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TRACKING LOOPS DESIGN AND ANALYSIS FOR INS-AIDED GPS RECEIVER

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Abstract. *The structural design of the INS-aided (Inertial Navigation System-aided) receiver tracking loops is the main part of implementing an SINS (Strap-down Inertial Navigation System)/GPS Ultra-Tightly Coupled Integrated Navigation System, and the loop bandwidth is the key factor to determine the performance of the tracking loop. Considering satellite signals are easy to be out of locking in high dynamic environment, the INS Velocity Information receiver tracking loops to increase the effective bandwidth of tracking loop is designed in this paper. The simulation results show the inter-restricted relationship between the thermal agitation noise error and the dynamic stress error in the carrier tracking loop.*
Keywords: GPS, INS-aided, Tracking loops, Ultra-Tightly Coupled Integrated Navigation System, High dynamic

1. **Introduction.** With the development of software receiver technology and data fusion algorithms, SINS/GPS Ultra-Tightly Coupled Integrated Navigation System has gradually become a research hotspot after loose coupling and tight coupling. Unlike loose coupling and tight coupling, it turns the research focus to the interior of the receiver, realizing the auxiliary of the SINS to the GPS receiver. In 2000, in [7,8], D. Gustafson et al. in the United States Draper Laboratory proposed the concept of a deep integrated adaptive GPS-based navigator based on GPS for the first time. And the simulation results show that Ultra-Tightly Coupled Integrated Navigation System has stronger anti-interference ability and dynamic adaptability. In 2003, according to the research of D. Gustafson, S. Alban in Stanford University and D. G. Egziabher in Minnesota University proposed the concept of Ultra-Tight Integration based on the tight coupling integrated navigation system for the first time, designing a new INS information aided tracking loop. The receiver can get the equivalent loop bandwidth and increases loss-of-lock threshold. In [3,4], a series of simulation results demonstrate the superiority of the integrated navigation system performance. Since then, scientists from various countries carried out relevant research on the SINS/GPS Ultra-Tightly Coupled Integrated Navigation System, which can be known from [5-12].

In an SINS/GPS Ultra-Tightly Coupled Integrated Navigation System, in order to maintain good tracking performance in high dynamic environment, it is necessary for the receiver to increase the loop bandwidth of the tracking loop to carry large dynamic stress; however, larger loop bandwidth can weaken the ability of the tracking loop to filter out external noise and the anti-interference performance. Given this, the carrier tracking loop structure aided is designed by the INS velocity information, in which the Doppler frequency shift of the satellite signal is estimated via the INS information to compensate for the loop bandwidth required by the dynamic characteristics of carriers. Meanwhile, according to the related formulas, loop tracking error is a function of the satellite signal carrier-to-noise ratio and loop bandwidth. This paper presents a method of estimating the carrier-to-noise ratio and the motion characteristics of carriers to optimize the loop bandwidth in order to maintain the tracking error minimum and improve the tracking accuracy of the system.

2. **Tracking Loops Design for INS-Aided.** Figure 1 shows the design of the carrier tracking loop aided by SINS subsystem speed. According to the Doppler frequency shift information estimated by the INS subsystem and the phase difference of the loop filter output, the carrier NCO can constantly adjust the parameters of the local copied carrier signal to make the carrier tracking loop track the satellite signal stably after the introduction of the INS velocity aided loop.

