ICIC Express Letters
Volume **10**, Number **6**, June **2016**

ICIC International ©2016 ISSN 1881-803X
pp. 1375–1381

A GREEN APPROACH FOR A BI-OBJECTIVE PROGRAMMING INVENTORY ROUTING PROBLEM

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Received November 2015; accepted February 2016

Abstract. *A bi-objective multi-product multi-period inventory routing problem (IRP) consists of constructing routes for multiple capacitated vehicles to meet the given demand of a single plant in a way that no vehicle carries load more than its capacity and each supplier is visited only once each period over a finite planning horizon. In this supply chain, the demand for each product is assumed to be deterministic, time varying and ready for collection when the vehicle arrives at the supplier site. The bi-objective optimization model pertaining to minimize transportation cost and the greenhouse gas emission is considered as a possible solution to achieve a balance of economic and environment. A numerical study is proposed to show the applicability of the model and underline the impact of the green approach on future supply chain performance.*

Keywords: Inventory routing problem, Green approach, Transshipment

1. **Introduction.** In the context of globalization, transportation has become the primary source of greenhouse gases (GHGs) emissions such as carbon dioxide (CO_2), leading to the recent expansion of green logistics investigation as a subset of the green supply chain [1,2]. Logistics are now widely considered to be value-adding components in supply chain structure, whose primary objective is to coordinate activities such as freight transport, storage, inventory management and materials handling at a lower cost. One of the well-known topics typically addressed in this regard is the inventory routing problem (IRP). The IRP in a supply chain simultaneously determines the optimal inventory levels, delivery routes and vehicle scheduling based on the minimal cost criterion. As a result of governmental regulations and huge social pressure, transportation companies start taking into serious consideration of the emissions reduction objective in defining their working plans. Accordingly, it is necessary for the generated working plans to minimize transportation costs and GHGs emissions with respect to different constraints.

[3,4] presented a classification and comprehensive literature review of inventory routing problems. IRP has been the subject of intensive research addressed from various categorized criteria such as single of multiple periods [5], single or multiple customers [6], and deterministic or stochastic demand [7]. [8] is the first to consider green logistics through incorporating a decision variable, while satisfying some 'green' constraint conditions. However, minimizing transportation costs and GHGs emissions are not necessarily positively correlated and even completely conflicting for some cases. [9] shows the effectiveness of explicitly considering emissions minimization as separate objectives to optimize and proves that short routes are not necessarily less pollutant. It also claims

that there is a need to develop a new optimization model with GHGs emissions as an objective to optimize.

This study attempts to reduce the emission of GHGs to achieve a balance of economic and environmental problems by proposing a novel model which extends current studies on the delivery IRP by taking transportation costs and GHGs emissions as two optimal objectives.

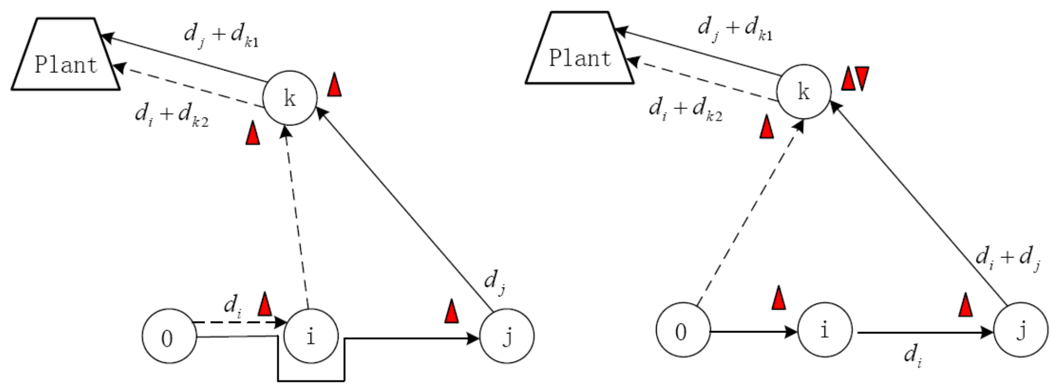
In Section 2, the inventory routing problem under study is described. In Section 3, its mathematical formulation is then provided. A numerical study is provided in Section 4 and Section 5 concludes the paper and proposes further research in this field.

