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A LOCATION-ALLOCATION MODEL FOR HIERARCHICAL HEALTHCARE FACILITIES UNDER GRADING TREATMENT

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Abstract. *The grading treatment and two-way referral are the main tasks of Chinese healthcare reform. According to the referral regulation, this paper proposes a location-allocation model to design the three-layer hospital network, in order to improve geographical access for patients while minimizing costs for hospitals. Genetic algorithm is used to solve the problems and we examine the effect caused by the parameter of referral rate. The result shows that referral rate has a significant influence on the location of hierarchical hospitals, and hence the care-seeking flow varies in different scenarios.*
Keywords: Hierarchical healthcare facilities, Location-allocation, Grading treatment, Referral

1. **Introduction.** With the rapid urbanization, and the improvement of traffic conditions and living standard, people's demand for medical services is rising [1], but problems caused by unordered health care-seeking are also increasingly serious. Therefore, the location research about hospitals of different levels is vital for optimizing allocation of regional medical resources [2]. And the reasonable optimization for hospital location can not only save costs effectively, but also improve residents' medical accessibility and the efficiency of medical service system.

Location problems in healthcare have been discussed by many authors. Daskin and Dean [3] reviewed three basic models for addressing the location problems in healthcare, including location set covering, maximal covering and P-median models. Smith et al. [4] used a Mixed Integer Program to determine the locations of sustainable community hospitals by considering both top-down and bottom-up community health schemes. As people have different requirements for hospitals with different levels, hierarchical location models for healthcare facilities are proposed in some literature. Hodgson [5] presented a hierarchical location-allocation model to ameliorate problems of rural accessibility to health care in Third World settings. Galvão et al. [6] firstly presented a three-level location model without capacity constraints, and applied it to the location of healthcare facilities in Rio de Janeiro, and then a capacitated model was proposed considering the imbalance in facility loading [7]. Teixeira and Antunes [8] put forward a discrete hierarchical model with capacity constraints, and discussed three allocation modes named single-assignment, closest-assignment and path-assignment. Wan et al. [9] introduced the piecewise utility function and proposed the nested public facility location model based on hierarchy model.

Location-allocation models also have been used as decision support tools for healthcare planning. Chu and Chu [10] developed a general modelling framework for supply and demand matching and studied the issues of hospital location and service allocation from the aspects of new service distribution and existing service redistribution. Günes and Yaman [11] presented an integer programming model to find the best re-allocation of resources among hospitals, the assignment of patients to hospitals and the service portfolio to minimize system costs. Mestre et al. [12] proposed a hierarchical multiservice mathematical programming model to address issues of location and supply of hospital services, so as to maximize patients' geographical access to a hospital, and then they made a further study by extending the demand to uncertainty [13]. Smith et al. [14] presented discrete hierarchical location models with the concern of bi-criteria efficiency and equity. Syam and Côté [15] proposed a location-allocation model for specialized healthcare services to minimize the total costs of health system and patients.

Although studies on the location of facilities and allocation of resources have gained much success from an academic standpoint, very few have considered the factor of referral on solving these problems. Based on the grading treatment and the two-way referral regulation in China, this paper presents a location-allocation model for hierarchical healthcare facilities, in order to improve geographical access for patients while minimizing costs of hospitals. And we analyze the influence of referral rate on locations of three-layer hospitals. The remainder of the paper is organized as follows. A general framework for modelling is described in Section 2. A mathematical model is presented in Section 3. Following the model, Section 4 describes an experiment that evaluates the impact of important parameters and discusses the results we obtained. In the last section, conclusions and future research are provided.

2. **A General Framework.** Some developed countries like America and England have executed the system of grading treatment for a long time and gained profits. However, in China, grading treatment and two-way referral are still in practice and are the main tasks of current healthcare reform. The aim is to form the regulation of "slight illness in the community hospital, serious illness to the high-level hospital, and rehabilitation back to the community". Figure 1 describes the system of grading treatment in China. It is composed of three levels of medical institutions, namely the first-level hospital, the second-level hospital and the third-level hospital. The first-level hospitals mainly refer to community clinics, which are responsible for basic public health services including prevention and treatment of chronic diseases. The second-level hospitals are responsible for diagnosis and treatment of common diseases. The third-level hospitals are mainly for the treatment of difficult and complicated diseases. Hospitals of each level provide outpatient and inpatient services. Patients can choose any level hospital for the initial diagnosis, but the referral must be executed by rules as Figure 1 shows. Overall, there are eight kinds of referrals, which will be considered in the following model.

