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PATH FOLLOWING FOR UNDERACTUATED SHIPS CONTROL AND SIMULATION BASED ON ACTIVE DISTURBANCE REJECTION WITH SLIDING MODE CONTROL

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Abstract. *A hybrid solution is proposed for motion control of underactuated surface ships with actuator constraints and external disturbances in this paper. Active disturbance rejection control (ADRC), in conjunction with sliding mode control (SMC), forms an effective and robust control structure to steer the ship to follow the desired path. The cross track error caused by wind and ocean current is removed via designing a coordinate transformation of nonlinear equation in the control law. Simulation study was performed based on a training ship to verify the validity of the proposed controller on a navigation simulator platform.*
Keywords: ADRC, Path following, Underactuated ships, Sliding mode

1. **Introduction.** The motion control problems of underactuated surface ships are challenging due to the fact that the ships possess more degrees of freedom to be controlled than the number of the independent controls under some nonintegrable second-order nonholonomic constraints [1]. Path following is defined as a control problem of forcing an underactuated ship to follow a specified path at a desired forward speed [2]. The problem of path-following was introduced in [3] where some local results were obtained using linearization techniques. A fourth-order ship model in Serret–Frenet frame was used in [4] to develop a control strategy to track both a straight line and a circumference under constant ocean current disturbance. Adaptive robust controllers are proposed based on hierarchical sliding mode method which suits various forms of curve [5]. In [6], the problem of two dimensional trajectory tracking for autonomous marine surface vehicles is addressed using model predictive control (MPC).

Aiming specially to the uncertain system, the ADRC technique which has been proved very effective due to its independence on accurate mathematical model of the plant was proposed by Han [7], and then the linear ADRC (LADRC) was developed to achieve the parameterization in [8]. Meanwhile, the sliding mode control (SMC) has also attractive features to keep the systems insensitive to uncertainties on the sliding surface [9]. The target of this paper is to develop a path following controller to improve the performance in adaptation and robustness by employing the combination of ADRC and SMC. The controller combines the advantages of ADRC with SMC to make the parameters' physical meaning more obvious and be tuned easily. The cross-track error resulting from wind and current is removed.

The rest of this paper is organized as follows. The ship model and ADRC theory are described in Section 2. Section 3 proposes the hybrid design of ADRC and SMC. The ship path following controller is designed in Section 4. Section 5 studies the simulations to validate the superior control performance on a navigation simulator platform. Finally, Section 6 contains the main conclusions.

2. Problem Formulation and Preliminaries.

2.1. The ship kinematics and dynamics model.

The kinematics and dynamics (MM-G) model of an underactuated ship in surge, sway and yaw in the earth-fixed and the body-fixed frames can be described as

$$\begin{cases} \dot{x} = u_r \cos\psi - v_r \sin\psi + V_c \cos\psi_c = V_X \cos\psi - V_Y \sin\psi \\ \dot{y} = u_r \sin\psi + v_r \cos\psi + V_c \sin\psi_c = V_X \sin\psi + V_Y \cos\psi \\ \dot{\psi} = r \\ (m + m_x)\dot{V}_X - (m + m_y)V_Y r = X_H + X_P + X_R(\delta) + X_E + (m_x - m_y)V_c \sin(\psi_c - \psi)r \\ (m + m_y)\dot{V}_Y + (m + m_x)V_X r = Y_H + Y_P + Y_R(\delta) + Y_E - (m_x - m_y)V_c \cos(\psi_c - \psi)r \\ (I_{zz} + J_{zz})\dot{r} = N_H + N_P + N_R(\delta) + N_E \end{cases} \tag{1}$$

where x, y and ψ are the longitudinal displacement, lateral displacement and heading angle, respectively, in the earth-fixed frame, V_X and V_Y are the longitudinal, lateral velocities over ground, and r is the yaw angular rate in the ship-fixed frame. u_r and v_r denote surge, sway velocities through water, V_c and ψ_c denote the speed and set of current in the earth-fixed frame. δ is rudder angle. m, m_x, m_y, I_{zz} and J_{zz} denote the ship inertia, added mass, and added moment of inertia. X, Y, N terms with subscripts H, P, R, and E, respectively, are longitudinal and lateral forces, and moments induced by hydrodynamic damping, propeller, rudder, and other external effects except current.

In practice, when an underactuated surface ship travels at sea, the rudder angle is the only control input to follow a desired path and to steer a comparatively steady course. However, cross track must be compensated by sideslip compensation when the ship is affected by wind and ocean current since no sway control means are available.

