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OUTPUT FEEDBACK TRACKING CONTROL FOR OMNIDIRECTIONAL REHABILITATIVE TRAINING WALKER WITH RANDOM PARAMETERS

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Abstract. *This paper investigates an output feedback tracking control method for an omnidirectional rehabilitative training walker (ODW) with random parameters. A stochastic model is constructed to describe the motion of the ODW subject to random parameters uncertainty. A high-gain observer to estimate the unknown states and a state feedback controller with a suitable parameter estimator are used to make the tracking error system exponentially practically mean-square stable. We show that the mean square of the tracking error can be made arbitrarily small by choosing appropriate design parameters. Simulation results demonstrate the feasibility and effectiveness of the proposed method.*
Keywords: Random parameters, Output tracking control, High-gain observer, Omnidirectional walker

1. Introduction.

An omnidirectional rehabilitative training walker [1,2] is being developed to support people with walking impairments for both walking rehabilitation and walking support. This walker allows omnidirectional movement, which includes not only forward and backward motions, but also right and left motions, oblique motions, rotations, and combinations of these motions. The training programs are stored in the walker so that rehabilitation can be carried out accurately without the presence of physical therapists.

The trajectory tracking task has been widely studied in the past few years [3,4] as an important issue in robotic control. Among the available results, several effective control approaches have been developed to improve tracking performance, for example, feedback linearization tracking control [5], adaptive tracking control [6], and backstepping tracking control [7]. A limitation of these previously published results is that they require at least the measurement of velocity on the link side or motor side. However, as pointed out in [8], in robotic applications today, velocity sensors are frequently omitted because of their considerable production cost, and the size and the weight of the servo-drives. Moreover, in practical robotic systems, the velocity measurements obtained through tachometers are easily perturbed by noise. Therefore, to align with the economic and/or physical constraints, the ability to control robots without velocity measurements is of great importance. Several research studies and approaches related to the problem of trajectory tracking in controlling robot motion via a suitable observer have been addressed in recent years [9,10].

As we know, robot systems are often subject to random disturbances and random parameters uncertainty in a vibration environment, which significantly affects their performance in an uncertain manner. [11,12] proposed tracking control approaches for robot manipulators with external disturbances; [13] presented a stochastic Hamiltonian dynamic model and produced a state feedback controller using an adaptive backstepping technique. Note that all the above stochastic models only considered random noise that came from control input channels. However, there are many unknown parameters that can be found in moving robots such as the variable arm-length of a robot manipulator [13], or the center of gravity shift of a rehabilitative training walker [14]. Under random parameters circumstances, it is not trivial to design a controller to achieve accurate tracking with a fast and smooth response. Considering theoretical study and engineering applications simultaneously, there are many problems worth investigating.

In the present paper, we will investigate the ODW with random parameters, and propose a high-gain observer based scheme to address the output feedback tracking control of the ODW in the presence of parametric uncertainties.

Motivated by the above observations, the main contributions of this paper are as follows.

(i) Since the effect of random parameters is a major challenge for mechanical control systems, in the present paper, by changing the random parameters into random disturbances, we construct a reasonable stochastic model to describe the motion of an ODW subject to random parameters uncertainty.

(ii) As the velocities of ODW are not measured in practice, we propose a high-gain observer based scheme to address the output feedback tracking control of the ODW in the presence of parametric uncertainties such that the tracking error system is exponentially practically stable in mean square. The tracking error tends to an arbitrarily small neighborhood of zero by tuning design parameters, and so does its derivative.

(iii) In the simulation, random noises are reasonably introduced to the ODW. A stochastic ODW model is constructed and an output feedback tracking controller is figured out to demonstrate the efficiency of the scheme of this paper. Tracking a given trajectory, the simulation test results indicate that performance of the novel observer and developed output feedback tracking controller are successful.

The remainder of this paper is organized as follows. The stochastic ODW model with uncertainty of random parameters is formulated in Section 2. The design of observer and tracking controller are proposed in Section 3. Simulation results are presented in Section 4, and concluding remarks are presented in Section 5.

2. **Stochastic Model of the ODW with Random Parameters.**

An image of the ODW is presented in Figure 1. The coordinate settings and structure used to develop the tracking control for the ODW are shown in Figure 2.