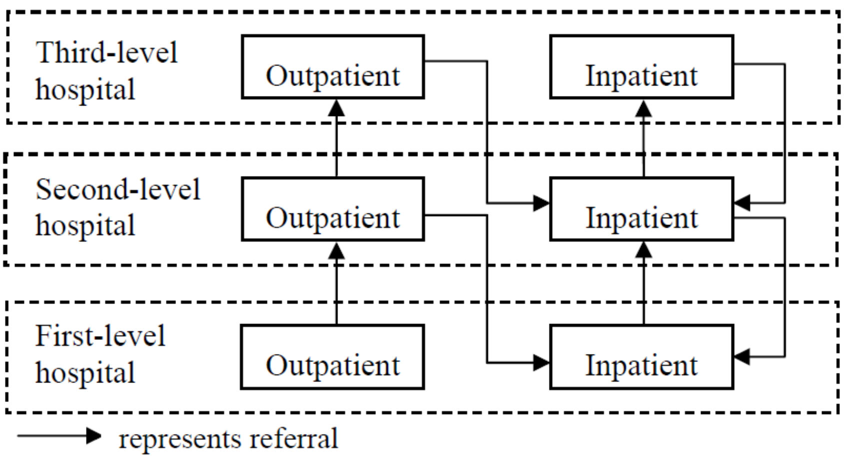

FIGURE 1. The system of grading treatment in China

3. **A Location-Allocation Model.** The mathematical formulation of the model according to the system of grading treatment in China is presented in this section. The basic assumptions of the model are as follows. (1) Demand points and facilities are discrete, generated by point. (2) Facilities can be established on demand points or be independent from them. (3) Demand for health services at each point is certain. (4) Distances between facilities and demand points can be measured. (5) Patients in an area see a doctor in this area. (6) Patients can have an entry to any level of hospitals initially, but the referral must be in strict accordance with the level of hospitals which means patients in a first-level hospital could not be transferred to a third-level hospital.

Notations used in the model are as follows.

Table 1. Indexes and sets

$s \in S$	Set of services in hospitals, indexed by s, $S = \{s, u\}$, s represents outpatient service, u represents inpatient service
$i \in I$	Set of demand points, indexed by i
$j, j' \in J$	Potential locations for first-level hospitals, indexed by j
$k, k' \in K$	Potential locations for second-level hospitals, indexed by k
$h, h' \in H$	Potential locations for third-level hospitals, indexed by h
$n, n' \in N$	Potential locations for all level hospitals, indexed by n, $N = J \cup K \cup H$
$l, l' \in L$	Set of hospital level, indexed by l, $L = \{A, B, C\}$, A represents the first level, B represents the second level, C represents the third level

Table 2. Parameters

d_{in}	Distance (Euclidean Distance) from demand point i to site n
d_{jk}, d_{kh}	Distance (Euclidean Distance) from j/k transferring to k/h
D_{is}	Demand for service s in location i
$P_{ss}^{ll'}$	Percent of patients transferring from service s in l level hospital to service s in l' level hospital
N_l	Maximum number of l level hospital
$d_{\max}^{l}$	Maximum distance allowed for a person to access service s in l level hospital
$cap\min{}_{ns}^{l}$	Minimum capacity required for service s in l level hospital at site n
$cap\max{}_{ns}^{l}$	Maximum capacity required for service s in l level hospital at site n
C_n^l	Fixed cost of establishing an l level hospital at site n
ρ_{is}	Penalty for unsatisfied demand in location i for service s, $\rho_{is} = \frac{d_{\max}^{A} + d_{\max}^{B} + d_{\max}^{C}}{3}$

Table 3. Variables

X_{ns}^{l}	=1 if l level hospital is located at site n providing service s =0 otherwise
Y_{ins}^{l}	Flow from demand point i to l level hospital at site n for service s
$Y_{jkss}^{AB}, Y_{jkuu}^{AB}, Y_{khss}^{BC}, Y_{khuu}^{BC}$ $Y_{kjsu}^{BA}, Y_{kjuu}^{BA}, Y_{hksu}^{CB}, Y_{hkuu}^{CB}$	Flow from one kind of hospital transferring to another kind of hospital for service s/u
$Y_{is\rho}$	Unsatisfied demand for service s in location i