2. **Problem Description.** The problem is defined on the assumptions that an assembly plant (Node F) demands some types of products transported by a rent truck company (depot) from a set of suppliers {1, 2, ..., N} in each period; each supplier provides one product type for the assembly plant. This rental truck company has several types of trucks associated with its own capacity, transportation cost rate and GHG emission index.

The aim of this problem is to find the best optimization configuration of vehicle types, routes, temporary storage supplier site, pickups, deliveries and transshipments in each period in a way that minimizes the transportation costs and GHGs emissions while satisfying all constraints.

A green transportation option is premised based on the proposed inventory routing problem. Under this policy, a vehicle may either provide a specific product for an assembly plant directly from the supplier which produces the product or from other suppliers which temporarily stored this product in the previous trips [10]. A simple illustrative example is used to explain this premise.

Figure 1 illustrates the case of 3 product suppliers and 2 trip periods to discuss the possible decrease of GHGs emissions by reducing travel distance.



Solution (a)		Solution (b)		Demand		
				Node	$Period_1$	$Period_2$
Trips in period 1	⟶	Load	▲	i	–	d_i
Trips in period 2	- - -▸	Unload	▼	j	d_j	–
Supplier	○			k	d_{k1}	d_{k2}

Figure 1. Two kinds of transport solutions

In solution (a), the vehicle visited nodes j and k picking up d_j and d_{k1} units of product type j and k in period 1 (solid arrows). In period 2 (dashed arrows), only nodes i and k are visited by the vehicle picking up d_i and d_{k2} units of product type i and k for no demanding of node j.

In solution (b), despite the fact that there is no current demand for product type i in period 1; the vehicle visits node i and picks up d_i units (thus meeting the demand for product i in the next period), and then visits node j and picks up the required number of product type $j(d_j)$, and then goes to node k and stores d_i units at this node (meaning that the d_i units are transshipped to node k) while picking up d_{k1} units of product type k (solid arrows). In the second period (dashed arrows), the vehicle goes directly to node k picking up the previously stored products (d_i) and simultaneously new required number of products (d_{k2}). It is distinct that the total travel distance will be reduced by the solution (b) if arc(i,k) has a greater distance than arc(i,j).

The original model is formulated on the opinion of solution (a) with the aim at finding the best configuration of the vehicle types, routes, pickups and delivers in each period in a manner that minimizes the total cost of the supply chain and GHGs emissions, while satisfying all constraints. In the first period the vehicle is not allowed to transship the product type i which is needed in the next period, and it is only allowed to transship the products needed in current period according to the planning; however, the green model is based on the opinion of solution (b) that the vehicle is allowed to arbitrarily store pickups at every code on its trip (transshipment). In this manner, the vehicle can pick up products from one model and store them temporarily at another node to reduce the total travel distance while meeting the current demand of the assembly plant.

The notation used in this framework is defined in Table 1.

Table 1. The notation defined in this framework

Sets		Parameters	
$\Omega = \{0, 1, \dots, N+1\}$ set of all nodes		D_{pt}	demand for product type $p\ (1, 2, \dots, P)$ in period $t\ (1, 2, \dots, T)$
$\omega = \{1, 2, \dots, N\}$ set of suppliers		c_{ij}	length of arc(i,j)
$O = \{0\}$ depot (rental vehicle company)		u_k	fixed transportation cost for vehicle type k per trip
$F = \{N+1\}$ assembly plant		NT_{kt}	the number of vehicle type k available in period t
Decision variables		cap_k	capacity of vehicle type k
I_{ipt}	the inventory level of product type p at supplier i ($i \in \omega$) or at assembly plant ($i \in F$) in period t	h_{ip}	inventory holding cost in node i for product type p per unit product per period
x_{ijkt}	a binary variable that determines if arc(i,j) is visited by vehicle type k in period t	v_k	variable transportation cost per unit distance for vehicle type $k\ (1, 2, \dots, K)$
y_{ikt}	a binary variable that determines if supplier i is visited by vehicle type k in period t	I_{ip0}	initial inventory level of product type p in node i
b_{ipt}	the quantity of product type p transshipped to supplier i in period t	GHL_t	allowed level of GHG emission in each period
a_{ipt}	the quantity of product type p picked up from supplier i in period t	GHG_k	GHG_s produced by vehicle type k per unit distance
Q_{ijpk}	the quantity of product type p transported by vehicle type k through arc(i,j) in period t		