The kinetics model is expressed as [15]

$$M_0 K\ddot{X}(t) + M_0\dot{K}\dot{X}(t) = B(\theta)u(t) \tag{1}$$

where

$$M_0 = \begin{bmatrix} M+m & 0 & 0 \\ 0 & M+m & 0 \\ 0 & 0 & I_0 + mr_0^2 \end{bmatrix}, \quad K = \begin{bmatrix} 1 & 0 & p \\ 0 & 1 & q \\ 0 & 0 & 1 \end{bmatrix}$$

$$B(\theta) = \begin{bmatrix} -\sin\theta_1 & \sin\theta_2 & \sin\theta_3 & -\sin\theta_4 \\ \cos\theta_1 & -\cos\theta_2 & \cos\theta_3 & \cos\theta_4 \\ \lambda_1 & -\lambda_2 & -\lambda_3 & \lambda_4 \end{bmatrix}, \quad \begin{array}{l} \lambda_1 = l_1\cos(\theta_1 - \varphi_1) \\ \lambda_2 = l_2\cos(\theta_2 - \varphi_2) \\ \lambda_3 = l_3\cos(\theta_3 - \varphi_3) \\ \lambda_4 = l_4\cos(\theta_4 - \varphi_4) \end{array}$$

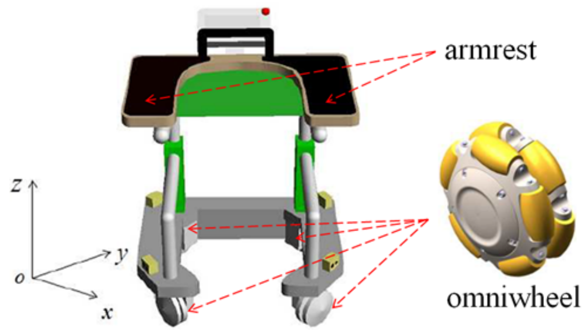

FIGURE 1. ODW and omniwheel

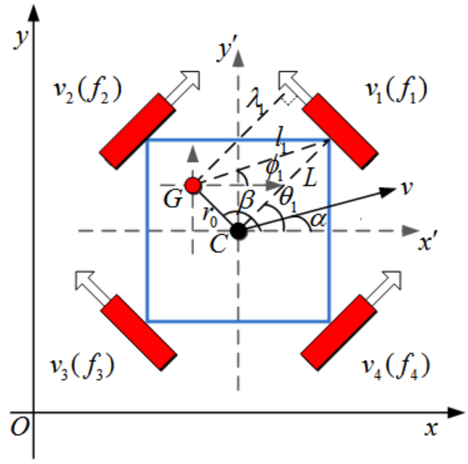

FIGURE 2. Structure of ODW

$$X(t)=\begin{bmatrix} x(t) \\ y(t) \\ \theta(t) \end{bmatrix},\ u(t)=\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix},\quad \begin{array}{l} p=\frac{1}{2}[(\lambda_1-\lambda_3)\sin\theta+(\lambda_2-\lambda_4)\cos\theta] \\ q=\frac{1}{2}[(\lambda_2-\lambda_4)\sin\theta-(\lambda_1-\lambda_3)\cos\theta] \end{array}$$

where M is the mass of the ODW, m is the user's equivalent mass; and I_0 is the inertia mass of the walker. mr_0^2 is the inertia mass caused by m. f_1, f_2, f_3 and f_4 are the input forces; r_0 and λ_1, λ_2, λ_3, λ_4 are the random parameters. θ is the angle between the x' axis and the position of the first omniwheel, $\theta=\theta_1$; then we have $\theta_2=\theta+\pi/2$, $\theta_3=\theta+\pi$, and $\theta_4=\theta+3\pi/2$. L is the distance from the center of the ODW to each omniwheel.