Objective functions:

$$\min \sum_{s\in S}\sum_{i\in I}\left(\sum_{j\in J} d_{ij}Y_{ijs}^{A}X_{js}^{A}+\sum_{k\in K} d_{ik}Y_{iks}^{B}X_{ks}^{B}+\sum_{h\in H} d_{ih}Y_{ihs}^{C}X_{hs}^{C}\right)$$
$$+\sum_{h\in H}\sum_{k\in K}\sum_{j\in J}\left[d_{jk}\left(Y_{jkss}^{AB}+Y_{jkuu}^{AB}\right)+d_{kh}\left(Y_{khss}^{BC}+Y_{khuu}^{BC}\right)+d_{jk}\left(Y_{kjsu}^{BA}+Y_{kjuu}^{BA}\right)\right.$$
$$\left.+\ d_{kh}\left(Y_{hksu}^{CB}+Y_{hkuu}^{CB}\right)\right]+\sum_{s\in S}\sum_{i\in I}\rho_{is}Y_{is\rho} \tag{1}$$

$$\min \frac{\sum_{s\in S}\left(\sum_{j\in J} C_j^{A}X_{js}^{A}+\sum_{k\in K} C_k^{B}X_{ks}^{B}+\sum_{h\in H} C_h^{C}X_{hs}^{C}\right)}{2} \tag{2}$$

Subject to:

$$\sum_{j\in J} Y_{ijs}^{A}+\sum_{k\in K} Y_{iks}^{B}+\sum_{h\in H} Y_{ihs}^{C}+Y_{is\rho}=D_{is} \qquad \forall i\in I, s\in S \tag{3}$$

$$\sum_{i\in I} Y_{ijs}^{A}P_{ss}^{AB}=\sum_{k\in K} Y_{jkss}^{AB} \qquad \forall j\in J \tag{4}$$

$$\left(\sum_{i\in I} Y_{iks}^{B}+\sum_{j\in J} Y_{jkss}^{AB}\right)P_{ss}^{BC}=\sum_{h\in H} Y_{khss}^{BC} \qquad \forall k\in K \tag{5}$$

$$\sum_{i\in I} Y_{iju}^{A}P_{uu}^{AB}=\sum_{k\in K} Y_{jkuu}^{AB} \qquad \forall j\in J \tag{6}$$

$$\left(\sum_{i\in I} Y_{iku}^{B}+\sum_{j\in J} Y_{jkuu}^{AB}\right)P_{uu}^{BC}=\sum_{h\in H} Y_{khuu}^{BC} \qquad \forall k\in K \tag{7}$$

$$\left(\sum_{i\in I} Y_{ihu}^{C}+\sum_{k\in K} Y_{khuu}^{BC}\right)P_{uu}^{CB}=\sum_{k\in K} Y_{hkuu}^{CB} \qquad \forall h\in H \tag{8}$$

$$\left(\sum_{i\in I} Y_{iku}^{B}+\sum_{h\in H} Y_{hkuu}^{CB}+\sum_{h\in H} Y_{hksu}^{CB}+\sum_{j\in J} Y_{jkuu}^{CB}\right)P_{uu}^{BA}=\sum_{j\in J} Y_{kjuu}^{BA} \qquad \forall k\in K \tag{9}$$

$$\left(\sum_{i\in I} Y_{ihs}^{C}+\sum_{k\in K} Y_{khss}^{BC}\right)P_{su}^{CB}=\sum_{k\in K} Y_{hksu}^{CB} \qquad \forall h\in H \tag{10}$$

$$\left(\sum_{i\in I} Y_{iks}^{B}+\sum_{j\in J} Y_{jkss}^{AB}\right)P_{su}^{BA}=\sum_{j\in J} Y_{kjsu}^{BA} \qquad \forall k\in K \tag{11}$$

$$cap\min{}_{js}^{A}X_{js}^{A}\le\sum_{i\in I} Y_{ijs}^{A}\le cap\max{}_{js}^{A}X_{js}^{A} \qquad \forall j\in J \tag{12}$$

