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FAST DISTRIBUTED CONSENSUS BASED MAXIMUM LIKELIHOOD ESTIMATION IN MULTIPLE VIEW LOCALIZATION

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Abstract. *In this paper, by using scalability property of projective space, we provide a fast consensus based maximum likelihood estimation in multi view object localization. We bring a proof that our proposed approach is converged and simulation results show that our proposed approach is faster in comparison with the traditional approach.*
Keywords: Maximum likelihood estimation, Distributed data fusion, Consensus algorithm, Homography

1. **Introduction.** With increasing interest in wireless sensor networks, the use of the multiple-view structure in the object tracking applications is increased [1, 2, 3, 4]. In these systems, data of a scene from all the views should be illustrated in the same coordination, namely global coordination. Relations between cameras' coordination and any arbitrary global coordination are described by homography which is a popular nonlinear transformation (in Euclidean geometry) in multi-view schemes. The use of homography implies that the cameras' data are represented in the projective geometry rather than Euclidean geometry (or any other geometries) [5]. The most important property of the projective geometry is that data in this form are unsensitive to scale. Though the relation between each camera coordination and global coordination in Euclidean geometry is nonlinear, this relation in the projective geometry has a linear form. With this linear model a maximum likelihood estimation is a good choice to estimate the objects' coordination in multi-view tracking systems.

Distributed data fusion is another important issue in the sensor networks applications [6, 7] which increases the system's ability in dealing with the failure in any part of the network. Consensus algorithm is an efficient solution in distributed approaches in many applications such as beam forming [8], spectral sensing in cognitive radio [9], target tracking [10, 11, 12] and adaptive filters [13]. In recent years, due to the improvements and the extension ideas in sensor and multi-agent networks, consensus algorithm is also used in data fusion [6, 14, 15]. In this procedure each node communicates only with its neighbors and after several iterations, nodes reach the consensus in the whole network. Actually, this consensus value (values) can play the role of an auxiliary variable or a cost function in the network and help us reach the final purpose [16]. Consensus algorithm is organized in Euclidean geometry and all iterations and its convergence criterion are described in this geometry [17, 18]. In [19] a new criterion for convergence in the Riemannian manifold is introduced. The projective geometry is a special type of Grassmann geometry [20] and Grassmann geometry itself is a kind of Riemannian geometry. However, the convergence of Riemannian consensus criterion is not guaranteed in every situation. Therefore, in this work we do not use Riemannian manifold criterion for consensus. Instead, we propose a new convergence criterion in the projective geometry.

In this paper, we propose a consensus based MLE for the object localization in the projective geometry. For this purpose, we apply a modification in the Euclidean consensus criteria and use the new criteria for consensus in the projective geometry. This paper is organized as follows. In the next section, a short explanation of the consensus procedure comes and the description of the distributed MLE algorithm which is presented in [6] is developed. The problem statement and our proposed procedure are introduced in Section 3. Finally, in Section 4, with the numerical results we show our proposed scheme performance with respect to the traditional consensus algorithms. Then the paper is terminated by a conclusion.

2. Consensus Procedure.

Consider a connected sensor network modeled by a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with N edges, where $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ is the set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges so that $(v_i, v_j) \in \mathcal{E}$ if there is an edge between the i^{th} and the j^{th} nodes and $\mathcal{A}$ is the digraph adjacency matrix. By assuming an arbitrary object on the plane as $\mathbf{p} = (x, y, z) \in \mathbb{P}^2$ [1] in homogeneous coordination (equal to $(x/z, y/z) \in \mathbb{R}^2$), each camera registers this object as $\tilde{\mathbf{p}}_i = (\tilde{x}_i, \tilde{y}_i, \tilde{z}_i) \in \mathbb{P}^2$, $i \in \mathcal{V}$ in its local coordination. Projective geometry $\mathbb{P}^N$ is equivalent to $\mathcal{G}_{N,1}$ [2], so the object location can be represented on the Riemannian Geometry. If $\mathbf{H}_i \in \mathbb{R}^{3\times3}$ denotes a homography between reference and the i^{th} camera coordination[3], then relation between the object coordinate in each camera ($\tilde{\mathbf{p}}_i$) and the reference coordinate ($\mathbf{p}_i$) is described as $\mathbf{p}_i = \mathbf{H}_i\tilde{\mathbf{p}}_i$ [5]. Based on the proposed approach in [6] for distributed consensus based MLE, Algorithm 1 shows the distributed MLE for object tracking in the plane and obtains $\mathbf{p}_{ML}$, where d_{ij} are the entry of the consensus matrix and $\mathbf{p}_{ML,i}$ is the MLE of position of the object in the i^{th} node.

