutions of linear matrix inequalities. We therefore derive the sufficient conditions for the existence of such a controller with safety velocity constraints. As an application, our simulation results confirmed the effectiveness of the proposed method and verified that the walker can provide a safe training velocity. **Keywords:** Rehabilitative training walker, Tracking control, Safety velocity, Velocity constraints performance

1. Introduction. With an aging society, an increasing number of people suffer from walking impairments due to illnesses or accidents. Therefore, demand for walking rehabilitation has increased in recent years, leading to an increased need for a walking training machine that can efficiently conduct a variety of training programs. Omnidirectional rehabilitation training walker (ODW) can help patients move within any radius in any direction and can also follow special training trajectories [1,2]. To operate effectively in real-world applications, the control algorithm must guarantee that the device can accurately follow a prescribed trajectory.

An important issue in robotics control, trajectory tracking has been widely studied [3-5]. Available results, such as adaptive tracking control [6,7], adaptive fuzzy tracking control [8], adaptive dynamic surface control [9], and backstepping tracking control [10], have been developed to improve tracking performance. However, a limitation of these previously published results is that robots are required to provide enough motion velocity without imposing velocity constraints during trajectory tracking.

In fact, the mechanical systems to be controlled are often subject to various types of physical constraints such as the input saturation and state constraints [11,12]. It is widely acknowledged that dealing with constraints is one of the basic challenges in most physical control problems [13]. From a practical view point, rehabilitative training robots are different from general mechanical systems operating at high speeds [14]. The tracking motion velocity of rehabilitative robots must not exceed the safety velocity. Here, the safety velocity refers to the actual velocity under the allowable maximum velocity of rehabilitative robots. Therefore, to guarantee the safety of rehabilitees, the controller for rehabilitative training robots with safety velocity tracking performance is crucial.

In our previous study [15], a reliable control method was proposed based on redundant degrees of freedom. The aim was to design an asymptotically stable controller that can guarantee the safety of the user when one wheel actuator fault occurs. In this paper, safety velocity control is considered in order to prevent motion velocity exceeding user's endurance capacity in practical applications. Therefore, determining how to maintain both stability and safety velocity performance is a worthwhile endeavor, which has motivated us to conduct the present study. The main contributions of this paper are summarized as follows.

(i) We propose a safety velocity trajectory tracking control algorithm for ODW and establish closed loop stability of the tracking error system based on a Lyapunov function.

(ii) We obtain an upper bound on the degradation of tracking velocity that guarantees trajectory tracking performance. The existence of controller parameter matrices is given in terms of solutions of linear matrix inequalities. A significant feature of our proposed approach is that a controller achieves velocity constraints performance via a simple design.

(iii) As an application, on the basis of the dynamic model, the safety velocity tracking control on the ODW is considered. The efficiency of the proposed scheme is demonstrated.

The remainder of this paper is organized as follows. In Section 2, dynamic model of the ODW is formulated. The main results that provide a solution to the safety velocity tracking control problem are presented in Section 3. Simulation results are given in Section 4, and concluding remarks are provided in Section 5.

2. Dynamic Model of the Omnidirectional Walker. Figure 1 shows the structure of the ODW with omniwheels, while the coordinate system and parameters used to develop the ODW model are shown in Figure 2.



FIGURE 1. Structure of the ODW

In Figure 2,

 $\Sigma(x, O, y)$ : Absolute coordinate system

- $\Sigma(x', G, y')$ : Translation coordinate system
- v: Speed of the ODW
- $v_i$ : Speed of an omnidirectional wheel
- $f_i$ : Force on each omnidirectional wheel
- L: Distance from the center of gravity of the walker to each omnidirectional wheel
- $\alpha$ : Angle between the x' axis and the direction of v