2.2. ADRC structure and its algorithm.

Consider a generally nonlinear time-varying 2nd order dynamic system

$$\ddot{y}(t) = f\left(\dot{y}(t), y(t), d(t)\right) + bu \tag{2}$$

where y and u are output and input, respectively, and $d(t)$ is the external disturbance. Here $f\left(\dot{y}(t), y(t), d(t)\right)$ represents the unknown nonlinear time-varying dynamics of the plant. b is control gain and unknown, although some knowledge of b can be got, i.e., $b \approx b_0$. Rewrite (2) as

$$\ddot{y}(t) = f\left(\dot{y}(t), y(t), d(t)\right) + (b - b_0)u + b_0 u = f + b_0 u \tag{3}$$

where $f = f\left(\dot{y}(t), y(t), d(t)\right) + (b - b_0)u$ is referred to as the generalized disturbance. Assuming f is differentiable, let $x_3 = f$, and an extend state observer (ESO) of the augmented state space form, which will estimate the derivatives of y and f, is given as

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 - l_1\left(\hat{x}_1 - x_1\right) \\ \dot{\hat{x}}_2 = \hat{x}_3 - l_2\left(\hat{x}_1 - x_1\right) + b_0 u \\ \dot{\hat{x}}_3 = -l_3\left(\hat{x}_1 - x_1\right) \end{cases} \tag{4}$$

where $\hat{x} = [\hat{x}_1 \;\; \hat{x}_2 \;\; \hat{x}_3]^T$ is the estimate of the state of x, and l_i, $i = 1, 2, 3$, are the observer gain parameters. The observer gains are chosen such that the characteristic polynomial

$s^3 + l_1 s^2 + l_2 s + l_3$ is Hurwitz. For tuning simplicity, all the observer poles are placed at $-\omega_o$ [8] . It results in the characteristic polynomial of (4) to be

$$s^3 + l_1 s^2 + l_2 s + l_3 = (s + \omega_o)^3 \tag{5}$$

where the observer bandwidth ω_o, is the sole turning parameter, and

$$L = [l_1 \;\; l_2 \;\; l_3] = \left[3\omega_o \;\; 3\omega_o^2 \;\; \omega_o^3\right]^T \tag{6}$$

The ADRC control law is given by

$$u = \frac{-k_p\left(\hat{x}_1 - v\right) - k_d\left(\hat{x}_2 - \dot{v}\right) + \ddot{v} - \hat{x}_3}{b_0} \tag{7}$$

where v is the reference input, k_p and k_d are the controller gain parameters selected to make $s^2 + k_d s + k_p$ be Hurwitz.

The closed loop system becomes

$$\ddot{y} = (f - \hat{x}_3) - k_p\left(\hat{x}_1 - v\right) - k_d\left(\hat{x}_2 - \dot{v}\right) + \ddot{v} \tag{8}$$

Note that with a well-designed ESO, the estimation error in $\hat{x}_i$, $i = 1, 2, 3$, is ignored, and let $e = x_1 - v$, then $\hat{x}_1 - v \approx e$ and $\hat{x}_2 - \dot{v} \approx \dot{e}$. The plant (3) is reduced to

$$\ddot{e} = -k_p e - k_d \dot{e} = u_0 \tag{9}$$

which is a classic Proportion Differentiation (PD) control law.

3. **Feedback Control Law Design with Sliding Mode Control.**

3.1. **Feedback control law with linear sliding mode.** The method of determining k_p and k_d was proposed using the bandwidth idea in [8] , but the physical meaning is not clear. Furthermore, it is hard to achieve perfect tracking performance. Therefore, the sliding mode idea is applicable to the design of error feedback control law in this section.

Let $k_1 = k_p/k_d$ and $k_2 = k_d$, Equation (3) or (9) becomes

$$\ddot{e} = -k_2\left(k_1 e + \dot{e}\right) \tag{10}$$

And let

$$\sigma = k_1 e + \dot{e} \tag{11}$$

While $\sigma \to 0$, $\dot{e}(t) = -k_1 e(t)$, $e(t)$ will converge in index law, and time constant is $1/k_1$. Therefore, σ is the phrase locus of e and $\dot{e}_1$ on the phrase plane. For Equation (11),

$$\dot{\sigma} = k_1\dot{e} + \ddot{e} = k_1\dot{e} + u_0 = k_1\dot{e} - k_2\sigma \tag{12}$$

Owing to $k_1\dot{e}(t) \to 0$ while $t \to \infty$, and $k_d > 0$, system (12) is a simple proportion negative feedback system, that is

$$\dot{\sigma} = -k_2\sigma \tag{13}$$

The essence of PD control for a second order system is similar to the design of only 1st order sliding mode in form. The control law of Equation (9) is written as

$$u_0 = -k_2\left(k_1 e + \dot{e}\right) \tag{14}$$

where $k_1 > 0$ and $k_2 > 0$ are the design parameters, in addition, $k_2 > k_1$ generally.