To obtain the stochastic model, random parameters r_0, λ_1, λ_2, λ_3, λ_4 are extracted from the differential Equation (1), and we have

$$\ddot{X}(t)=M_1^{-1}B^*(\theta)u(t)+M_1^{-1}N(\theta)\xi(t) \tag{2}$$

where

$$M_1=\begin{bmatrix} M+m & 0 & 0 \\ 0 & M+m & 0 \\ 0 & 0 & I_0 \end{bmatrix}$$

$$N(\theta)=\begin{bmatrix} -(M+m)\sin\theta & \dot{\theta}^2(M+m)\sin\theta & -\dot{\theta}^2(M+m)\cos\theta & -(M+m)\cos\theta & 0 \\ (M+m)\cos\theta & -\dot{\theta}^2(M+m)\cos\theta & -\dot{\theta}^2(M+m)\sin\theta & -(M+m)\sin\theta & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B^*(\theta)=\begin{bmatrix} -\sin\theta_1 & \sin\theta_2 & \sin\theta_3 & -\sin\theta_4 \\ \cos\theta_1 & -\cos\theta_2 & \cos\theta_3 & \cos\theta_4 \\ L & -L & -L & L \end{bmatrix}$$

$$\xi(t)=\begin{bmatrix} \ddot{\theta}(\lambda_1-L) \\ (\lambda_2-L) \\ (L-\lambda_3) \\ \ddot{\theta}(L-\lambda_4) \\ \left((\lambda_1-L)(f_1+f_3)-(\lambda_2-L)(f_2+f_4)-\ddot{\theta}mr_0^2\right) \end{bmatrix}$$

The purpose of this study is to solve the tracking control problem under the uncertain parameters r_0, λ_1, λ_2, λ_3, λ_4. From the physical significance, the parameters uncertainty $\xi(t)$ exhibits the properties of random disturbances that are caused by white noise as τ^e in [16]. According to Section 5.5 in [17], by replacing $\xi(t)$ with "$\frac{d\bar{B}}{dt}$", the Stratonovich stochastic differential equation (SDE) of (1) can be obtained

$$d\dot{X}(t)=M_1^{-1}B^*(\theta)u(t)dt+M_1^{-1}N(\theta)\circ d\bar{B} \tag{3}$$

where $\bar{B}$ is a 5-dimensional independent Wiener process.

For the convenience of narration, we denote $N(\theta) = [\alpha_{ij}]_{i\times j}$ $(i = 1, 2, 3; j = 1, 2, \cdots, 5)$. The Wong-Zakai correction term [18] equals $\frac{1}{2}\left[\sum_{j=1}^{6} \alpha_{1j} \cdot \frac{\partial \alpha_{1j}}{\partial \dot{x}(t)} \ \sum_{j=1}^{6} \alpha_{2j} \cdot \frac{\partial \alpha_{2j}}{\partial \dot{y}(t)} \ \sum_{j=1}^{6} \alpha_{3j} \cdot \frac{\partial \alpha_{3j}}{\partial \dot{\theta}(t)}\right]^T = [0 \quad 0 \quad 0]^T$, and the *Itô* SDE of the ODW is described by

$$d\dot{X}(t) = M_1^{-1} B^*(\theta) u(t) dt + M_1^{-1} N(\theta) d\bar{B} \tag{4}$$

Suppose that the power spectral density of white noise $\xi(t)$ equals $\frac{\Sigma}{2\pi}$, i.e., $d\bar{B} = \Sigma dw$ holds. By defining $v(t)=\dot{X}(t)$ and viewing $\left(X^T(t), v^T(t)\right)^T$ as a state, the stochastic model with random parameters uncertainty can be described as

$$\begin{aligned} dX(t) &= v(t)dt \\ dv(t) &= M_1^{-1} B^*(\theta) u(t) dt + M_1^{-1} N(\theta) \Sigma dw \end{aligned} \tag{5}$$

Assumption 2.1. *From the physical significance, the angular velocity $\dot{\theta}$ is bounded, so there exists a constant h such that*

$$2\dot{\theta}^4 (M + m)^2 + 2(M + m)^2 + 1 \le h \tag{6}$$

3. **The Design of the Observer and Tracking Controller.** To estimate the unmeasurable state v, a novel observer is designed as

$$\begin{aligned} d\bar{v}(t) &= \left[M_1^{-1} B^*(\theta) u(t) - k\hat{v}\right] dt \\ \hat{v}(t) &= \bar{v}(t) + kX(t) \end{aligned} \tag{7}$$

where $\hat{v}(t) \in R^n$ is the estimate of $v(t)$, $\bar{v}(t)$ is the internal state, and $k > 0$ is the gain to be designed later. From (5), (7), and observer errors $\tilde{v}(t) = v(t) - \hat{v}(t) = [\tilde{v}_1, \tilde{v}_2, \tilde{v}_3]^T$, we have