$$cap\min{}_{ju}^{A}X_{ju}^{A}\le\sum_{i\in I} Y_{iju}^{A}+\sum_{k\in K} Y_{kjsu}^{BA}+\sum_{k\in K} Y_{kjuu}^{BA}\le cap\max{}_{ju}^{A}X_{ju}^{A} \quad \forall j\in J \tag{13}$$

$$cap\min{}_{ks}^{B}X_{ks}^{B}\le\sum_{i\in I} Y_{iks}^{B}+\sum_{j\in J} Y_{jkss}^{AB}\le cap\max{}_{ks}^{B}X_{ks}^{B} \qquad \forall k\in K \tag{14}$$

$$cap\min{}_{ku}^{B}X_{ku}^{B}\le\sum_{i\in I} Y_{iku}^{B}+\sum_{h\in H} Y_{hkuu}^{CB}+\sum_{h\in H} Y_{hksu}^{CB}+\sum_{j\in J} Y_{jkuu}^{AB}$$
$$\le\ cap\max{}_{ku}^{B}X_{ku}^{B} \qquad \forall k\in K \tag{15}$$

$$cap\min{}_{hs}^{C}X_{hs}^{C} \leq \sum_{i\in I} Y_{ihs}^{C} + \sum_{k\in K} Y_{khss}^{BC} \leq cap\max{}_{hs}^{C}X_{hs}^{C} \qquad \forall h \in H \tag{16}$$

$$cap\min{}_{hu}^{C}X_{hu}^{C} \leq \sum_{i\in I} Y_{ihu}^{C} + \sum_{k\in K} Y_{khuu}^{BC} \leq cap\max{}_{hu}^{C}X_{hu}^{C} \qquad \forall h \in H \tag{17}$$

$$Y_{ins}^{l} + D_{is}X_{n's}^{l} \leq D_{is} \qquad \forall i \in I, l \in L, n \in N, n' \in \{n' | d_{in'} < d_{in}\}, s \in S \tag{18}$$

$$Y_{ihs}^{C} + D_{is}\left(X_{j's}^{A} + X_{k's}^{B}\right) \leq D_{is}$$
$$\forall i \in I, h \in H, j' \in \{j' | d_{ij'} < d_{ij}\}, k' \in \{k' | d_{ik'} < d_{ik}\}, s \in S \tag{19}$$

$$Y_{iks}^{B} + D_{is}X_{j's}^{A} \leq D_{is} \qquad \forall i \in I, k \in K, j' \in \{j' | d_{ij'} < d_{ij}\}, s \in S \tag{20}$$

$$Y_{ins}^{l} = 0 \qquad \forall i \in I, l \in L, n \in \{n | d_{in} > d_{\max}^{l}\}, s \in S \tag{21}$$

$$0 \leq Y_{ins}^{l} \leq D_{is}X_{ns}^{l} \qquad \forall i \in I, n \in N, l \in L, s \in S \tag{22}$$

$$\sum_{n\in N} X_{ns}^{l} \leq N_{l} \qquad \forall s \in S, l \in L \tag{23}$$

$$X_{ns}^{l} = X_{nu}^{l} = \{0, 1\} \qquad \forall n \in N, l \in L \tag{24}$$

$$Y_{jkss}^{AB}, Y_{jkuu}^{AB}, Y_{khss}^{BC}, Y_{khuu}^{BC}, Y_{kjsu}^{BA}, Y_{kjuu}^{BA}, Y_{hksu}^{CB}, Y_{hkuu}^{CB} \geq 0 \quad \forall j \in J, k \in K, h \in H \tag{25}$$

The model considers two objective functions. Equation (1) minimizes the travel distances to hospitals weighted by demand. In this objective function, the first term is the sum of distances that patients enter any level hospitals directly. The second term is the sum of transferring distances between different levels of hospitals. The third term is a penalty for unsatisfied demand expressed by distance. Equation (2) minimizes the fixed costs for hospital construction.

For the constraints, Equation (3) states that all demands are considered by the model, either satisfied or unsatisfied demand. Equations (4)-(11) are flow conservation constraints that determine the balance between two levels of hospitals. Equations (12)-(17) state that flows to each hospital are constrained by hospital capacity. Equations (18)-(20) ensure that demand points are allocated to the closest facility. Equation (21) ensures that distance for patients to access different level hospitals should not be over the maximum distance. Equation (22) ensures that any demand point can only be served by an opened facility. Equation (23) limits the number of hospitals at different levels. Equation (24) ensures that a hospital provides inpatient service and outpatient service at the same time. Equation (25) states that decision variables are nonnegative.