Algorithm 1 Distributed MLE for Object Localization

for $i = 1 \to N$ **do**
  $\mathbf{S}_i(0) = \mathbf{H}_i^T \mathbf{\Sigma}_i^{-1} \mathbf{H}_i$
  $\mathbf{q}_i(0) = \mathbf{H}_i^T \mathbf{\Sigma}_i^{-1} \tilde{\mathbf{p}}_i(0)$
end for
while Reaching to consensus in $\mathbf{p}_{ML,i}(k)$s **do**
  for $i = 1 \to N$ **do**
    $\mathbf{S}_i(k) = \sum_{j=1}^{N} d_{ij}\mathbf{S}_j(k-1)$
    $\mathbf{q}_i(k) = \sum_{j=1}^{N} d_{ij}\mathbf{q}_j(k-1)$
    $\mathbf{p}_{ML,i}(k) = \mathbf{S}_i^{-1}(k)\mathbf{q}_i(k)$
  end for
end while

3. The Proposed Approach.

In this section, we introduce an approach to increase the speed of the consensus convergence. We propose a modified Euclidian consensus on Projective geometry. Our algorithm uses this property of homogenous geometry that (x, y, z) and $(\alpha x, \alpha y, \alpha z)$, where α is a nonzero coefficient, show the same point in $\mathbb{R}^2$ geometry (equal to $(x/z, y/z)$). As mentioned in the introduction, the consensus algorithm and convergence to consensus in the network are based in the Euclidean geometry, while our problem is stated in the projective geometry. Therefore, because of "up to scale" property of projective geometry, there are infinity options to represent location of object in the projective geometry. For example, suppose the location of object in the plane in the projective space is (x, y, z). In this situation, there is no difference whether the consensus algorithm reaches to (x, y, z) or $(\alpha x, \alpha y, \alpha z)$, as both of these points show the

[1] Projective geometry
[2] Grassmann Geometry
[3] It depends on calibration matrix and the position of each camera [5].

same point in the Euclidean geometry. It is worthwhile noting that location of object in the Euclidean is our favorite and representation in projective geometry is only used for mathematical computation.

Based on the above mentioned point, let an object localization problem with $\mathbf{p}_C$ be as the final consensus point. In each iteration, the consensus algorithm induces to each node's value $\mathbf{p}_i(k)$ tended to $\mathbf{p}_C$ ($\mathbf{p}_i(k) \to \mathbf{p}_C$, $i \in \{1, 2, \ldots, N\}$). In this work, as data represented in the homogenous coordinates, at each moment, tending $\mathbf{p}_i(k)$ to any multiplied value of $\mathbf{p}_C$ causes the same point in $\mathbb{R}^2$. Moreover, tending any multiplied value of $\mathbf{p}_i(k)$ to any multiplied value of $\mathbf{p}_C(k)$ has the same results. So, in our approach, the consensus procedure can be summarized in an attempt to establish a relationship $\alpha_i(k)\mathbf{p}_i(k) \to \gamma_i(k)\mathbf{p}_C$, or $\frac{\alpha_i(k)}{\gamma_i(k)}\mathbf{p}_i(k) \to \mathbf{p}_C$. Based on what was mentioned in the above, we can compute coefficients $\beta_i(k) = \alpha_i(k)/\gamma_i(k)$, $i \in \{1, 2, \ldots, N\}$, so that algorithm converges to the final values faster. For better imagination, consider a problem in $\mathbb{P}^1$ geometry by a set of two tuple vectors as $\mathbf{p}_i(k)$, $i \in \{1, 2, 3\}$ and consensus matrix initialized below:

$$\mathbf{p}_1(0) = \begin{bmatrix} 6 \\ 7 \end{bmatrix}, \quad \mathbf{p}_2(0) = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \quad \mathbf{p}_3(0) = \begin{bmatrix} 4 \\ 13 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \end{bmatrix}$$

According to this configuration, the final consensus vector equals $\mathbf{p}_C = \begin{bmatrix} 4 & 8 \end{bmatrix}^T$. Figure 1 shows the iterative changes in $\mathbf{p}_i(k)$ for nodes 2 and 3 under the consensus algorithm. As shown in Figure 1 at first iteration $0.8\mathbf{p}_3(1) \approx \mathbf{p}_C$ and in second iteration $0.48\mathbf{p}_2(2) \approx 0.42\mathbf{p}_C$. In other words, $\mathbf{p}_3$ reaches the final consensus value after first iteration and $\mathbf{p}_2$ reaches the final consensus after two iterations. All of the lying points on solid line are equal in $\mathbb{R}$ geometry and only have a difference in an scale factor. More consideration on Figure 1 shows that initial value of $\mathbf{p}_2$ is equal to $\mathbf{p}_C$. So, $\mathbf{p}_2$ does not need to be changed in iterative consensus algorithm and it can be introduced as the final consensus value. Therefore, we can propose a new stop condition in 5$^{\text{th}}$ row of Algorithm 1 and we consider reaching consensus for $\beta_i(k)\mathbf{p}_{ML,i}(k)$ instead of $\mathbf{p}_{ML,i}(k)$.