FIGURE 2. The ODW coordinate system

- $\beta$ : Angle between the x' axis and  $r_0$
- $\theta_i$ : Angle between the x' axis and the position of each omnidirectional wheel
- $l_i$ : Distance from the center of gravity to the middle of each omnidirectional wheel
- $\phi_i$ : Angle between the x' axis and  $l_i$
- G: Center of gravity of the walker

 $r_0$ : Distance between G and the center of gravity due to the load Based on [16], the dynamic model is expressed as follows:

$$M_0 K(\theta) \ddot{X}(t) + M_0 \dot{K}\left(\theta, \dot{\theta}\right) \dot{X}(t) = B(\theta) u(t)$$
(1)

We let

$$\begin{aligned} x_1(t) &= X(t) \\ x_2(t) &= \dot{X}(t) \end{aligned} \tag{2}$$

such that Equation (1) can be expressed as follows:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -[M_0 K(\theta)]^{-1} \left[ M_0 \dot{K}(\theta, \dot{\theta}) \right] x_2(t) + U(t) \end{cases}$$
(3)

where  $U(t) = [M_0 K(\theta)]^{-1} B(\theta) u(t)$ .

The actual motion trajectory is X(t), and the desired motion trajectory is  $X_d(t)$ ; therefore, trajectory tracking error  $e_1(t)$  and velocity tracking error  $e_2(t)$  are

$$e_1(t) = X(t) - X_d(t)$$
 (4)

$$e_2(t) = \dot{X}(t) - \dot{X}_d(t) = x_2(t) - \dot{X}_d(t)$$
(5)

where  $e_1^T(t) = \begin{bmatrix} e_{1x}^T(t) & e_{1y}^T(t) & e_{1\theta}^T(t) \end{bmatrix}$ , which are the x, y axes, and orientation angle trajectory tracking errors, respectively; likewise,  $e_2^T(t) = \begin{bmatrix} e_{2x}^T(t) & e_{2y}^T(t) & e_{2\theta}^T(t) \end{bmatrix}$  are the x, y axes, and orientation angle velocity tracking errors, respectively.

Next, we design the control input as follows:

$$U(t) = [M_0 K(\theta)]^{-1} \left[ M_0 \dot{K}(\theta, \dot{\theta}) \right] x_2(t) + \ddot{X}_d(t) + u_c(t)$$
(6)

$$u_c(t) = K_d e_2(t) + K_p e_1(t)$$
(7)

where  $K_d = diag \{K_{dx}, K_{dy}, K_{d\theta}\}$  and  $K_p = diag \{K_{px}, K_{py}, K_{p\theta}\}$ .

Thus, the error dynamics equation is obtained as:

$$\begin{cases} \dot{e}_1(t) = e_2(t) \\ \dot{e}_2(t) = K_d e_2(t) + K_p e_1(t) \end{cases}$$
(8)

It can easily be verified that the tracking error sub-system dynamics equation about the x, y axes and orientation angle are given by the following:

$$\begin{cases} \dot{e}_{1x}(t) = e_{2x}(t) \\ \dot{e}_{2x}(t) = K_{dx}e_{2x}(t) + K_{px}e_{1x}(t) \end{cases}$$
(9)

$$\begin{cases} \dot{e}_{1y}(t) = e_{2y}(t) \\ \dot{e}_{2y}(t) = K_{dy}e_{2y}(t) + K_{py}e_{1y}(t) \end{cases}$$
(10)

$$\begin{cases} \dot{e}_{1\theta}(t) = e_{2\theta}(t) \\ \dot{e}_{2\theta}(t) = K_{d\theta}e_{2\theta}(t) + K_{p\theta}e_{1\theta}(t) \end{cases}$$
(11)

Furthermore, the cost function associated with Equation (8) is

$$J = \int_0^\infty \left( e_2^T(t) Q e_2(t) \right) dt \tag{12}$$

Here, Q > 0 is a symmetric constant matrix.