4. **Experiment.**

4.1. **A case study.** Taking a district in S city as an example, residents in this area are clustered into 20 demand points, i.e., $I = 20$. The sets of potential locations for three levels of hospitals are $J = 20$, $K = 8$, $H = 5$. Demands are obtained from historical data. As distance data are too large, they will not be listed in the paper. Some other main parameters are set as follows: $N_A = 20$, $N_B = 5$, $N_C = 2$, $cap\max{}_{js}^{A} = 24000$, $cap\max{}_{ks}^{B} = 40000$, $cap\max{}_{hs}^{C} = 60000$, $d_{\max}^{A} = 2$, $d_{\max}^{B} = 5$, $d_{\max}^{C} = 9$, $cap\min{}_{js}^{A} = 120$, $cap\min{}_{ks}^{B} = 8000$, $cap\min{}_{hs}^{C} = 30000$, $C_{j}^{A} = 100$, $C_{k}^{B} = 1000$, $C_{h}^{C} = 10000$, $cap\max{}_{ju}^{A} = 600$, $cap\max{}_{ku}^{B} = 1000$, $cap\max{}_{hu}^{C} = 1500$, $\rho_{is} = 16/3$, $\rho_{iu} = 16/3$, $cap\min{}_{ju}^{A} = 60$, $cap\min{}_{ku}^{B} = 300$, $cap\min{}_{hu}^{C} = 750$, $P_{ss}^{AB} = 20\%$, $P_{uu}^{AB} = 10\%$, $P_{ss}^{BC} = 10\%$, $P_{uu}^{BC} = 5\%$, $P_{su}^{BA} = 5\%$, $P_{uu}^{BA} = 3\%$, $P_{su}^{CB} = 3\%$, $P_{uu}^{CB} = 2.5\%$.

Genetic algorithm is used to solve this optimization problem. The possible solutions are coded into vectors or chromosomes and since the constraints cannot be expressed by simple codes, we incorporate them into the fitness function. An initial group of solutions is generated randomly first and the fitness of every chromosome is calculated according to the fitness function. The population experiences reproduction, crossover and mutation

TABLE 4. Computational results

Level	**Site**	**Amount**
First-level hospital	0 1 0 0 0 1 1 1 1 1 0 0 0 0 1 0 0 1 1 1	10
Second-level hospital	0 0 0 0 1 0 1 0	2
Third-level hospital	1 1 0 1 1	4

TABLE 5. Computational results with referral rate changing

Parameters	**Level**	**Amount**
$P_{ss}^{AB} = 20\%, P_{uu}^{AB} = 10\%$ $P_{ss}^{BC} = 10\%, P_{uu}^{BC} = 5\%$ $P_{su}^{BA} = 5\%, P_{uu}^{BA} = 3\%$ $P_{su}^{CB} = 3\%, P_{uu}^{CB} = 2.5\%$	First-level hospital	10
	Second-level hospital	2
	Third-level hospital	4
$P_{ss}^{AB} = 15\%, P_{uu}^{AB} = 8\%$ $P_{ss}^{BC} = 8\%, P_{uu}^{BC} = 3\%$ $P_{su}^{BA} = 3\%, P_{uu}^{BA} = 2.5\%$ $P_{su}^{CB} = 2.5\%, P_{uu}^{CB} = 2\%$	First-level hospital	11
	Second-level hospital	1
	Third-level hospital	2
$P_{ss}^{AB} = 10\%, P_{uu}^{AB} = 6\%$ $P_{ss}^{BC} = 6\%, P_{uu}^{BC} = 2.5\%$ $P_{su}^{BA} = 2.5\%, P_{uu}^{BA} = 2\%$ $P_{su}^{CB} = 2\%, P_{uu}^{CB} = 1.5\%$	First-level hospital	12
	Second-level hospital	3
	Third-level hospital	0

operation and a new one is generated. In this study, the parameters are set as follows: maximum iterations = 100, crossover rate = 0.9 and mutation rate = 0.1. When the number of iterations is 43, the function converges and we get an optimal solution as shown in Table 4.