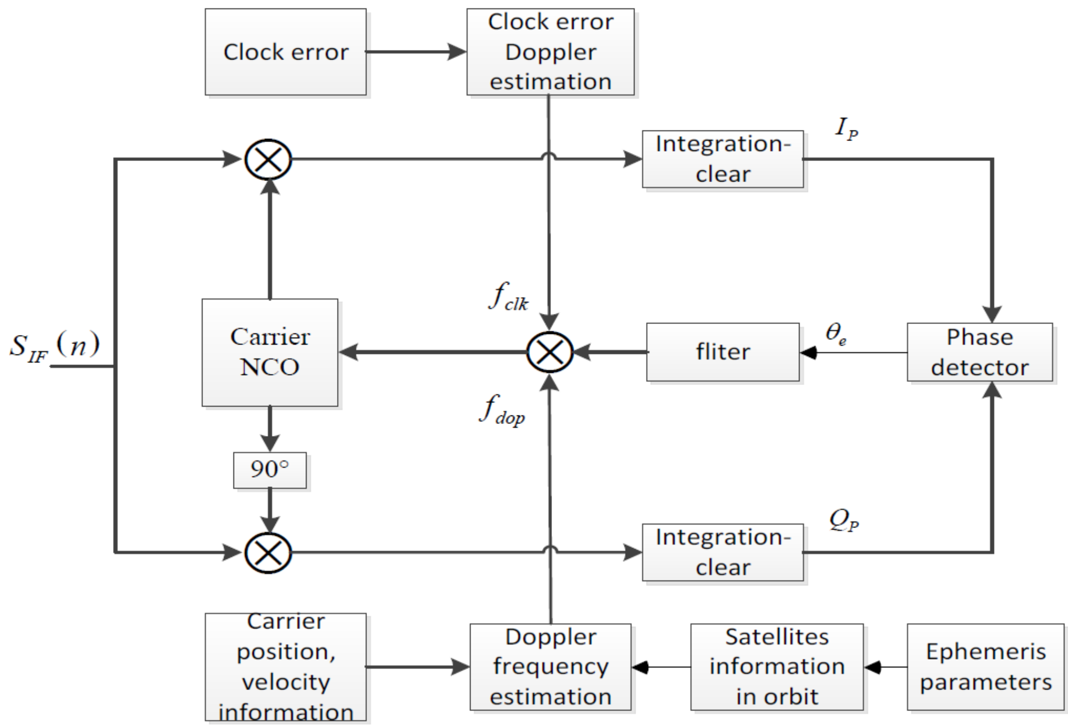

FIGURE 1. INS-aided receiver carrier tracking loop design

According to the relationship between the signal tracking parameters and the position and velocity information, from [13], local copied orthogonal carrier signals can be expressed as

$$i(t) = \cos(2\pi(f_{IF} + f_{dop})t + \varphi(t)) \tag{1}$$

$$q(t) = \sin(2\pi(f_{IF} + f_{dop})t + \varphi(t)) \tag{2}$$

where f_{IF} is IF digital signal frequency, and f_{dop} is Doppler frequency between the carrier and satellite based on the position, velocity information and satellite in-orbit information of INS subsystem. φ is the carrier phase of the loop filter's output; f_{dop} is mainly used to offset the dynamic stress error caused by high dynamic environment.

Figure 2 shows the schematic diagram of the INS Velocity Information receiver carrier tracking loop in the s-domain. A tracking loop includes an information fusion auxiliary circuit and a carrier tracking loop. The basic PLL architecture is in the dashed lines, $\theta(s)$ is the carrier phase of the received signal, $w(s)$ is phase noise, the loop filter is a second-order filter ($F(S) = K \cdot \frac{S+a}{S}$), and the NCO is an ideal integrator ($\frac{1}{S}$). By using the velocity information calculated by the INS subsystem, the INS velocity-aided loops can get the Doppler frequency f_{dop} that the carrier is relative to the satellite and use it to characterize the change rate of the phase of the reference carrier as the carrier tracking loop auxiliary information. $\frac{1-k}{\tau s+1}$ represents the imperfection of the speed of the measuring carrier, τ is the delay time constant, k is the scale factor error and the sum of the INS velocity-aided frequency in the forward path and the output of the PLL loop filter is used to control the NCO.

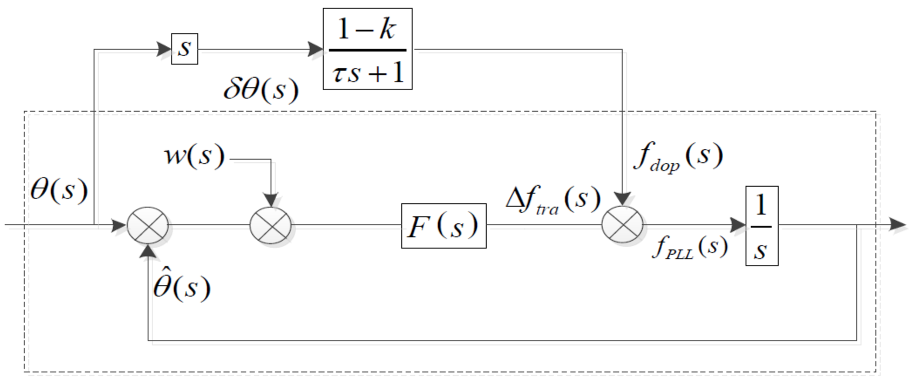

Figure 2. Aided tracking loop s-domain schematics

From Figure 2, the loop output phase can be expressed as

$$\hat{\theta}(s) = H_1(s)\theta(s) + H_2(s)w(s) \tag{3}$$

In this formula,

$$H_1(s) = \frac{[(1-k) + K\tau]s^2 + (K + Ka\tau)s + Ka}{\tau s^3 + (1 + K\tau)s^2 + (K + Ka\tau)s + Ka} \tag{4}$$

$$H_2(s) = \frac{Ks + Ka}{s^2 + Ks + Ka} \tag{5}$$

Formula (4) shows that the values of the INS speed information are ideal, namely $\tau = 0$, $k = 0$, $H_1(S) = 1$, which means that the receiver tracking loop can track the carrier with high dynamic movement without errors under ideal conditions. The auxiliary speed cannot be completely accurate because of the existence of the velocity deviation and the clock deviation. However, in the design of INS-aided tracking loop, the loop bandwidth required by the dynamic nature of the carrier can be completely compensated by INS velocity information.