$$d\hat{v}(t) = \left[M_1^{-1} B^*(\theta) u(t) + k\tilde{v}(t)\right] dt \tag{8}$$

$$d\tilde{v}(t) = -k\tilde{v}(t)dt + M_1^{-1} N(\theta) \Sigma dw \tag{9}$$

By giving a reference signal $X_d(t) \in C^2(R^n)$, the output tracking error is defined as $e_1(t) = X(t) - X_d(t)$. Performing the transform $e_2(t) = \hat{v}(t) - \dot{X}_d(t) - \alpha e_1(t)$, the error system gives

$$\begin{aligned} de_1(t) &= [e_2(t) + \alpha e_1(t) + \tilde{v}(t)]dt \\ de_2(t) &= \left[M_1^{-1} B^*(\theta) u(t) + (k - \alpha)\tilde{v}(t) - \ddot{X}_d - \alpha e_2(t) - \alpha^2 e_1(t)\right] dt \end{aligned} \tag{10}$$

where the parameter α will be designed later.

Theorem 3.1. *For the stochastic model of the ODW with random parameters uncertainty (5), given the reference signal $X_d(t) \in C^2(R^n)$, under the observer (7) and the controller:*

$$u(t) = B^T(\theta)\left(B(\theta)B^T(\theta)\right)^{-1} M_1 \left(\ddot{X}_d + \left(\alpha - c_2 - \frac{1}{4\varepsilon_1^4} - \frac{3}{4}\varepsilon_3^{\frac{4}{3}}(k - \alpha)^{\frac{4}{3}}\right) e_2 + \alpha^2 e_1\right) \tag{11}$$

where $\varepsilon_i > 0$ $(i = 1, 2, 3, 4)$ and $c_j > 0$ $(j = 1, 2, 3)$ are design parameters, $\alpha = -c_2 - \frac{3}{4}\varepsilon_1^{\frac{4}{3}} - \frac{3}{4}\varepsilon_2^{\frac{4}{3}}$, and $k = -c_1 + \frac{9}{4}\varepsilon_4^{\frac{4}{3}} + \frac{3}{4\varepsilon_2^4} + \frac{3}{4\varepsilon_3^4}$. The closed-loop system $\left(\tilde{v}^T, e_1^T, e_2^T\right)^T$ has a unique solution on $[t_0, \infty)$ and is exponentially stable in mean square for initial values $\tilde{v}(t_0) > 0$, $e_1(t_0) \in R^n$ and $e_2(t_0) \in R^n$. The tracking errors e_1 and $\dot{e}_1$ satisfy

$$\lim_{t\to\infty} E\left|e_1\right| \le \left(\frac{4d}{c}\right)^{\frac{1}{4}} \tag{12}$$

$$\lim_{t\to\infty} E\left|\dot{e}_1\right| \le 2\left(1 + c_1^2\right)\left(\frac{4d}{c}\right)^{\frac{1}{4}} \tag{13}$$

Moreover, the right-hand side of (12) and (13) can be made arbitrarily small by choosing appropriate design parameters.

Proof: We define the Lyaounov function

$$V = \frac{1}{4}\left(\tilde{v}^T\tilde{v}\right)^2 + \frac{1}{4}\left(e_1^Te_1\right)^2 + \frac{1}{4}\left(e_2^Te_2\right)^2 \tag{14}$$

The infinites generator of V along the system $\left(\tilde{v}^T, e_1^T, e_2^T\right)^T$ satisfies

$$\begin{aligned} LV \;=\; & \tilde{v}^T\tilde{v}\tilde{v}^T(-k\tilde{v}) + e_1^Te_1e_1^T(e_2 + \alpha e_1 + \tilde{v}) + e_2^Te_2e_2^T\Big(M_1^{-1}B^*(\theta)u(t) + (k-\alpha)\tilde{v}(t) \\ & -\ddot{X}_d - \alpha e_2(t) - \alpha^2 e_1(t)\Big) + \frac{1}{2}Tr\left\{\sum\nolimits^T N^T(\theta)M_1^{-1}\left(2\tilde{v}\tilde{v}^T + \tilde{v}^T\tilde{v}I\right)M_1^{-1}N(\theta)\sum\right\} \end{aligned} \tag{15}$$