In this paper, our focus is to design controller u(t) considering velocity constraint performance such that the following two requirements are simultaneously satisfied:

(1) Trajectory error  $e_1(t)$  and velocity error  $e_2(t)$  are asymptotically stable.

(2) Actual tracking motion velocity is  $x_2(t)$  and  $x_2^T(t) = \begin{bmatrix} x_2^x(t) & x_2^y(t) & x_2^\theta(t) \end{bmatrix}$ , noting that  $|x_2^T(t)| = \begin{bmatrix} |x_2^x(t)| & |x_2^y(t)| & |x_2^\theta(t)| \end{bmatrix}$ , with  $x_2^x(t)$ ,  $x_2^y(t)$  being the velocity of x, y axes, and  $x_2^\theta(t)$  being the angle velocity;  $v_{\text{max}}$  is the allowable maximum motion velocity, and  $v_{\text{max}}^T = \begin{bmatrix} v_{x \max} & v_{y \max} & v_{\theta \max} \end{bmatrix}$ , where  $v_{x \max}$ ,  $v_{y \max}$  are the maximum velocities of the x, y axes, and  $v_{\theta \max}$  is the maximum angle velocity. The performance of  $x_2(t)$  satisfies the following inequalities:

$$|x_2^x(t)| \le v_{x\max}, \quad |x_2^y(t)| \le v_{y\max}, \quad |x_2^\theta(t)| \le v_{\theta\max}$$
 (13)

3. Controller Design with Velocity Constraints Performance. We aim to design a safety velocity controller based on Lyapunov stability theory.

**Theorem 3.1.** Considering the error Equation (8), if there exist symmetric positive matrices  $P = diag\{P_{11}, P_{22}, P_{33}\}$  and  $Q = diag\{Q_{11}, Q_{22}, Q_{33}\}$ , as well as matrices  $S = diag\{S_{11}, S_{22}, S_{33}\}$  and  $R = diag\{R_{11}, R_{22}, R_{33}\}$ , such that the following LMIs hold:

$$\begin{bmatrix} S_{11} + Q_{11} & R_{11} & 0 & 0\\ P_{11} & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & v_{x \max} & 0 \end{bmatrix} \le 0$$
(14)

$$\begin{bmatrix} S_{22} + Q_{22} & R_{22} & 0 & 0 \\ P_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & v_{y \max} & 0 \end{bmatrix} \le 0$$
(15)

$$\begin{bmatrix} S_{33} + Q_{33} & R_{33} & 0 & 0 \\ P_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & v_{\theta \max} & 0 \end{bmatrix} \le 0$$
(16)

Then, control input Equations (6) and (7) solve the problem of asymptotic stability. **Proof:** Define the Lyapunov function as

$$V(t) = V_1(t) + V_2(t) + V_3(t) = \frac{1}{2}e_1^T(t)Pe_1(t) + \frac{1}{2}e_2^T(t)Qe_2(t)$$
(17)

Here, we have the following:

$$V_1(t) = \frac{1}{2} e_{1x}^T(t) P_{11} e_{1x}(t) + \frac{1}{2} e_{2x}^T(t) Q_{11} e_{2x}(t)$$
(18)

$$V_2(t) = \frac{1}{2} e_{1y}^T(t) P_{22} e_{1y}(t) + \frac{1}{2} e_{2y}^T(t) Q_{22} e_{2y}(t)$$
(19)

$$V_3(t) = \frac{1}{2}e_{1\theta}^T(t)P_{33}e_{1\theta}(t) + \frac{1}{2}e_{2\theta}^T(t)Q_{33}e_{2\theta}(t)$$
(20)

The time derivative of V(t) along the trajectory of Equation (8) is given as follows:

$$\dot{V}(t) = e_1^T(t)P\dot{e}_1(t) + e_2^TQ\dot{e}_2(t)$$
(21)