3. **Mathematical Formulation.** The mixed integer programming for the bi-objective IRP is modeled as follows:

$$Min\, Z_1 = \sum_{(i,j)\in\Omega} \sum_{k,t} v_k c_{ij} x_{ijkt} + \sum_{i\in\omega\cup F,p,t} h_{ip} I_{ipt} + \sum_{i\in\omega,k,t} u_k x_{oikt} \quad (1)$$

$$Min\, Z_2 = \sum_{(i,j)\in\Omega k,t} \sum GHG_k c_{ij} x_{ijkt} \tag{2}$$

Subject to

$$I_{ipt} = I_{ip(t-1)} + b_{ipt} - a_{ipt} \quad \forall i \in \omega,\ p \neq i, t \tag{3}$$

$$I_{(N+1)pt} = I_{(N+1)p(t-1)} + \sum_{i\in\omega,k} Q_{i(N+1)pkt} - D_{pt} \quad \forall p, t \tag{4}$$

$$\sum_{j\in\Omega} x_{ijkt} = \sum_{j\in\Omega} x_{jikt} = y_{ikt} \quad \forall i \in \omega, k, t \tag{5}$$

$$\sum_k y_{ikt} \leq 1 \quad \forall i \in \omega, t \tag{6}$$

$$\sum_{j\in\omega\cup O,k} Q_{jipkt} + a_{ipt} - b_{ipt} = \sum_{j\in\omega\cup F,k} Q_{ijpkt} \quad \forall i \in \omega, p, t \tag{7}$$

$$\sum_p Q_{ijpkt} \leq cap_k x_{ijkt} \quad \forall (i,j) \in \Omega, k, t \tag{8}$$

$$a_{ipt} \leq I_{ip(t-1)} \quad \forall i \in \omega,\ p \neq i, t \tag{9}$$

$$\sum_{i\in\omega} x_{0ikt} \leq NT_{kt} \quad \forall k, t \tag{10}$$

$$\sum_{i\in\omega,k} x_{0ikt} \geq 1 \quad \forall t \tag{11}$$

$$\sum_{i\in\omega} x_{i(N+1)kt} \geq 1 \quad \forall k, t \tag{12}$$

$$x_{i0kt} = 0 \quad \forall i \in \omega, k, t \tag{13}$$

$$x_{(N+1)ikt} = 0 \quad \forall i \in \omega, k, t \tag{14}$$

$$x_{iikt} = 0 \quad \forall i \in \Omega, k, t \tag{15}$$

$$x_{0(N+1)kt} = 0 \quad \forall k, t \tag{16}$$

$$Q_{0ipkt} = 0 \quad \forall i \in \omega, p, k, t \tag{17}$$

$$y_{ikt}, x_{ijkt} \in \{0,1\} \quad \forall (i,j) \in \Omega, k, t$$

$$Q_{ijpkt},\ a_{ipt},\ b_{ipt} \geq 0,\ \text{integer} \tag{18}$$

Equation (1) is the objective function of the proposed model that aims at minimizing total supply chain cost, including inventory holding cost as well as transportation costs. Equation (2) is the objective function of the proposed model to reduce GHGs emissions. Constraint (3) balances the inventory level of product type p at the supplier i in period t. Constraint (4) is an inventory level equation for product p at the assembly plant. Constraints (5) and (6) guarantee that each supplier should be visited only once by the vehicle in each period. Constraint (7) is an inventory balance equation for the supplier i in the current period. Constraint (8) shows that it is not allowed to exceed the vehicle's capacity. Constraint (9) ensures that the quantity of products transshipped is not more than that of the previous periods and products cannot be picked up from the suppliers not producing the product. Constraint (10) guarantees the number of the type k vehicles should be in a given quantity. Constraints (11) and (12) ensure that each trip should start at depot node 0 and end at assembly plant node $N+1$. Constraints (13)-(16) show the impossible arcs. Constraint (17) specifies that the quantity should not be returned to the depot. Finally, Constraint (18) shows the definition of the variable types.