where I denotes the identity matrix with an appropriate dimension. Using Young's inequality, we have

$$e_1^Te_1e_1^Te_2 \le \frac{3}{4}\varepsilon_1^{\frac{4}{3}}|e_1|^4 + \frac{1}{4\varepsilon_1^4}|e_2|^4 \tag{16}$$

$$e_1^Te_1e_1^T\tilde{v} \le \frac{3}{4}\varepsilon_2^{\frac{4}{3}}|e_1|^4 + \frac{1}{4\varepsilon_2^4}|\tilde{v}|^4 \tag{17}$$

$$(k-\alpha)e_2^Te_2e_2^T\tilde{v} \le \frac{3}{4}\varepsilon_3^{\frac{4}{3}}(k-\alpha)^{\frac{4}{3}}|e_2|^4 + \frac{1}{4\varepsilon_3^4}|\tilde{v}|^4 \tag{18}$$

Further, according to the definition of the Frobenius norm, the norm compatibility, (6), and using Young's inequality, we have

$$\begin{aligned} & \frac{1}{2}Tr\left\{\sum\nolimits^T N^T(\theta)M_1^{-1}\left(2\tilde{v}\tilde{v}^T + \tilde{v}^T\tilde{v}I\right)M_1^{-1}N(\theta)\sum\right\} \\ \le\; & \frac{3}{2}(\tilde{v}^T\tilde{v})\left\|M_1^{-1}\right\|_F^2\|N(\theta)\|_F^2\left\|\sum\right\|_F^2 \\ \le\; & \frac{3}{2}(\tilde{v}^T\tilde{v})\left\|M_1^{-1}\right\|_F^2\left[2\dot{\theta}^4(M+m)^2 + 2(M+m)^2 + 1\right]\left\|\sum\right\|_F^2 \\ \le\; & \frac{3}{2}\left(\tilde{v}^T\tilde{v}\right)h \le \frac{9\varepsilon_4^2}{4}\left(\tilde{v}^T\tilde{v}\right)^2 + \frac{1}{2\varepsilon_4^2}h^2 \end{aligned} \tag{19}$$

Substituting (11), (14) and (16) through (19) to (15), we have

$$LV \le -c_1|\tilde{v}|^4 - c_2|e_1|^2 - c_3|e_2|^2 + d \le -cV + d \tag{20}$$

where $c = \min\{c_1, c_2, c_3\}$, $d = \frac{1}{2\varepsilon_4^2}h^2$.

From the physical significance of the inertia matrix M_1, i.e., it is symmetric and positive definite, then it is smooth, which implies that the local Lipschitz condition holds, and does so in the functions $u(t)$ and $B^*(\theta)$. Therefore, the closed-loop system $\left(\tilde{v}^T, e_1^T, e_2^T\right)^T$ satisfies the local Lipschitz condition. From (14) and (20), following the same line as the proof of Lemma 1 in [16], there exists a unique strong solution to the closed-loop system $\left(\tilde{v}^T, e_1^T, e_2^T\right)^T$ on $[t_0, \infty)$ for initial values $\tilde{v}(t_0) > 0$, $e_1(t_0) \in R^n$ and $e_2(t_0) \in R^n$, and the closed-loop system $\left(\tilde{v}^T, e_1^T, e_2^T\right)^T$ is exponentially practically mean-square stable.

Moreover, using $e^{ct} > 0$ to multiply the inequality (20), we find

$$L\left(e^{ct}V(x,t)\right) = e^{ct}\left(LV(x,t) + cV(x,t)\right) \le e^{ct}d_c \tag{21}$$

Hence, by Lemma 3.3.1 in [17] integrating (21) from t_0 to t, we have

$$E\left(e^{ct}V(x,t)\right) \le e^{ct_0}V(x_0,t_0) + E\int_{t_0}^{t} e^{cs}d_c \cdot ds \quad \forall t \ge t_0 \tag{22}$$