3. **Aided Tracking Loops Bandwidth Analysis.** Supposed that the closed-loop transfer function of the tracking loop is $H(S)$, according to the definition of the loop bandwidth,

$$B_L = \int_0^\infty |H(jw)|^2 df \tag{6}$$

In this formula, $w = 2\pi f$.

Here shows the difference of the effective bandwidth between the traditional carrier tracking loop and INS-aided carrier tracking loop.

In Figure 2, the basic two phase-locked loop is in the dashed line, and for a traditional carrier tracking loop, the closed-loop transfer function is

$$H(s) = \frac{Ks + Ka}{s^2 + Ks + Ka} \tag{7}$$

Ignoring the interference noise, for an INS velocity-aided carrier tracking loops, the closed-loop transfer function is

$$H(s) = \frac{[(1-k) + K\tau]s^2 + (K + Ka\tau)s + Ka}{\tau s^3 + (1 + K\tau)s^2 + (K + Ka\tau)s + Ka} \tag{8}$$

If Formulas (7) and (8) are put into Equation (6) and give the parameters (K and a) of the filter different values and also give the parameters (K and τ) of the INS subsystem different values, then the bandwidth both in a traditional tracking loop and an INS velocity-aided tracking loop can be got. Table 1 shows different tracking loop bandwidths under different filter parameters and the velocity aided accuracy.

Table 1. Tracking loop bandwidths under different filter parameters

parameter / aided or not	$K = 66.31$, $a = 33.53$ $k = 0.01$, $\tau = 0.01$	$K = 5.34$, $a = 2.66$ $k = 0.01$, $\tau = 0.01$	$K = 66.31$, $a = 33.35$ $k = 0.001$, $\tau = 0.01$
no-aid	27	2	27
velocity aided	59.3437	27.1699	281.0246

From the first column and the second column of Table 1, when the equivalent bandwidth of the INS velocity-aided tracking loop is required to be equal to the bandwidth of the traditional tracking loop, the actual bandwidth of the former is much smaller than the latter. So the INS velocity-aided tracking loop can filter out interference noise well and enhance the anti-interference ability of receivers while it tracks dynamic signal with high stability.

From the first column and the third column of Table 1, the more precise (the smaller K and τ) the velocity values the INS subsystem provides, the wider the equivalent bandwidth and adjustable range will be. The actual tracking loop bandwidth more can be reduced to filter the interference noise better.

4. **Tracking Performance Analysis.** For a linear system its transfer function is

$$H(s) = \frac{b_1 s^{n-1} + b_2 s^{n-2} + \cdots + b_n}{s^n + a_1 s^{n-1} + \cdots + a_n} \tag{9}$$

According to the method of analyzing a linear system, the state equation and observation equation of the system can be got,

$$\begin{aligned} \boldsymbol{X}(t) &= \begin{bmatrix} 0 & 1 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \dots & \dots & & \dots \\ -a_n & -a_{n-1} & \dots & -a_1 \end{bmatrix} \boldsymbol{X}(t) + \begin{bmatrix} 0 \\ 0 \\ \dots \\ 1 \end{bmatrix} \boldsymbol{U}(t) \\ \boldsymbol{Y}(t) &= [b_n \quad b_{n-1} \quad \dots \quad b_1]\boldsymbol{X}(t) \end{aligned} \tag{10}$$

The transfer function of traditional tracking loop is given below as

$$H(s) = \frac{Ks + Ka}{s^2 + Ks + Ka}$$

The corresponding state equation is

$$\begin{aligned} &\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -Ka & -K \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ &y = \begin{bmatrix} Ka & K \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{aligned} \tag{11}$$

For an INS-aided tracking loop, its transfer function is Equation (8), and the corresponding equations of the system are

$$\begin{aligned} &\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{Ka}{\tau} & -\frac{K\tau a+K}{\tau} & -\frac{1+K\tau}{\tau} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \\ &y = \begin{bmatrix} \frac{Ka}{\tau} & \frac{K\tau a+K}{\tau} & \frac{1-k+K\tau}{\tau} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \end{aligned} \tag{12}$$

Set $K = 20$, $a = 10$, k and τ take different values, for Formulas (11) and (12), the 4^{th}-order Runge-Kutta method is used to do numerical integration and get the curves of the transient process of the system, as is shown in Figure 3 and Figure 4. According to Figure 3 and Figure 4, the INS velocity information can reduce the transient time of tracking loop effectively, and with the decrease of k and τ, the transient time that the phase-locked loop required to reach the stable state also decreases rapidly. Set error band as 2%, and then the transient time of the tracking loop without velocity-aided is about 0.35s when the velocity-aided tracking is added.