4. **Experimental Result.** Assumed that a company is going to plan its IRP, the planning time horizon of which is divided into two periods. The assembly plant demands 5 types of products transported from 5 suppliers ($S_1, \ldots, S_5$) by a rental truck company (depot) in each period, and each supplier can provide only one product type. The information on the truck types and capacity associated with other data is summarized in Table 2.

Table 3 shows the travel distance. Assumed that the unit inventory holding cost at the assembly (equal to 20) is higher than that of the suppliers (equal to 5), and in each period every supplier has the same unit inventory holding cost. We also assumed that the depot is not allowed to store products and the initial inventories are assumed to be zero at all nodes. The demand for each product per period is provided in Table 4.

The bi-objective IRP programming model is implemented by Matlab R2013b. The transportation cost and GHGs emission of the original model are computed without considering the premised green transportation opinion and the objective of Z_2. The comparison results are reported in Tables 5 and 6.

In Table 5, the GHG emission level of both original model and green model during each period is compared. According to this table, a 39.5% average savings is achieved by

Table 2. Vehicle characteristics

Vehicle type k	v_k	u_k	NT_{kt}		cap_k	GHG_k
			$t = 1$	$t = 2$		
1	13	1000	3	3	500	1.3
2	11	3000	3	3	1000	5.1

Table 3. Travel distance between nodes (c_{ij})

	Depot	S_1	S_2	S_3	S_4	S_5	Assembly plant
Depot	0	30	25	50	60	90	90
S_1	30	0	35	50	45	70	65
S_2	25	35	0	30	60	70	95
S_3	50	50	30	0	50	45	120
S_4	60	45	60	50	0	40	45
S_5	90	70	70	40	40	0	60
Assembly plant	90	65	90	45	45	60	0

Table 4. Demand for each product in each period

Product type p	Period t	
	1	2
1	0	500
2	500	0
3	0	100
4	200	200
5	300	100

Table 5. Greenhouse gas emission level (comparison)

	Original model	Green model	Δ%
Period 1	918.0	943.5	2.8
Period 2	1071.0	260.0	−75.7
Average	994.5	601.75	−39.5

Table 6. Objective function components (comparison)

	Original model	Green model	Δ%
Inventory holding cost	00.0	1000.0	–
Transportation cost	10290.0	9635.0	−6.4
Total cost	10290.0	10635.0	3.4

applying the bi-objective programming model based on the green transportation option. Compared to the first period, the GHGs emissions in the green model solution are 75.7% lower, more than covering the previous increase. As seen in Table 6, the transportation cost for the green model solution is less than the original model solution, but the total cost is 3.4% higher. This increase in total supply chain cost can be interpreted as an extra charge incurred by making use of more fuel-efficient (but more expensive) vehicles to minimize the GHGs emissions. In addition, this increase is a rational result of the inventory holding costs of the products that must be temporarily held at the suppliers (transshipment) to reduce the number of trips. This is to say that the increased total supply chain cost is mainly caused by the inventory cost, and the total cost of green model can have a lower level than the original model if the inventory cost is lower than the saved transportation cost.

5. Conclusions.

In this paper, a bi-objective IRP mathematical model is presented in which one of the objectives is related to transportation cost, and the other to GHGs emissions in a many-to-one supply chain network. The proposed mathematical model is based on two distinct features. Firstly, the green transshipment option is a basic approach to improve the supply chain performance. Under this opinion, a vehicle can transport the product either from the supplier who produced the product or from temporary stored suppliers for the assembly plant. Secondly, vehicles associated with different types and capacities and GHGs emission indices are taken into consideration. The features can enable the proposed model to select an appropriate transportation model to reduce the transportation costs and be more environmental friendly. The result shows the method can be put into practice directly. The further promising areas include applying it to other supply chain structures, developing models under uncertain conditions and considering more detailed parameters (e.g., vehicle speed, loading) that have directly impact on GHGs emission indices.

Acknowledgement. This work is partially supported by the National Key Technology R&D Program (No. 2012BAF09B01) and the Liaoning Province Science and Technology Plan Program (No. 2011402003).

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