Using  $x^{T}(t) = \begin{bmatrix} e_{2x}^{T}(t) & e_{1x}^{T}(t) & |\dot{x}_{2}^{x}(t)|^{T} & 1 \end{bmatrix}$  pre-multiply (14) and x(t) post-multiply (14), we obtain the following:

$$e_{1x}^{T}(t)P_{11}e_{2x}(t) + e_{2x}^{T}(t)S_{11}e_{2x}(t) + e_{2x}^{T}(t)R_{11}e_{1x}(t) + v_{x\max}|\dot{x}_{2}^{x}(t)| + e_{2x}^{T}(t)Q_{11}e_{2x}(t) \le 0$$
 (22)  
Here,  $S_{11} = Q_{11}K_{dx}$  and  $R_{11} = Q_{11}K_{px}$ . Therefore, we have the following:

$$\dot{V}_1(t) \le -v_{x\max} |\dot{x}_2^x(t)| - e_{2x}^T(t)Q_{11}e_{2x}(t)$$
(23)

Likewise, according to Equations (16) and (17), we have

$$\dot{V}_2(t) \le -v_{y\max} |\dot{x}_2^y(t)| - e_{2x}^T(t)Q_{22}e_{2x}(t)$$
 (24)

$$\dot{V}_3(t) \le -v_{\theta \max} \left| \dot{x}_2^{\theta}(t) \right| - e_{2x}^T(t) Q_{33} e_{2x}(t)$$
 (25)

From Equations (23), (24), and (25), we yield the following:

$$e_1^T(t)Pe_2(t) + e_2^T(t)QK_de_2(t) + e_2^T(t)QK_pe_1(t) \le -v_{\max}^T |\dot{x}_2(t)| - e_2^T(t)Qe_2(t)$$
(26)

And from Equation (26), we yield

$$\dot{V}(t) \le -v_{\max}^T |\dot{x}_2(t)| - e_2^T(t)Qe_2(t)$$
(27)

According to Equation (27), we know that  $\dot{V}(t) < 0$ , which further implies that  $\dot{V}(t) = 0$ with  $\dot{x}_2(t) = 0$  and  $e_2(t) = 0$ . Therefore, trajectory tracking error  $e_1(t)$  and velocity tracking error  $e_2(t)$  are asymptotically stable on the basis of the LaSalle principle.

**Theorem 3.2.** Considering asymptotically stable the error Equation (8), if we have symmetric positive matrices  $P = diag\{P_{11}, P_{22}, P_{33}\}$  and  $Q = diag\{Q_{11}, Q_{22}, Q_{33}\}$  such that the following LMIs hold:

$$\begin{aligned}
\text{Mustrices } I &= \operatorname{unag}\left[1_{11}, 1_{22}, 1_{33}\right] \text{ una } Q = \operatorname{unag}\left[Q_{11}, Q_{22}, Q_{33}\right] \text{ such unac} \\
\text{MIs hold:} \\
\begin{bmatrix}
-\frac{1}{2}Q_{11} & 0 & 0 \\
0 & -\frac{1}{2}P_{11} & 0 \\
0 & 0 & V_1(0) + v_{x\max} |x_2^x(0)| - v_{x\max}^2
\end{bmatrix} &\leq 0 \quad (28) \\
\begin{bmatrix}
-\frac{1}{2}Q_{22} & 0 & 0 \\
0 & -\frac{1}{2}P_{22} & 0 \\
0 & 0 & V_2(0) + v_{y\max} |x_2^y(0)| - v_{y\max}^2
\end{bmatrix} &\leq 0 \quad (29) \\
\begin{bmatrix}
-\frac{1}{2}Q_{33} & 0 & 0 \\
0 & -\frac{1}{2}P_{33} & 0 \\
0 & 0 & V(0) + v_{x\max} |x_2^{\theta}(0)| - v_2^2
\end{bmatrix} &\leq 0 \quad (30)
\end{aligned}$$