3.2. **Feedback control law with nonlinear sliding mode.** Linear sliding mode requests that the system state has a large convergence rate when it is at a great deviation, which can be achieved by high speed and large control input. The state of an actual system has constraint condition due to the limitation of control input, so the form of nonlinear sliding mode (NLSM) in the phase plane can be used. The monotone bounded hyperbolic tangent function can be selected as the nonlinear sliding mode function, so we define

$$\sigma = k_1 \tanh(k_0 e) + \dot{e} \tag{15}$$

The feedback control law of system (8) becomes

$$u_0 = -k_2 \left(k_1 \tanh(k_0 e) + \dot{e}\right) \tag{16}$$

where $k_i \in \mathbb{R}^+$, $i = 0, 1, 2$, are the parameters to be tuned. If $\sigma \to 0$, then $\dot{e} \to -k_1 \tanh(k_0 e)$, where $\max(|\dot{e}|) \to k_1$ and $|\dot{e}| < k_1$. So the maximum system convergence rate is less than k_1.

4. Ship Path Following Control Design.

To design an ADRC ship tracking controller, the path following design model is derived from Equation (1) as Equation (17),

$$\begin{cases} \dot{y} = V_X \sin\psi + V_X \cos\psi \\ \dot{\psi} = r \\ \dot{r} = f(r, w) + b\delta \end{cases} \tag{17}$$

where, $f(r, w)$ is a multivariable function of both the states and external disturbances. w is the external disturbances. $b > 0$, is the control signal gain. The first formula of system (17) can be written as

$$\dot{y} = V_X \sin\psi + V_Y \cos\psi = \sqrt{V_X^2 + V_Y^2}\sin(\psi + \beta) \tag{18}$$

where $\beta = \arctan(v/u)$, is called the ship leeway angle or drift angle, which can be measured by GPS in real time on a modern ship. $\psi + \beta$ is the ship course over the ground.

Let $y_e = y - y_d$, where y_d is the reference lateral displacement, a nonlinear function of y_e and $\psi + \beta$ by designing a coordinate transformation equation is defined as

$$z = c_1 \tanh(c_0 y_e) + (\psi + \beta) \tag{19}$$

where, $c_0 > 0$ and $\pi > c_1 > 0$ are design parameters.

Theorem 4.1. *For the system (17), if the nonlinear function $z = c_1 \tanh(c_0 y_e) + (\psi + \beta) \to 0$, then both $y_e \to 0$ and $\psi + \beta \to 0$.*

Proof: There exists a Lyapunov function $V = \frac{1}{2}y_e^2$, then $\dot{V} = y_e\dot{y}_e = y\sqrt{V_X^2 + V_Y^2}\sin(z - c_1 \tanh(c_0 y_e))$, if $z = 0$, then $\dot{V} = y_e\sqrt{V_X^2 + V_Y^2}\sin(-c_1 \tanh(c_0 y_e))$, while $y_e > 0$ and $\pi > c_1 > 0$, then $-c_1 < -c_1 \tanh(c_0 y_e) < 0$, we have $\sin(-c_1 \tanh(c_0 y_e)) < 0$, so $\dot{V} < 0$; in addition, while $y_e < 0$ and $\pi > c_1 > 0$, then $\sin(-c_1 \tanh(c_0 y_e)) > 0$, so $\dot{V} < 0$; while $y_e = 0$, $\dot{V} = 0$. Thus, while $V = \frac{1}{2}y_e^2 \geq 0$, then $\dot{V} \leq 0$. While $z \to 0$, $y_e \to 0$, meanwhile, $\psi + \beta \to 0$. □

In Equation (19), c_0 is used to compress coordinate, and c_1 is used to adjust ship track convergence rate. Meanwhile, c_1 can limit the maximum course angle over ground to be used when the ship returns to the planned route.

Let $\psi^* = -c_1 \tanh(c_0(y - y_d)) - \beta$, ψ^* is the desired ship heading angle that makes the ship cross track error converge to zero. Hence, the ship track control problem is transformed into ship course control. Path following can be achieved by letting ship heading angle track the specified ship heading angle ψ^* which is regarded as reference signal. System (19) can be transformed in the following system

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = f(z_1, z_2) + bu \\ z = z_1 \to 0 \end{cases} \tag{20}$$

where z is output, and u is input, which is rudder angle.

