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ADAPTIVE NEURAL BACKSTEPPING CONTROL FOR UNCERTAIN UNIFIED CHAOTIC SYSTEMS WITH INPUT SATURATION

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Abstract. *In this paper, a neural network adaptive backstepping control scheme for uncertain unified chaotic systems is proposed. During the controller design, radial basis function (RBF) neural networks are used to approximate packaged unknown nonlinearities, and then an adaptive neural controller is proposed based on backstepping technique. The developed control scheme guarantees that all the signals involoved are bounded. Simulation results are applied to demonstrating the feasibility of the suggested control scheme.*
Keywords: Adaptive neural control, Unified chaotic system, Input saturation

1. **Introduction.** In the past decades, many researchers have been actively studying the control or synchronization of chaotic systems since a series of electronic, mechanical and chemical systems exhibit chaotic dynamics. So far, many remarkable control strategies have been proposed. In [1-5], several adaptive control schemes were presented for uncertain Lorenz system with unknown parameters. Furthermore, the problem of adaptive control of uncertain Lü system is reported in [1, 2], and in [3, 4], the adaptive control for uncertain unified chaotic system was proposed. To reduce the number of update laws, Chen et al. [5] presented an adaptive fuzzy control scheme with two controllers to control unified chaotic systems. In addition, the control of nonlinear systems preceded by saturation nonlinearities has been an active topic since the saturation nonlinearities are common in many practical systems. The existence of input saturation gravely limits the system performance or gives rise to undesirable inaccuracy. Therefore, the advanced control techniques to handle the effects of saturation have been called upon and have been studied for decades. In [7], Chen et al. proposed a robust adaptive neural control for a class of MIMO nonlinear systems with input nonlinearities. Wen et al. [8] investigated the problem of adaptive control for a class of uncertain nonlinear systems in the presence of input saturation and external disturbance, in which two new schemes are developed to compensate for the effects of the saturation nonlinearity and disturbances.

Based on the above observations, we consider the problem of robust adaptive neural control for uncertain unified chaotic systems with input saturation. During the controller design, RBF neural networks are used to approximate the uncertain nonlinear function and backstepping technique is employed to construct an adaptive controller. The proposed neural-based adaptive controller ensures the boundedness of all signals in the closed-loop system. Finally, the theoretic results are further illustrated through a numerical example.

The remainder of this paper is organized as follows. The problem formulation and preliminaries are given in Section 2. An adaptive neural control scheme is presented in Section 3. The simulation example is given in Section 4, followed by Section 5 which concludes the work.

2. **Problem Statement and Preliminaries.** In this paper, we consider the uncertain unified chaotic system described by

$$\begin{cases} \dot{s}_1 = a_1(s_2 - s_1), \\ \dot{s}_2 = a_2 s_1 - s_1 s_3 - a_3 s_2, \\ \dot{s}_3 = s_1 s_2 - a_4 s_3 + u, \end{cases} \tag{1}$$

where $a_1 = 25\beta + 10$, $a_2 = 28 - 35\beta$, $a_3 = 1 - 29\beta$ and $a_4 = \frac{\beta+8}{3}$ with $\beta \in [0, 1]$ being an uncertain parameter, s_i, $i = 1, 2, 3$, are the system states, and u denotes the system input subject to nonlinear saturation given by

$$u = \mathrm{sat}(v) = \begin{cases} \mathrm{sign}(v) u_M, & |v| \geq u_M, \\ v, & |v| < u_M, \end{cases} \tag{2}$$

where u_M is an unknown parameter of input saturation, and v is input signal of the saturation nonlinearity.

From (2), the backstepping technique cannot be applied to designing controller in a direct manner since there exist sharp corners when $|v| = u_M$. In order to handle this problem, a smooth function is applied to estimating the saturation function with a bounded error and defined as

$$g(v) = u_M * \tanh(v/u_M) = u_M * \frac{e^{v/u_M} - e^{-v/u_M}}{e^{v/u_M} + e^{-v/u_M}}. \tag{3}$$

Further, $\mathrm{sat}(v)$ in (2) is described as

$$\mathrm{sat}(v) = g(v) + d(v), \tag{4}$$

where $d(v) = \mathrm{sat}(v) - g(v)$ can be bounded as

$$|d(v)| = |\mathrm{sat}(v) - g(v)| \leq u_M(1 - \tanh(1)) = D. \tag{5}$$

By mean-value theorem [6], there exists a constant μ $(0 < \mu < 1)$ such that

$$g(v) = g(v_0) + g_{v_\mu}(v - v_0), \tag{6}$$

where $g_{v_\mu} = \frac{\partial g(v)}{\partial v}|_{v=v_\mu}$, $v_\mu = \mu v + (1 - \mu) v_0$. Select $v_0 = 0$. We rewrite (6) in the following form:

$$g(v) = g_{v_\mu} v. \tag{7}$$

To facilitate the controller design, the following assumption is imposed.