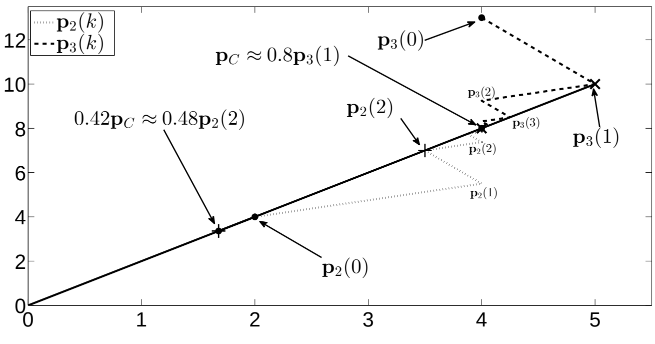


FIGURE 1. Up to scale property in $\mathbb{P}^1$. $\mathbf{p}_3$ gets consensus, almost, at $k = 1$ by setting $\alpha_3(1) = 0.8$ and $\gamma_3(1) = 1$. Also $\mathbf{p}_2$ gets consensus, approximately, at $k = 2$ by setting $\alpha_2(2) = 0.48$ and $\gamma_2(2) = 0.42$.

3.1. **Obtaining $\boldsymbol{\beta}$.** In order to obtain β, two local error functions are considered as follows:

$$E_i(k) = \sum_{j \in N_i} \|\beta_i(k)\mathbf{p}_i(k) - \beta_j(k)\mathbf{p}_j(k)\|^2 \tag{1}$$

$$\Delta_i(k) = \|\beta_i(k)\mathbf{p}_i(k) - \hat{\mathbf{p}}_{C,i}(k)\|^2 \tag{2}$$

$$\varphi_i(k) = E_i(k) + \Delta_i(k) \tag{3}$$

where N_i is the set of i^{th} node's neighbors.

In (2) $\hat{\mathbf{p}}_{C,i}(k)$ is an approximation of $\mathbf{p}_C$ in the i^{th} node. An estimation value for $\hat{\mathbf{p}}_{C,i}(k)$ is demonstrated in the next section.

Now, we calculate $\partial\varphi_i(k)/\partial\beta_i(k) = 0$ to obtain proportion $\beta_i(k)$. After some calculations, we have:

$$\beta_i(k) = \frac{\frac{1}{|N_i|}\sum_{j\in N_i}\beta_j(k)\mathbf{p}_i^T(k)\mathbf{p}_j(k) + \mathbf{p}_i^T(k)\hat{\mathbf{p}}_{C,i}}{2\mathbf{p}_i^T(k)\hat{\mathbf{p}}_{C,i}} \tag{4}$$

The first term in the numerator of (4) is induced by E_i and the second term is induced by Δ_i. By using (4) in each iteration we can obtain appropriate coefficients to reach the consensus.

3.2. **Approximation of $\mathbf{p}_C$.** To approximate $\mathbf{p}_C$, different strategies such as local averaging of each node and local weighted averaging of each node (based on the consensus matrix's rows values) can be used. Simulation results verified the performance of these choices. In this paper, we use the latter. So, the estimated value is obtained as below:

$$\hat{\mathbf{p}}_{C,i}(k) = \sum_{j=1}^{N} d_{ij}\mathbf{p}_j(k) \tag{5}$$

Obviously, when k increases, the above equation tends to $\mathbf{p}_C$.