From (14), we can deduce that

$$\begin{aligned} E\left|e_1\right|^2 \le\ & 2e^{\frac{1}{2}c(t_0-t)}\left(\frac{1}{4}\left(e_1^T(t_0)e_1(t_0)\right)^2+\frac{1}{4}\left(e_2^T(t_0)e_2(t_0)\right)^2+\frac{1}{2\gamma_1}h_1(t_0)\tilde{\Delta}^2(t_0)\right.\\ &\left.+\frac{1}{2\gamma_2}\tilde{h}^2(t_0)+\frac{1}{2\gamma_3}\tilde{\rho}_i^2(t_0)\right)^{\frac{1}{2}}+\left(\frac{4d_c}{c}\right)^{\frac{1}{2}} \end{aligned} \tag{23}$$

$$\begin{aligned} E\left|e_2\right|^2 \le\ & 2e^{\frac{1}{2}c(t_0-t)}\left(\frac{1}{4}\left(e_1^T(t_0)e_1(t_0)\right)^2+\frac{1}{4}\left(e_2^T(t_0)e_2(t_0)\right)^2+\frac{1}{2\gamma_1}h_1(t_0)\tilde{\Delta}^2(t_0)\right.\\ &\left.+\frac{1}{2\gamma_2}\tilde{h}^2(t_0)+\frac{1}{2\gamma_3}\tilde{\rho}_i^2(t_0)\right)^{\frac{1}{2}}+\left(\frac{4d_c}{c}\right)^{\frac{1}{2}} \end{aligned} \tag{24}$$

Using (23) and (24), and in view of $\left|\dot{e}_1\right|^2 = \left(\left|e_2\right| + c_1\left|e_1\right|\right)^2 \le 2(1+c_1^2)\left(\left|e_2\right|^2+\left|e_1\right|^2\right)$, it follows that (12) and (13) hold. Noting that $c = \min\{c_1, c_2, c_3\}$ and $d = \frac{1}{2\varepsilon_4^2}h^2$, the right-hand side of (12) and (13) can be made small enough by choosing ε_4 and c, since the parameters c_1, c_2 and c_3 are independent.

Remark 3.1. *According to Markov's inequality, we have* $E\left|e_1(t)\right|^2 < \infty$ *and* $E\left|e_2(t)\right|^2 < \infty$*, for any* $\delta_i > 0$ $(i = 1, 2)$ *such that*

$$P\{|e_1(t)| > \delta_1\} \le \frac{1}{\delta_1^2}E\left|e_1(t)\right|^2 \tag{25}$$

$$P\{|\dot{e}_1(t)| > \delta_2\} \le \frac{1}{\delta_2^2}E\left|\dot{e}_1(t)\right|^2 \tag{26}$$

By substituting (12), (13) into (25), (26) respectively, we can obtain

$$\lim_{t\to\infty} P\{|e_1(t)| > \delta_1\} \le \frac{1}{\delta_1^2}\left(\frac{4d_c}{c}\right)^{\frac{1}{2}} \tag{27}$$

$$\lim_{t\to\infty} P\{|\dot{e}_1(t)| > \delta_2\} \le \frac{2}{\delta_2^2}(1+c_1^2)\left(\frac{4d_c}{c}\right)^{\frac{1}{2}} \tag{28}$$

The right-hand side of (27) and (28) can be made small enough by tuning parameters c_1, c_2, c_3, and ε_4, which imply that asymptotical tracking can be achieved in all probability.

4. **Simulation Results.** In this section, the proposed output feedback tracking control algorithm is verified by a simulation of the ODW with random parameters uncertainty.

Here, to verify the tracking performance of the proposed method rigorously, we assume that the walker follows an elliptical path and that the random parameters of center of gravity shift $r_0 = 0.16(1+\sin t)$m, $\lambda_1 = L - r_0 \sin t$m, $\lambda_2 = L + r_0 \cos t$m, $\lambda_3 = L + r_0 \sin t$m, and $\lambda_4 = L - r_0 \cos t$m. The physical parameters of the ODW used in the simulation are $M = 58$kg, $m = 80$kg, $L = 0.4$m, $I_0 = 27.7\text{kg.m}^2$ and the design parameters are $c_1 = 1.5$, $c_2 = c_3 = 3$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 1$. The path X_d is described by

$$\begin{aligned} x_d &= 20\cos(0.1t) \\ y_d &= 10\sin(0.1t) \\ \theta_d &= \frac{\pi}{4} \end{aligned}$$

The simulation results are given in the following.