4.2. **Discussion of the effects of referral rate.** In this part, we demonstrate how referral rate affects the location decision by varying the value of referral rate. As shown in Table 5, the lower the referral rate is, the more first-level hospitals will be constructed. And higher referral rate also contributes to increase in the number of third-level hospitals.

We also analyze the flows from each demand point to different level hospitals under diverse referral rates. As shown in Figure 2, when the referral rate decreases, more patients choose first-level medical institutions both for outpatient service and inpatient service. Specifically, when the referral rate is in high level among the three situations, in each of the 20 demand points, approximately 60 percent of patients go to first-level hospitals for treatment. While the proportion goes beyond 80 percent on average as the referral rate falls to the low level.

5. **Conclusions.** A mathematical model has been presented in the context of grading treatment and two-way referral regulation, and a case study was used to test the applicability of the model. The experiment analysis shows that the referral rate has a significant influence on the location of hierarchical hospitals. Specifically, as referral rate declines, the quantity of first-level hospitals rises which means more people will go to community clinics. This implies that controlling the referral rate will assist to guide people to seek medical treatment in order and push the implementation of grading treatment.

The future research could be focused on changing the referral regulation, because this study is under a certain frame of referral. Allowing patients in a first-level hospital to be transferred to a third-level hospital may cause new findings. Another consideration is extending the deterministic demand into uncertain demand, to build a long term programming model from planning horizon. Finally, the effect of medical insurance leverage can be studied and residents' medical costs can be set in the objective function.