Assumption 2.1. *For the function g_{v_μ} in (6), there exists an unknown positive constant g_m such that*

$$0 < g_m \leq g_{v_\mu} \leq 1. \tag{8}$$

The objective is to construct a robust adaptive neural controller v for the system (1) such that all signals in the closed-loop system remain bounded.

In the following, RBF neural networks will be used to model any continuous function $f(Z): R^n \to R$,

$$f_{nn}(Z) = W^T S(Z), \tag{9}$$

where $Z \in \Omega_Z \subset R^q$ is the input vector with q being the neural networks input dimension, weight vector $W = [w_1, w_2, \ldots, w_l]^T \in R^l$, $l > 1$ is the neural networks node number, and $S(Z) = [s_1(Z), s_2(Z), \ldots, s_l(Z)]^T$ means the basis function vector with $s_i(Z)$ being chosen as the commonly used Gaussian function of the form $s_i(Z) = \exp\left[-(Z - \mu_i)^T (Z - \mu_i)/\eta_i^2\right]$, $i = 1, 2, \ldots, l$, where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{iq}]^T$ is the center of the receptive field and η_i is the width of the Gaussian function. In [9], it has been indicated that with sufficiently large node number l, the RBF neural networks (9) can approximate any continuous function $f(Z)$ over a compact set $\Omega_Z \subset R^q$ to any accuracy $\varepsilon > 0$ as $f(Z) = W^{*T} S(Z) + \delta(Z)$, $\forall z \in \Omega_z \in R^q$, where W^* is the ideal constant weight vector and $\delta(Z)$ denotes the approximation error and satisfies $|\delta(Z)| \leq \varepsilon$.

3. **Adaptive Neural Control.** An adaptive neural backstepping control scheme will be proposed, which is developed based on the following coordinate transformation:

$$z_i = x_i - \alpha_{i-1}, \quad i = 1, 2, 3, \tag{10}$$

where $\alpha_0 = 0$, and α_i is a virtual control signal. The real controller v will be constructed in the final step to stabilize the whole systems.

Step 1: Based on $z_1 = s_1$, one has

$$\dot{z}_1 = a_1(s_2 - s_1). \tag{11}$$

Consider a Lyapunov function $V_1 = \frac{1}{2}z_1^2$. Then, the time derivative of V_1 is

$$\dot{V}_1 = z_1 a_1(s_2 - s_1) = a_1 z_1 z_2 + z_1 a_1(\alpha_1 - s_1), \tag{12}$$

where $z_2 = s_2 - \alpha_1$. Noted that $s_1 = z_1$, we construct a virtual control input α_1 as

$$\alpha_1 = -k_1 z_1 \tag{13}$$

with $k_1 > -1$ being a design parameter. By substituting (13) into (12), one has

$$\dot{V}_1 = -a_1(k_1 + 1)z_1^2 + a_1 z_1 z_2, \tag{14}$$

where the term $a_1 z_1 z_2$ will be handled in Step 2.

Step 2: In this step, the singularity caused by $-s_1 s_3$ in the second subsystem of (1) is dealt with. Based on $z_2 = s_2 - \alpha_1$, one has

$$\begin{aligned}\dot{z}_2 &= \dot{s}_2 - \dot{\alpha}_1 \\ &= -s_1 s_3 - a_3 s_2 + a_2 s_1 + k_1 a_1(s_2 - s_1) \\ &= -s_1 s_3 - (a_3 - k_1 a_1)z_2 + (a_3 - k_1 a_1)k_1 s_1 + a_2 s_1 - k_1 a_1 s_1.\end{aligned} \tag{15}$$

Choose a Lyapunov function candidate as

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2\gamma_1}\tilde{p}^2, \tag{16}$$

where γ_1 is a positive design constant and $\tilde{p} = p - \hat{p}$ with $\hat{p}$ being the estimation of unknown constant p.

Then, we have

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + z_2\{-s_1 s_3 - (a_3 - k_1 a_1)z_2 + (a_3 - k_1 a_1)k_1 s_1 + a_2 s_1 - k_1 a_1 s_1\} - \frac{1}{\gamma_1}\tilde{p}\dot{\hat{p}} \\ &= -a_1(k_1 + 1)z_1^2 - (a_3 - k_1 a_1)z_2^2 - z_1 z_2 z_3 + a_1 z_1 z_2 + z_2\{-s_1\alpha_2 + k_1(a_3 - k_1 a_1)s_1 \\ &\quad + a_3 s_1 - k_1 a_1 s_1\} - \frac{1}{\gamma_1}\tilde{p}\dot{\hat{p}} \\ &= -a_1(k_1 + 1)z_1^2 - (a_3 - k_1 a_1)z_2^2 - z_1 z_2 z_3 - z_1 z_2\{\alpha_2 - p\} - \frac{1}{\gamma_1}\tilde{p}\dot{\hat{p}},\end{aligned} \tag{17}$$