According to the analysis above, the INS velocity-aided receiver tracking loop can be a good solution to the restrictive relation of the dynamic stress error and thermal noise on the loop bandwidth. By using the Doppler frequency shift of the satellite signal estimated by the INS velocity information, the aided tracking loop can eliminate the dynamic stress error of the tracking loop largely. Meanwhile, the actual bandwidth of the tracking loop becomes narrow and the ability of filtering out external noise becomes stronger, thus improving the performance of the tracking loop.

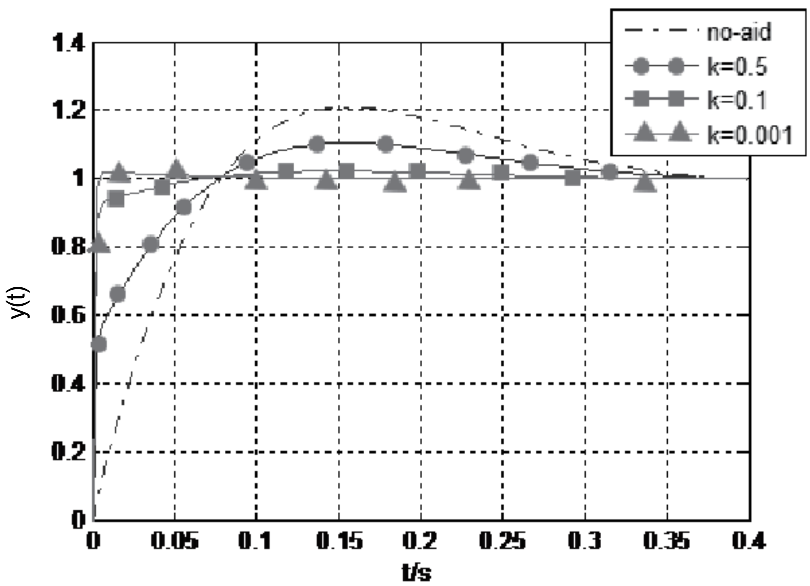

Figure 3. Transition curve with k ($\tau = 0.001$)

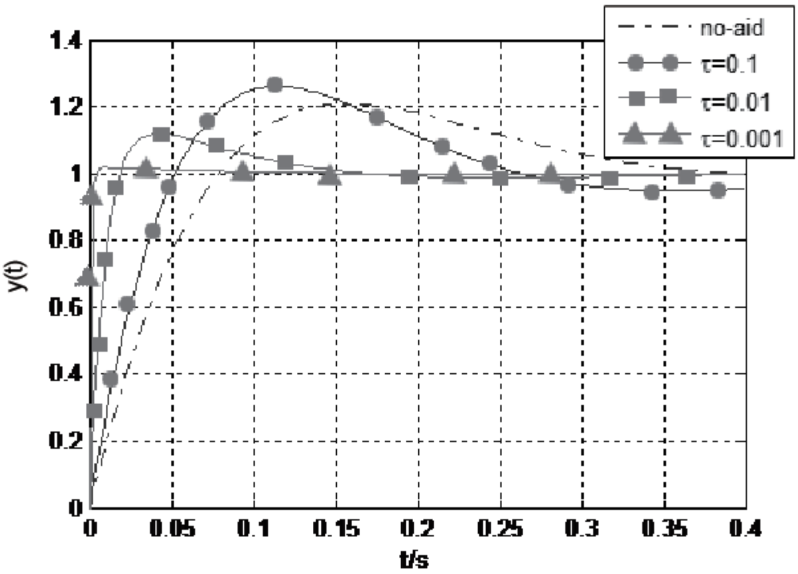

FIGURE 4. Transition curve with τ ($k = 0.001$)

5. **Conclusions.** The design of INS-aided tracking loop is the key to achieve Ultra-Tightly Coupled Integrated Navigation System. For high dynamic navigation-related issues, the inertial velocity information aided receiver tracking loop structure is designed, and the satellite signal Doppler shift is estimated via the inertial velocity information, largely eliminating the dynamic stress error in the tracking loop.

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