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COUPLING ANALYSIS AND LOOP PAIRING METHOD VIA AN INFORMATION-THEORETIC APPROACH

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Abstract. *It is essential to investigate coupling and loop pairing to determine control system configurations. This paper presents a new control-loop configuration criterion for multi-input multi-output (MIMO) processes based on relevant information-theoretic knowledge. Firstly, we propose a new variable pairing rule based on mutual information rate (MIR) among input and output variables. The information coupling strength is formulated in terms of mutual information rates. Then, the straightforward MIR calculation using frequency formula in MIMO linear time invariant (LTI) systems is introduced briefly under information-theoretic framework. Finally, two numerical examples are provided to demonstrate the effectiveness of the above coupling measure and the variable pairing rules. In addition, we analyze the simulation MIR results, which are consistent with RNGA method.*

Keywords: MIMO processes, Variable pairing, Coupling measure, Mutual information rates

1. **Introduction.** Decentralized control strategies are widely utilized in practical multivariable industrial processes due to its easy design, tuning, implementation and maintenance [1-4]. Because the control performance of multi-input multi-output (MIMO) systems will deteriorate with increasing loop interactions, it is necessary to reduce interactions among loops by properly pairing the manipulated variables (MVs) and controlled variables (CVs).

Some efforts have been conducted to analyze interactions and pair loops; the control system configurations can then be determined. The concept of relative gain array (RGA) put forward by Bristol is the most popular method to determine the interactions because of its simple calculation [5]. RGA may lead to unreasonable variable pairing results due to only taking into account steady-state gain. Several improved RGA methods were then proposed. Witcher and McAvoy proposed the dynamic relative gain matrix, which uses the transfer function to calculate dynamic RGA rather than the steady-state gain matrix [6]. Xiong et al. introduced the effective relative gain array (ERGA) to comprehensively reflect the dynamic and static characteristics by employing bandwidth of the transfer function element to amend steady-state matrix [7]. Xiong et al. also presented an alternative method to calculate ERGA for control systems with delay by replacing the bandwidth with the cut-off frequency [8]. Although the cut-off frequency based approach is more accurate and comprehensive for describing loop interactions, the cut-off frequency can hardly be calculated for the process with large delay. He et al. put forward a relative normalized gain array (RNGA), which uses the normalized integrated error to measure the transient information and the steady-state gain of the process [9].

Up till now, very few efforts have been made to investigate interaction and loop pairing of MIMO dynamic systems in the framework of information learning. Wiener pointed out many complicated control systems can be investigated by combining information and control theories [10]. Chen et al. presented several system parameter identification algorithms via information criteria [11,12]. Wang and Guo proposed a series of approaches to modeling, filtering and control of stochastic distribution systems [13,14]. Zhang et al. [15] proposed a minimum entropy-based performance assessment method for the feedback control loops subjected to non-Gaussian disturbances. Ren et al. [16] investigated nonlinear multivariate and non-Gaussian systems using the generalized density evolution equation and presented a control strategy based on an improved entropy criterion.

Following the concept of the mutual information rate (MIR) among variables of an MIMO dynamic system [12], an alternative loop pairing method based on MIRA, RGA and Niederlinski index (NI) is proposed, which considers the information coupling of transfer function comprehensively under information theoretic framework different from some traditional methods. With the aid of the simple frequency calculation of input-output mutual information rate, the relative variables pairing rules can then be formulated. Owing to the completely new angle, simple calculation and combining with the advantages of RGA and NI method, the MIRA based method is comprehensive and simple for field engineers to understand and employ in practical applications. Several typical examples are provided to testify the proposed loop pairing decisions.

This paper is organized as follows. Section 2 presents the loop pairing criteria based on MIR-RGA-NI. Section 3 summarizes the procedures to implement the proposed loop pairing method. In Section 4, the proposed loop pairing method is applied in two illustrative examples to verify its effectiveness. Finally, the last section concludes this paper.

2. Loop Pairing Criteria.

In this section, incorporating the information coupling strength array into the classical RGA-NI loop pairing method, a novel loop pairing method is presented.

2.1. RGA-NI loop pairing method.

The relative gain for an MIMO process is defined by [5]

$$\lambda_{ij} = \frac{(\partial y_i/\partial u_j)_{u_{k\neq j}=c}}{(\partial y_i/\partial u_j)_{y_{l\neq i}=c}} \tag{1}$$

where λ_{ij} is the relative gain of loop $y_i - u_j$; $(\partial y_i/\partial u_j)_{u_{k\neq j}=c}$ is the open-loop steady-state gain and $(\partial y_i/\partial u_j)_{y_{l\neq i}=c}$ the gain u_j to y_i when all other loops are closed.