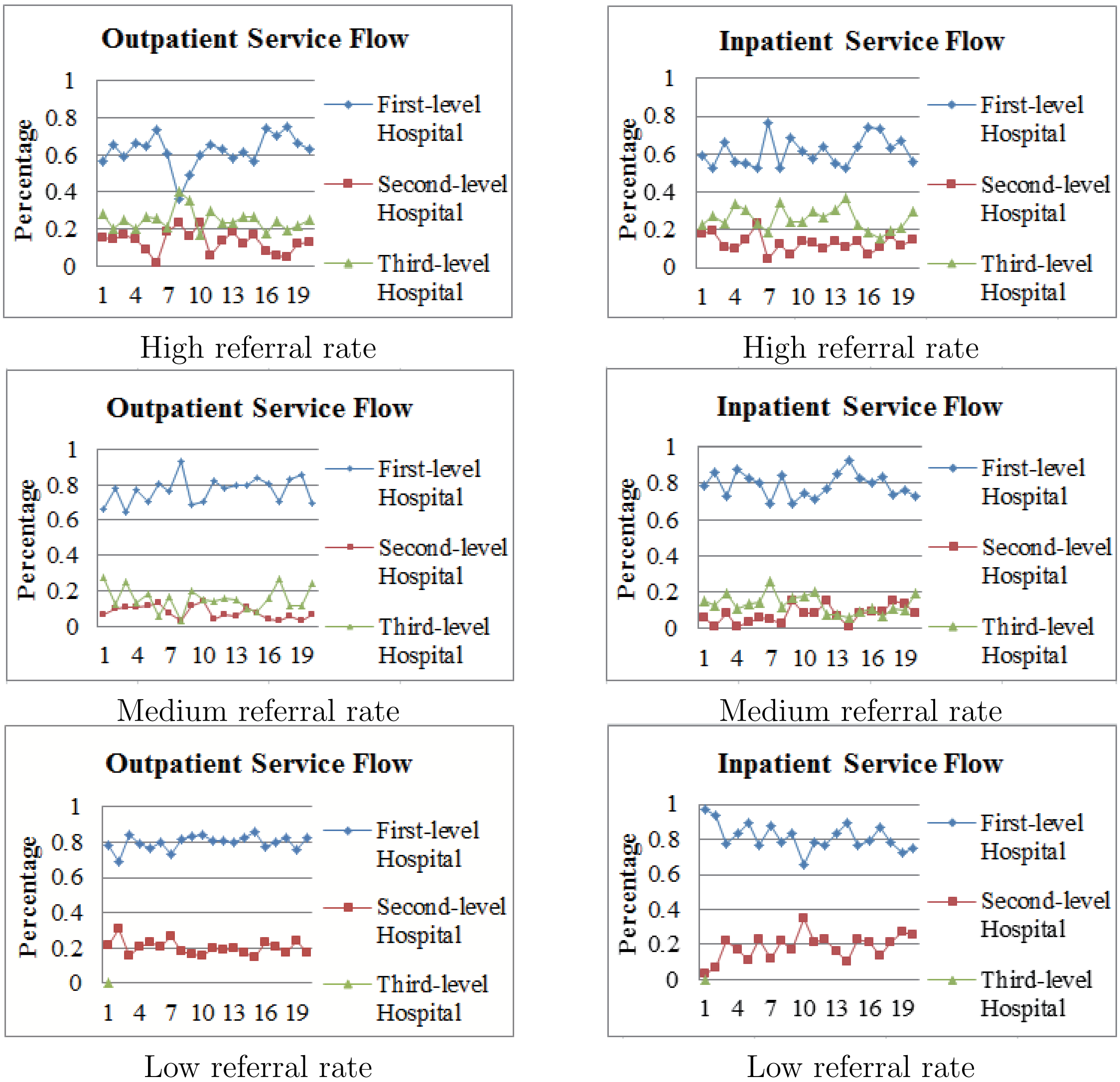

FIGURE 2. The percentage of choosing hierarchical hospitals with various referral rates

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REFERENCES

[1] S. Du, J. Xie and Z. Yi, Healthcare operation management: Emerging research hotspots and progress, *Journal of Management Sciences in China*, vol.16, no.8, pp.1-19, 2013.
[2] Y. H. Diao and Z. B. Jiang, A stochastic programming based method for hospital site selection, *Journal of Shanghai Jiaotong University*, vol.44, no.3, pp.433-436, 2010.
[3] M. S. Daskin and L. K. Dean, Location of health care facilities, in *Operations Research and Health Care: A Handbook of Methods and Applications*, M. L. Brandeau, F. Sainfort and W. P. Pierskalla (eds.), Dordrecht, Kluwer, 2004.
[4] H. K. Smith, P. R. Harper, C. N. Potts and A. Thyle, Planning sustainable community health schemes in rural areas of developing countries, *European Journal of Operational Research*, vol.193, no.3, pp.768-777, 2009.
[5] M. J. Hodgson, An hierarchical location-allocation model for primary health care delivery in a developing area, *Social Science & Medicine*, vol.26, no.1, pp.153-161, 1988.
[6] R. D. Galvão, L. G. A. Espejo and B. Boffey, A hierarchical model for the location of perinatal facilities in the municipality of Rio de Janeiro, *European Journal of Operational Research*, vol.138, no.3, pp.495-517, 2002.

[7] R. D. Galvão, L. G. A. Espejo, B. Boffey and D. Yates, Load balancing and capacity constraints in a hierarchical location model, *European Journal of Operational Research*, vol.172, no.2, pp.631-646, 2006.

[8] J. C. Teixeira and A. P. Antunes, A hierarchical location model for public facility planning, *European Journal of Operational Research*, vol.185, no.1, pp.92-104, 2008.

[9] B. Wan, C. Yang and S. Huang, Nested public facility location research based on hierarchy model, *Journal of WUT: Information & Management Engineering*, vol.34, no.2, pp.218-222, 2012.

[10] S. C. K. Chu and L. Chu, A modeling framework for hospital location and service allocation, *International Transactions in Operational Research*, vol.7, no.6, pp.539-568, 2000.

[11] D. E. Güne and H. Yaman, Health network mergers and hospital re-planning, *Journal of the Operational Research Society*, vol.61, no.2, pp.275-283, 2010.

[12] A. M. Mestre, M. D. Oliveira and A. Barbosa-Póvoa, Organizing hospitals into networks: A hierarchical and multiservice model to define location, supply and referrals in planned hospital systems, *OR Spectrum*, vol.34, no.2, pp.319-348, 2012.

[13] A. M. Mestre, M. D. Oliveira and A. Barbosa-Póvoa, Location-allocation approaches for hospital network planning under uncertainty, *European Journal of Operational Research*, vol.240, no.3, pp.791-806, 2015.

[14] H. K. Smith, P. R. Harper and C. N. Potts, Bicriteria efficiency/equity hierarchical location models for public service application, *Journal of the Operational Research Society*, vol.64, no.4, pp.500-512, 2012.

[15] S. S. Syam and M. J. Côté, A comprehensive location-allocation method for specialized healthcare services, *Operations Research for Health Care*, vol.1, no.4, pp.73-83, 2012.