$$\begin{bmatrix} -\frac{1}{2}Q_{22} & 0 & 0\\ 0 & -\frac{1}{2}P_{22} & 0\\ 0 & 0 & V_2(0) + v_{y\max} |x_2^y(0)| - v_{y\max}^2 \end{bmatrix} \le 0$$
(29)

$$\begin{bmatrix} -\frac{1}{2}Q_{33} & 0 & 0 \\ 0 & -\frac{1}{2}P_{33} & 0 \\ 0 & 0 & V_3(0) + v_{\theta \max} \left| x_2^{\theta}(0) \right| - v_{\theta \max}^2 \end{bmatrix} \le 0$$
(30)

Then, the safety velocity constraints performance of Equation (13) can be guaranteed using the suitable controller in the form of Equations (6) and (7).

Furthermore, the controller parameter matrices are  $K_d = Q^{-1}S$  and  $K_p = Q^{-1}R$ . Moreover,

$$J^* = V(t_0) + v_{\max}^T |x_2(0)|$$
(31)

**Proof:** According to Equation (23), we have  $V_1(t) \leq -v_{x \max} |\dot{x}_2^x(t)|$ .

Integrating inequality  $\dot{V}_1(t) \leq -v_{x \max} |\dot{x}_2^x(t)|$  from 0 to t on both sides, it follows that

$$\int_{0}^{t} \dot{V}_{1}(t)dt \leq \int_{0}^{t} (-v_{x\max} |\dot{x}_{2}^{x}(t)|)dt$$
(32)

From this, we can obtain the following:

$$V_1(t) - V_1(0) \le -v_{x\max}(|x_2^x(t)| - |x_2^x(0)|)$$
(33)

Inequality (33) can be further rewritten as follows:

$$v_{x\max} |x_2^x(t)| \le V_1(0) + v_{x\max} |x_2^x(0)| - V_1(t)$$
(34)

Using  $y^{T}(t) = \begin{bmatrix} e_{2x}^{T}(t) & e_{1x}^{T}(t) & 1 \end{bmatrix}$  pre-multiply (28) and y(t) post-multiply (28), we have the following:

$$-\frac{1}{2}e_{2x}^{T}(t)Q_{11}e_{2x}(t) - \frac{1}{2}e_{1x}^{T}(t)P_{11}e_{1x}(t) + V_{1}(0) + v_{x\max}|x_{2}^{x}(0)| - v_{x\max}^{2} \le 0$$
(35)

According to Equations (18) and (35), it follows that

$$-V_1(t) + V_1(0) + v_{x\max} |x_2^x(0)| \le v_{x\max}^2$$
(36)

Combining Equations (34) and (36), we yield

$$|x_2^x(t)| \le v_{x\max} \tag{37}$$

Likewise, according to Equations (29) and (30), we have  $|x_2^y(t)| \leq v_{y \max}$  and  $|x_2^\theta(t)| \leq$  $v_{\theta \max}$ .

Furthermore, by Theorem 3.1, it follows that  $K_d = Q^{-1}S$  and  $K_p = Q^{-1}R$ . The cost performance upper bound satisfies the following

$$J = \int_0^\infty \left( e_2^T(t) Q e_2(t) \right) dt = \int_0^\infty \left[ e_2^T(t) Q e_2(t) + \dot{V}(t) - \dot{V}(t) \right] dt$$
  
$$\leq -v_{\max}^T(|x_2(t)| - |x_2(0)|) - \int_0^\infty \dot{V}(t) dt = V(t_0) + v_{\max}^T |x_2(0)|$$

Therefore, we have  $J^* = V(t_0) + v_{\max}^T |x_2(0)|$  as a cost performance upper bound of Equation (8).

**Remark 3.1.** A new velocity constraints control method is proposed, and the safety velocity tracking performance (13) can be obtained from a Lyapunov function (17). This approach (13) has the advantage of providing a simple way to restrict motion velocity for ODW.