The RGA denoted by Λ can be expressed by

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{n1} & \lambda_{n2} & \cdots & \lambda_{nn} \end{bmatrix} \tag{2}$$

and calculated directly by open-loop steady-state gain

$$\Lambda = G(0) \otimes G^{-T}(0) \tag{3}$$

where Hadamard product, $\otimes$, is the element-by-element product. $G(0)$ is the steady-state gain matrix. $G^{-T}(0)$ is the transpose of the inverse of $G(0)$.

In addition, NI is often considered when designing MIMO control systems, because the stability of a multiple-loop system can be judged by the positive-negative of NI when all loops are closed [6]. NI provides a necessary condition for a stable paired system. If

$NI < 0$, the system is unstable.

$$NI = \frac{|G(0)|}{\prod_{i=1}^{n} g_{ii}(0)} \tag{4}$$

where $|G(0)|$ is the determinant of matrix $G(0)$; $g_{ii}(0)$ are the main diagonal elements of matrix $G(0)$.

2.2. **MIRA-based loop pairing method.** Mutual information rate (MIR) can be utilized to represent not only steady coupling but also dynamic coupling. In particular, it is natural to deal with stochastic disturbances in MIMO systems as well.

Each MIMO dynamic system can be regarded as an information or uncertainty transmission channel under information theoretic framework. Figure 1 shows the information flow of an MIMO dynamic system. The mutual information rate can be employed to represent information coupling strength because $\bar{I}_{ij}(u_j; y_i)$ measures the average amount of information between the manipulated variable (MV) u_j and the controlled variable (CV) y_i. The information or uncertainty transmission of the channel $u_j \to y_i$ increases with $\bar{I}_{ij}$. The controlled variable (CV) y_i contains no information about u_j when $\bar{I}_{ij} = 0$.

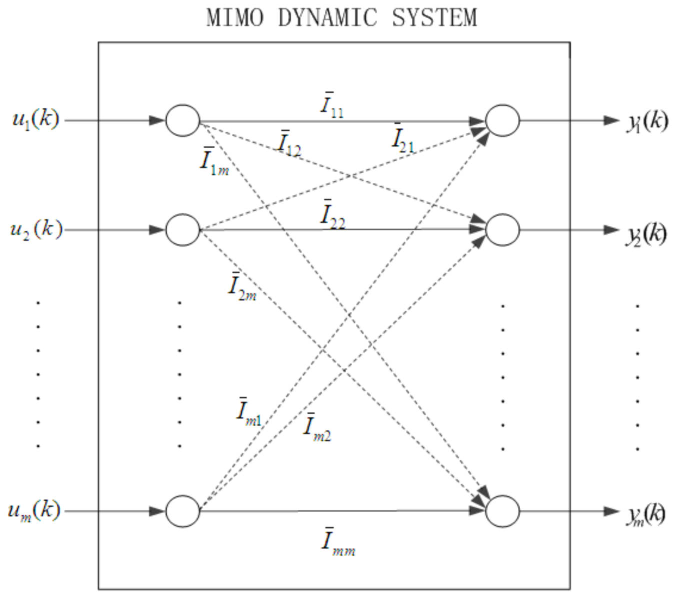

Figure 1. Information flow in the MIMO dynamic system

The mutual information rate array (MIRA) for an MIMO system can be expressed by

$$\bar{I} = \begin{bmatrix} \bar{I}_{11} & \bar{I}_{12} & \cdots & \bar{I}_{1n} \\ \bar{I}_{21} & \bar{I}_{22} & \cdots & \bar{I}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{I}_{n1} & \bar{I}_{n2} & \cdots & \bar{I}_{nn} \end{bmatrix} = \begin{bmatrix} \bar{I}(u_1; y_1) & \bar{I}(u_2; y_1) & \cdots & \bar{I}(u_n; y_1) \\ \bar{I}(u_1; y_2) & \bar{I}(u_2; y_2) & \cdots & \bar{I}(u_n; y_2) \\ \vdots & \vdots & \ddots & \vdots \\ \bar{I}(u_1; y_n) & \bar{I}(u_2; y_n) & \cdots & \bar{I}(u_n; y_n) \end{bmatrix} \tag{5}$$

The loop pairing method is then proposed based on MIR and Niederlinski index (NI). MVs and CVs in an MIMO system should be paired so that the following rules hold:

Rule 1: The elements of the paired RGA are positive.

Rule 2: NI is positive for paired loops.

Rule 3: The diagonal elements of the paired MIRA are the biggest one in its row or column.

Rule 1 guarantees the paired loops have small steady coupling. Rule 2 provides a necessary condition to maintain stability of the paired MIMO system. Compared with existing variable pairing methods, the MIRA uses the information theory to make comprehensive consideration for the total system not for separating steady-state and transient information. Besides, owing to the above properties of $\bar{I}_{ij}(u_j; y_i)$, the paired elements are convenient to be determined. So the MIR-based pairing rules can provide simpler, more accurate and more comprehensive coupling measures and pairing results.