The ESO of (20) is

$$\begin{cases} \dot{\hat{z}}_1 = \hat{z}_2 - l_1 \hat{z}_1 \\ \dot{\hat{z}}_2 = \hat{z}_3 - l_2 \hat{z}_1 + b_0 u \\ \dot{\hat{z}}_3 = -l_3 \hat{z}_1 \end{cases} \tag{21}$$

where $\hat{z}_1 \to z_1$, $z_2 \to \dot{z}_1$ and $\hat{z}_3 \to \hat{f} = f(z_1, z_2)$, $l_1 = 3\omega_o$, $l_2 = 3\omega_o^2$ and $l_3 = \omega_o^3$, and then the ADRC controller is

$$u = -\left(u_0 + \hat{f}\right)/b_0 \tag{22}$$

The 1st order nonlinear sliding mode feedback control u_0 is

$$u_0 = -k_2\left(k_1 \tanh(k_0 \hat{z}_1) + \dot{\hat{z}}_1\right) \tag{23}$$

5. **Simulation Study.** In this section, simulation study is performed on a navigation simulator platform with electronic chart displaying system (ECDIS). The simulation result is based on an oceangoing training vessel "Yulong" of Dalian Maritime University. The principal particulars of the ship are as follows: length is 126m, breadth is 20.8m, mean draft is 8.80m, diameter of propeller is 4.6m, and block coefficient is 0.681. In the simulation research, the ship path following controller is trained and tested in the entrance fairway of Dalian Port, where the effects of wind and ocean current are considered. The constant wind direction is NE with a speed of 5m/s, and the constant current set is SE with a velocity of 1 knot. The initial speed of ship is set 13 knots, and initial heading angle is 15°. Main engine is set full ahead at the beginning of the simulation.

The parameters of ADRC are chosen as $b_0 = 0.0022$, $\omega_o = 0.4$. The parameters of NSML control law are chosen as $k_0 = 3$, $k_1 = 0.02$, $k_2 = 0.03$, where $k_1 = 0.02$ means the maximum turning rate is 0.02rad/s. The parameters of the transformation Equation (19) are chosen as $c_0 = 0.03$ and $c_1 = \pi/6$, respectively, where $c_1 = \pi/6$ indicates $\max(\psi + \beta) = \pi/6$, which may occur when ship is turning. The simulation results are shown in Figure 1.

In Figure 1, the solid line indicates the planning route and the dashed line relates to the actual path that the ship navigates. We can see that the dashed line and the solid line overlap basically except at the waypoint where the ship alters the course. By observing the data of tracking error, the actual tracking error is less than 10 meters, which is less than the ship breadth, barring some exceptions at the waypoint. This illustrates that the ship travels on the planning route with high precision, and the simulation results indicate

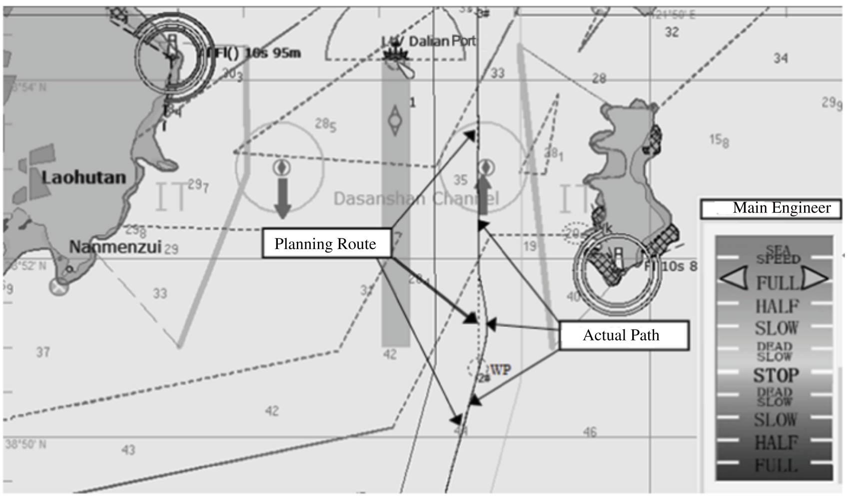

FIGURE 1. Simulation results of path following in Dalian entrance

that the ADRC controller has a powerful robustness to the environment disturbances and system uncertainties.

6. **Conclusions.** This paper has presented a novel path following control approach to underactuated vessels under disturbances of ocean current and wind. A compound control approach of ADRC with sliding mode has been applied to the design of ship path following control. The leeway angle has been compensated in the controller by means of designing a coordinate transformation equation. The highly precise ship tracking controller is robust in presence of the internal uncertainties of system and the external disturbance. To improve the tracking precision at the waypoint is our future work.

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