**Remark 3.2.** According to the inequalities (14)-(16) and (28)-(30), the method of adjusting parameters is designed using LMIs, and it is convenient to obtain the controller matrices  $K_p$  and  $K_d$ .

4. Simulation Results. In this section, we verify our proposed safety velocity tracking control algorithm via the ODW tracking simulation. Here, the  $X_d(t)$  is described as follows:

$$x_d(t) = 5 (1 - e^{-0.1t})$$
  

$$y_d(t) = 5 (1 - e^{-0.1t})$$
  

$$\theta_d(t) = \frac{\pi}{4}$$

The physical parameters of the ODW used in the simulation are: M = 58kg; L = 0.4m;  $I_0 = 27.7$ kg · m<sup>2</sup>; the trainer load m = 60kg; center of gravity shift  $r_0 = 0.1$ m; and  $\beta = \frac{\pi}{4}$ rad. Suppose we have maximum motion velocity  $v_{\text{max}}^T = \begin{bmatrix} 0.25 \text{m/s} & 0.25 \text{m/s} & \frac{\pi}{6} \text{rad/s} \end{bmatrix}$ , initial motion velocity  $x_2^T(0) = \begin{bmatrix} 0 \text{m/s} & 0 \text{m/s} & 0 \text{rad/s} \end{bmatrix}$ , initial position x(0) = 1m, y(0) = 1m, and  $\theta(0) = 0$  rad. By solving the inequalities of Equations (14) and (28), Equations (15) and (29), and Equations (16) and (30), our simulation results are shown in Figures 3-8, described in further detail below.

Figures 3, 4, and 5 plot the trajectories of the x, y axes, and orientation angle, respectively. The ODW can complete trajectory tracking in limited time. Therefore, the tracking error state equation can realize asymptotic stability. The ODW can provide line tracking for rehabilitee training, as shown in Figure 6. The tracking velocity along the xand y axes and the orientation angle are shown in Figures 7 and 8, respectively. Here, we can observe that our proposed control method can guarantee the ODW's continuous motion within the given safety velocity.



FIGURE 3. Tracking performance of x position

FIGURE 4. Tracking performance of y position



FIGURE 5. Tracking performance of angle



FIGURE 7. Motion velocity



FIGURE 6. Path tracking of line



FIGURE 8. Angular velocity





FIGURE 10. Motion velocity

To verify the effectiveness of tracking control with safety velocity performance, we conducted comparative simulations with the normal tracking control without velocity constraints performance. Using the controller (6), (7) and Lyapunov function (17), the parameter matrices  $K_p = diag\{-20, -10, -10\}$  and  $K_d = diag\{-10, -2.5, -10\}$  are adjusted manually to realize the trajectory tracking with the same initial values. The simulation results are presented in Figures 9 and 10.

Figure 9 plots the path tracking performance of the walker. It is evident that the walker can realize asymptotic stability and path tracking. However, the motion velocity exceeds the user's endurance capacity in initial period time as shown in Figure 10. The user may be in danger because the actual motion velocity is far from the safety velocity. Therefore, the controller with safety velocity performance is important.

5. **Conclusions.** In this paper, we proposed a new safety velocity tracking control approach for an ODW. Using the common Lyapunov function, the obtained safety velocity controller can stabilize the ODW and realize velocity constraints performance. Simulation results for our new synthesis design to resolve safety velocity tracking issues demonstrated the effectiveness of our proposed method. Tracking results were consistent with a pre-programmed training path designed by a medical professional.

Acknowledgment. The authors would like to thank the reviewers for their constructive comments to improve the quality of this article. This study was supported by JSPS KAK-ENHI Grant Numbers 24300203, 23240088, and 22300197, and the Canon Foundation, as well as the program for Liaoning Excellent Talents at the University of China under Grant LJQ2014013 and the Liaoning Natural Science Foundation of China under Grant 2015020066.