Remark 2.1. *Due to the non-negative property of the mutual information rate, we do not need to emphasize that the elements of the paired MIRA are positive.*

3. **MIRA Calculation of LTI MIMO Systems.** In this section, the frequency formula of calculating MIRA of an LTI MIMO system is introduced briefly. The generality of this algorithm is applied to the special case study of the LTI MIMO systems expressed by

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \tag{6}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$, $u = [u_1, u_2, \ldots, u_m]^T \in R^m$ and $y = [y_1, y_2, \ldots, y_m]^T \in R^m$ are system state, input vector and output vector respectively. A, B and C are the matrices with proper dimensions.

The transfer function of the system is

$$\bar{G}(z) = C(zI - A)^{-1}B \tag{7}$$

The frequency domain ω form is obtained by replacing z with $e^{j\omega}$

$$\bar{G}(\omega) = C\left(Ie^{j\omega} - A\right)^{-1} B \tag{8}$$

where $\bar{G}(\omega)$ is an $m \times m$ rational fractional matrix and $\bar{G}(z) \in RH_\infty$ if the system is stable. Denote the ith row jth column element of $\bar{G}(\omega)$ by $\bar{G}_{ij}(\omega)$. Before introducing the calculation process of the frequency formula of the input-output MIRA, the following lemmas and assumption should be given.

Lemma 3.1. *Let $\bar{G}(z) \in RH_\infty$ be the $m \times n$ transfer function matrix of a discrete-time MIMO LTI system. If the stationary input process $u(k) \in R^n$ has the spectral density Φ_x, the spectral density of the output will be $\Phi_y = \bar{G}(\omega)\Phi_u\bar{G}^*(\omega)$ where $\bar{G}^*(\omega)$ is the conjugate of $\bar{G}(\omega)$.*

Lemma 3.2. [17] *Under the stationary Gaussian processes, the mutual information rate can be expressed as*

$$\bar{I}(\{X_t\}; \{Y_t\}) = \frac{1}{4\pi}\int_{-\pi}^{\pi} \ln \frac{\det \Phi_X(\omega) \det \Phi_Y(\omega)}{\det \Phi_Z(\omega)} d\omega \tag{9}$$

where $\{X_t \in R^n, t \in Z\}$ and $\{Y_t \in R^m, t \in Z\}$ are two joint Gaussian stationary processes respectively having spectral densities $\Phi_X(\omega)$ and $\Phi_Y(\omega)$, and $\Phi_Z(\omega)$ is the spectral density of $\{Z_t = [X_t^T, Y_t^T] \in R^{n+m}, t \in Z\}$.

Assumption 3.1. *In the system (6), the input vector $u(k)$ is a zero-mean unit Gaussian white noise with spectral density of $m \times m$ identity matrix I.*

According to the above assumption and Lemma 3.1, the spectral density of output vector $y_i(k)$ is

$$\Phi_{yi}(\omega) = \bar{G}_i(\omega)\Phi_{uj}(\omega)\bar{G}_i^*(\omega) = \bar{G}_i(\omega)\bar{G}_i^*(\omega) = \sum_{j=1}^{m} |\bar{g}_{ij}(\omega)|^2 \tag{10}$$

Utilizing this property, construct a novel transfer function as follows:

$$T_{ij}(\omega) = \begin{bmatrix} 0 & \cdots & 1 & \cdots & 0 \\ \bar{g}_{i1}(\omega) & \cdots & \bar{g}_{ij}(\omega) & \cdots & \bar{g}_{im}(\omega) \end{bmatrix} \tag{11}$$

Then the new fictitious output vector can be denoted by

$$\eta_{ij}(\omega) = T_{ij}(\omega)\bar{U}(\omega) = \begin{bmatrix} \bar{u}_j(\omega) \\ \bar{y}_i(\omega) \end{bmatrix} \tag{12}$$

where the input vector $\bar{U}(\omega) = [\bar{u}_1(\omega) \cdots \bar{u}_j(\omega) \cdots \bar{u}_m(\omega)]^T$.