Figures 3, 4, 5 and 6 plot the tracking performance of the ODW for the x position, y position, orientation angle, and the path tracking of the ellipse, respectively. We can see that the closed-loop system can realize exponentially mean-square stable and that the ODW can realize trajectory tracking with the controller (11). It is evident that the novel stochastic model of the ODW (5) is an effective method to deal with random parameters. Observer errors are given in Figure 7. It is seen that the designed observer (7) functions

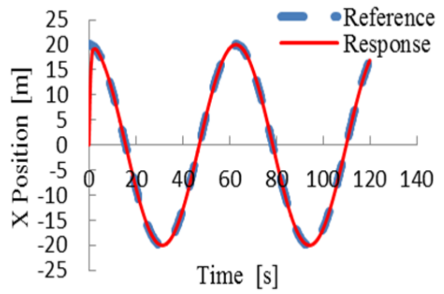

Figure 3. Tracking performance of x position

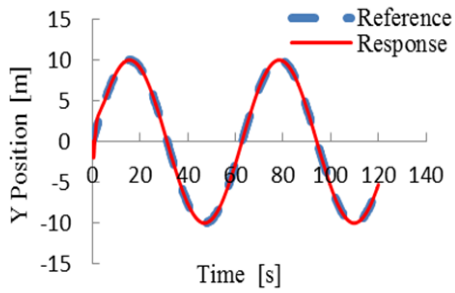

Figure 4. Tracking performance of y position

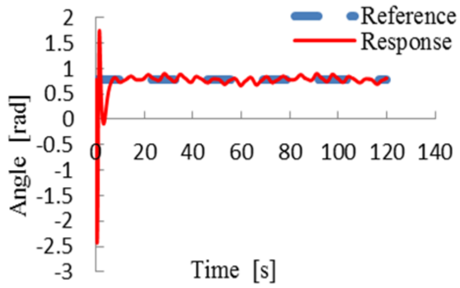

Figure 5. Tracking performance of angle

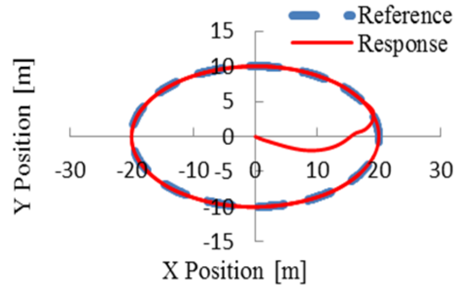

Figure 6. Path tracking of ellipse

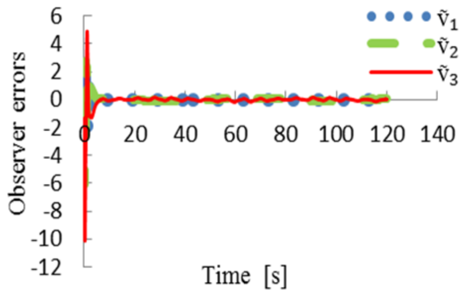

Figure 7. Observer errors

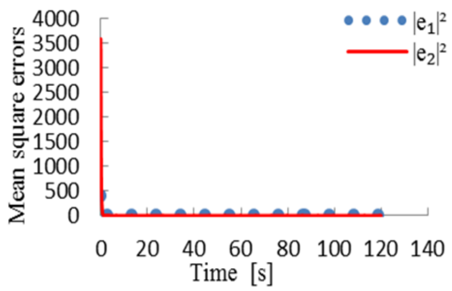

Figure 8. Mean square errors

well in the output feedback tracking controller (11) proposed. Figure 8 shows that the mean square of the errors can be made arbitrarily small by choosing appropriate design parameters.

5. **Conclusions.** In this paper, we propose a high-gain observer-based scheme to address the output feedback tracking control of an ODW in the presence of parametric uncertainties. To accomplish this, we first construct a reasonable stochastic model to describe the motion of an ODW subject to random parameters uncertainty. Then, we design a high-gain observer to estimate the unknown states and a state feedback controller with suitable parameter estimators to achieve trajectory tracking. The stability of the system is analyzed and established using a Lyapunov approach. Finally we demonstrate the effectiveness of our proposed controllers with simulation studies.

There are other problems under current investigation such as tracking control with incomplete measurements, bounded control force aiming to control the ODW's trajectory, and their application to other mechanical systems.

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