## REFERENCES

- Y. L. Jiang, S. Y. Wang and K. Ishida, Control of an omnidirectional walking support walker by forearm pressures, *The 33rd Annual International Conference of the IEEE EMBS*, Boston, Massachusetts, USA, pp.7466-7469, 2011.
- [2] R. P. Tan, S. Y. Wang and Y. L Jiang, Adaptive control method for path-tracking control of an omnidirectional walker compensating for center-of-gravity shifts and load changes, *International Journal of Innovative Computing, Information and Control*, vol.7, no.7(B), pp.4423-4434, 2011.
- [3] S. J. Yoo and B. S. Park, Formation tracking control for a class of multiple mobile robots in the presence of unknown skidding and slipping, *IET Control Theory and Applications*, vol.7, no.5, pp.635-645, 2013.
- [4] Z. Y. Wang and D. B. Gu, Cooperative target tracking control of multiple robots, *IEEE Trans. Industrial Electronics*, vol.59, no.8, pp.3232-3239, 2012.
- [5] M. Y. Cui, X. J. Xie and Z. J. Wu, Dynamic modeling and tracking control of robot manipulators in random vibration environment, *IEEE Trans. Automatic Control*, vol.58, no.6, pp.1540-1545, 2013.
- [6] B. K. Sahu and B. Subudhi, Adaptive tracking control of an autonomous underwater vehicle, International Journal of Automation and Computing, vol.11, no.3, pp.299-307, 2014.
- [7] X. Wang and J. Zhao, Switched adaptive tracking control of robot manipulators with friction and changing loads, *International Journal of System Science*, vol.44, no.1, pp.1-12, 2013.
- [8] K. D. Sharma, A. Chatterjee and A. Rakshit, A PSO-Lyapunov hybrid stable adaptive fuzzy tracking control approach for vision-based robot navigation, *IEEE Trans. Instrumentation and Measurement*, vol.61, no.7, pp.1908-1914, 2012.
- [9] M. Y. Li, S. C. Tong and T. S. Li, Fuzzy adaptive dynamic surface control for a single-link flexiblejoint robot, *Nonlinear Dynamic*, vol.70, no.3, pp.2035-2048, 2012.
- [10] Y. X. Simon, Z. Anmin and G. F. Yuan, A bioinspired neurodynamics-based approach to tracking control of mobile robot, *IEEE Trans. Industrial Electronics*, vol.59, no.8, pp.3211-3220, 2012.
- [11] L. Lu and B. Yao, A performance oriented multi-loop constrained adaptive robust tracking control of one-degree-of-freedom mechanical systems: Theory and experiments, *Automatica*, vol.50, no.4, pp.1143-1150, 2014.
- [12] M. Harmouche, S. Laghrouche and Y. Chitour, Global tracking for underactuated ships with bounded feedback controllers, *International Journal of Control*, vol.87, no.10, pp.2035-2043, 2014.
- [13] S. I. Han and J. M. Lee, Output tracking error constrained robust positioning control for a nonsmooth nonlinear dynamic system, *IEEE Trans. Industrial Electronics*, vol.61, no.12, pp.6882-6891, 2014.
- [14] Z. Chen, B. Yao and Q. F. Wang, Accurate motion control of linear motors with adaptive robust compensation of nonlinear electromagnetic field effect, *IEEE/ASME Trans. Mechatronics*, vol.18, no.3, pp.1122-1129, 2013.
- [15] P. Sun, S. Y. Wang and H. Reza Karimi, Robust redundant input reliable tracking control for omnidirectional rehabilitative training walker, *Mathematical Problems in Engineering*, vol.2014, pp.1-10, 2014.
- [16] R. P. Tan, S. Y. Wang and Y. L. Jiang, Adaptive controller for omnidirectional walker: Improvement of dynamic model, *Proc. of the IEEE International Conference on Mechatronics and Automation*, Beijing, China, pp.325-330, 2011.