According to Lemma 3.1, the spectral density of $\eta_{ij}(\omega)$ is derived as follows

$$\Phi_{\eta ij}(\omega) = T_{ij}(\omega)T_{ij}^*(\omega) = \begin{bmatrix} 1 & \bar{g}_{ij}^*(\omega) \\ \bar{g}_{ij}(\omega) & \Phi_{yi}(\omega) \end{bmatrix} \tag{13}$$

Because $\det\Phi_{uj}(\omega) = 1$ and $\det\Phi_{\eta ij}(\omega) = \Phi_{yi}(\omega) - \bar{g}_{ij}(\omega)\bar{g}_{ij}^*(\omega) = \Phi_{yi}(\omega) - |\bar{g}_{ij}(\omega)|^2$, the mutual information rate $\bar{I}(u_j; y_i)$ can be calculated according to Lemma 3.2.

$$\begin{aligned} \bar{I}(u_j; y_i) &= \frac{1}{4\pi}\int_{-\pi}^{\pi} \ln \frac{\det\Phi_{uj}(\omega)\det\Phi_{yi}(\omega)}{\det\Phi_{\eta ij}(\omega)} d\omega \\ &= \frac{1}{4\pi}\int_{-\pi}^{\pi} \ln \frac{\Phi_{yi}(\omega)}{\Phi_{yi}(\omega) - |\bar{g}_{ij}(\omega)|^2} d\omega \end{aligned} \tag{14}$$

Remark 3.1. *The mutual information rate between input and output variables reflects dynamic and steady coupling.*

4. **Illustrative Case Studies.** A potential weakness of RGA or RGA-NI methods is that they only use the steady state gains which are based on the assumption of perfect loop control to determine loop pairing. Considering the information of transfer function comprehensively, MIRA based method can show better dynamic applicability. We use the following typical examples to illustrate this point, where the RGA method fails and gives erroneous results.

Case 1. Consider the following two-input two-output transfer function [9]:

$$\begin{bmatrix} \dfrac{5e^{-s}}{100s+1} & \dfrac{e^{-4s}}{10s+1} \\ \dfrac{-5e^{-4s}}{10s+1} & \dfrac{5e^{-s}}{100s+1} \end{bmatrix} \tag{15}$$

The results obtained from RGA, RNGA and MIRA loop pairing methods are shown in Table 1.

Table 1. Loop pairing results

Tools	RGA	RNGA
Calculated results	$\begin{bmatrix} 0.8333 & 0.1667 \\ 0.1667 & 0.8333 \end{bmatrix}$	$\begin{bmatrix} 0.0867 & 0.9124 \\ 0.9124 & 0.0867 \end{bmatrix}$
Conclusions	Diagonal pairing	Off-diagonal pairing
Tools	MIRA	NI
Calculated results	$\begin{bmatrix} 0.1293 & 2.2650 \\ 0.7778 & 0.0070 \end{bmatrix}$	5.9989
Conclusions	Off-diagonal pairing	Positive

Case 2. Consider the following three-input three-output transfer function [9]:

$$\begin{bmatrix} \dfrac{e^{-9s}}{6s^2+17s+1} & \dfrac{-9e^{-5s}}{s^2+4s+1} & \dfrac{13e^{-3s}}{3s^2+35s+1} \\ \dfrac{-5e^{-13s}}{2s^2+19s+1} & \dfrac{8e^{-2s}}{s^2+33s+1} & \dfrac{7e^{-5s}}{s^2+3s+1} \\ \dfrac{-16e^{-3s}}{s^2+5s+1} & \dfrac{3e^{-7s}}{s^2+14s+1} & \dfrac{e^{-11s}}{3s^2+25s+1} \end{bmatrix} \tag{16}$$

The results obtained from RGA, RNGA and MIRA loop pairing methods are shown in Table 2.

Table 2. Loop pairing results

Tools	RGA	RNGA
Calculated results	$\begin{bmatrix} -0.0054 & 0.3981 & 0.6073 \\ -0.0992 & 0.6912 & 0.4080 \\ 1.1046 & -0.0893 & -0.0153 \end{bmatrix}$	$\begin{bmatrix} -0.0024 & 0.9237 & 0.0787 \\ -0.0063 & 0.0829 & 0.9235 \\ 1.0088 & -0.0066 & -0.0022 \end{bmatrix}$
Conclusions	Off-diagonal pairing	1-2, 2-3, 3-1
Tools	MIRA	NI
Calculated results	$\begin{bmatrix} 0.0003 & 1.5824 & 0.0286 \\ 0.0141 & 0.0165 & 1.5487 \\ 2.5178 & 0.0034 & 0.0001 \end{bmatrix}$	2.3998
Conclusions	1-2, 2-3, 3-1	Positive

It can be observed from Table 1 and Table 2 that MIRA based method can determine the loop pairs. Compared with RGA based method, the results obtained based on MIRA method are more accurate and comprehensive due to taking dynamic information into consideration. Owing to the monotonicity and non-negative properties of MIRA method, it is very convenient to determine the pairing relationship compared with RNGA based method. In addition, MIRA can be directly calculated according to the derived frequency formula.

5. **Conclusion.** In this paper, an MIRA based loop pairing criterion is proposed. MIR is introduced to measure the information couplings among multiple variables. The presented method is then applied into two illustrative examples to testify its effectiveness. The general MIRA based loop pairing method will be investigated for MIMO nonlinear systems in further research.

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