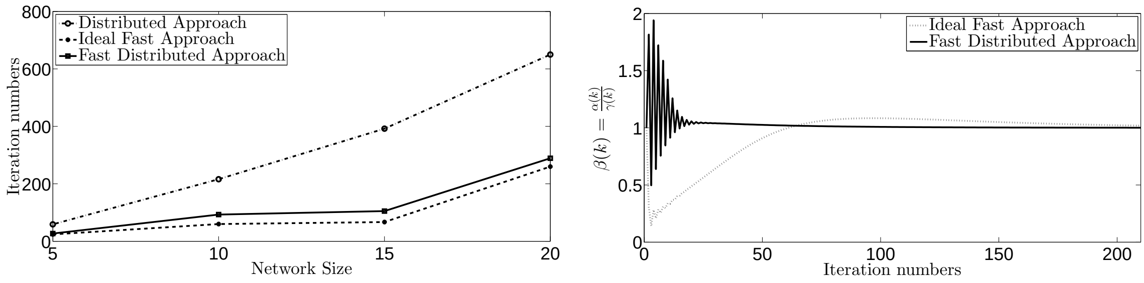
3.3. **Convergence analysis.** The proposed algorithm in our proposed approach does not have any stability and convergence issue. Since we do not modify traditional iterative consensus algorithm, it can be resulted that:

$$\lim_{k\to+\infty}\hat{\mathbf{p}}_{C,i}(k) = \lim_{k\to+\infty}\mathbf{p}_i(k) = \mathbf{p}_C \tag{6}$$

In the convergence condition, with respect to (1), (2) and (3), it is obvious that the only condition which can minimize (3) is that all values of $\beta_i(k)$ are equal to one. Therefore, stop condition in our proposed approach will be identical to Algorithm 1 and algorithm reaches to consensus, definitely. In the next section, simulation results of the convergence procedure would be shown.

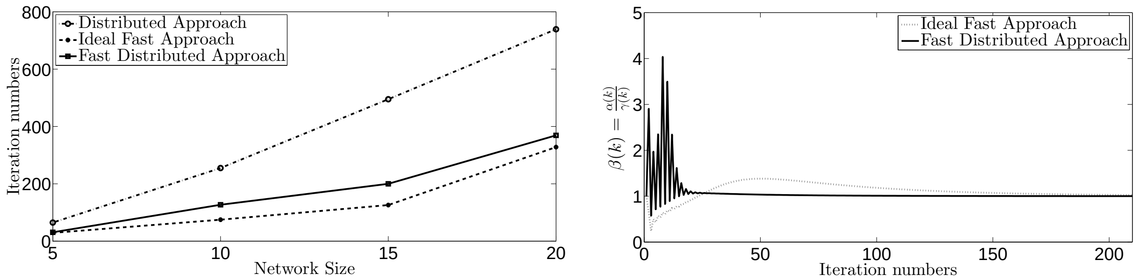
4. **Experimental Results.** In this section, we present simulation results of our proposed approach. For this purpose, we construct a static random connected network. Also, we select maximum-degree weighted matrix [6] as the consensus matrix. We use random 3×3 matrices for simulating calibration matrix of cameras and add independent white Gaussian noise to the measurements. The results are obtained by averaging over 100 epoches by different random initial values.

Figure 2(a) and Figure 3(a) show the iteration numbers which needed to reach consensus for $\mathbf{S}$ and $\mathbf{q}$ in different sizes of camera networks. As seen in these figures, in distributed approach, number of iterations which is needed for convergency increases exponentially, but our proposed fast distributed approach and ideal fast approach reach to consensus faster than distributed ones and also their number of iterations increases linearly. The ideal fast approach uses real value of the final consensus vector ($\mathbf{p}_C$). Therefore, it has the best results, but it cannot be realized. As shown in the figures, our proposed method (fast distributed approach) has significant improvement with respect to the standard consensus algorithm. Finally, in Figure 2(b) and Figure 3(b), the ratio of α and γ in the 19^{th} node for a 20 cameras network is shown. Our approach has a fluctuational behavior in the first iterations, as we used approximated function instead of $\mathbf{p}_C$; however, by passing the time and tending $\hat{\mathbf{p}}_{C,i}(k)$ to $\mathbf{p}_C$, the distributed approach ratio is converged to the ideal one.



(a) (b)

FIGURE 2. (a), (b) Iteration numbers versus different network sizes and $\frac{\alpha}{\gamma}$ in the 19$^{\text{th}}$ node for $\mathbf{S}$



(a) (b)

FIGURE 3. (a), (b) Iteration numbers versus different network sizes and $\frac{\alpha}{\gamma}$ in the 19$^{\text{th}}$ node for $\mathbf{q}$

5. **Conclusion.** In this paper, we introduced a fast and distributed maximum likelihood estimation using the consensus algorithm. Our approach utilizes "up to scale" property in projective geometry to reach the consensus quickly. The difference between nodes' values and meanwhile the difference between nodes' values and consensus values are evaluated by two error functions. To estimate consensus value in the second error function, we used local weighted average of each node. Experimental results show that this estimation can improve the convergence speed.

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