where $p = k_1(a_3 - k_1 a_1) + a_3 - k_1 a_1 + a_1$ is an unknown constant. Now, we design a virtual control α_2 and adaption law $\dot{\hat{p}}$ as

$$\alpha_2 = \hat{p}, \tag{18}$$

$$\dot{\hat{p}} = -\lambda_1 \hat{p} + \gamma_1 z_1 z_2. \tag{19}$$

Furthermore, we can rewrite (17) as

$$\begin{aligned}\dot{V}_2 &= -a_1(k_1 + 1)z_1^2 - (a_3 - k_1 a_1)z_2^2 - z_1 z_2 z_3 + \frac{\lambda_1}{\gamma_1}\tilde{p}\hat{p} \\ &\le -a_1(k_1 + 1)z_1^2 - (a_3 - k_1 a_1)z_2^2 - \frac{\lambda_1}{2\gamma_1}\tilde{p}^2 + \frac{\lambda_1}{2\gamma_1}p^2 - z_1 z_2 z_3,\end{aligned} \tag{20}$$

where $(a_3 - k_1 a_1) > 0$.

Step 3: Actual control v will be constructed in this step. By $z_3 = s_3 - \alpha_2$, (4) and (7), one has

$$\dot{z}_3 = s_1 s_2 - a_4 s_3 + g_{v_\mu} v + d(v) - \dot{\alpha}_2. \tag{21}$$

Consider a Lyapnov function as

$$V_3 = V_2 + \frac{1}{2} z_3^2 + \frac{g_m}{2\gamma_2} \tilde{\theta}^2, \tag{22}$$

where γ_2 is a positive design parameter and $\tilde{\theta} = \theta - \hat{\theta}$ is parameter error.

From (20) and (21), the time derivative of (22) is described by

$$\begin{aligned} \dot{V}_3 \le & -a_1(k_1+1)z_1^2 - (a_3 - k_1 a_1) z_2^2 - \frac{\lambda_1}{2\gamma_1}\tilde{p}^2 \\ & + \frac{\lambda_1}{2\gamma_1} p^2 + z_3(g_{v_\mu} v + d(v) + f(Z)) - z_3^2 - \frac{g_m}{\gamma_2}\tilde{\theta}\dot{\hat{\theta}}, \end{aligned} \tag{23}$$

where $f(Z) = s_1 s_2 - a_4 s_3 - \dot{\alpha}_2 - z_1 z_2 + z_3$ with $Z = [s_1, s_2, s_3]^T \in \Omega_Z \subset R^3$. Since $a_4 = \frac{\beta+8}{3}$ with β being an unknown constant, $f(Z)$ cannot be employed to design an actual controller v. To deal with this problem, RBF neural networks $W^T S(Z)$ can be used to estimate $f(Z)$ such that, for any given positive constant ε,

$$f(Z) = W^T S(Z) + \delta(Z), \quad |\delta(Z)| \le \varepsilon \tag{24}$$

where $\delta(Z)$ is approximation error. Further, we can obtain

$$\begin{aligned} z_3 f(Z) &= z_3 W^T S(Z) + z_3 \delta(Z) \\ &\le \frac{g_m}{2a^2} z_3^2 \theta S^T(Z) S(Z) + \frac{1}{2}a^2 + \frac{1}{2} z_3^2 + \frac{1}{2}\varepsilon^2, \end{aligned} \tag{25}$$

where the unknown constant $\theta = \frac{\|W\|^2}{g_m}$ and a is a design parameter. Substituting (24) into (23) and using (25) result in

$$\begin{aligned} \dot{V}_3 \le & -a_1(k_1+1)z_1^2 - (a_3 - k_1 a_1) z_2^2 - \frac{\lambda_1}{2\gamma_1}\tilde{p}^2 + \frac{\lambda_1}{2\gamma_1} p^2 \\ & + z_3\left(g_{v_\mu} v + d(v)\right) - \frac{1}{2} z_3^2 + \frac{g_m}{2a^2} z_3^2 \theta S^T(Z) S(Z) + \frac{1}{2}a^2 + \frac{1}{2}\varepsilon^2 - \frac{g_m}{\gamma_2}\tilde{\theta}\dot{\hat{\theta}}. \end{aligned} \tag{26}$$

At the present stage, construct an actual controller as

$$v = -k_2 z_3 - \frac{1}{2a^2} z_3 \hat{\theta} S^T(Z) S(Z), \tag{27}$$

where $k_2 > 0$ is a design constant and $\hat{\theta}$ is the estimation of θ. Then, we can obtain

$$z_3(g_{v_\mu} v + d(v)) \le -k_2 g_m z_3^2 - \frac{\hat{\theta}}{2a^2} g_m z_3^2 S^T(Z) S(Z) + \frac{1}{2} z_3^2 + \frac{1}{2} D^2. \tag{28}$$

Furthermore, combining (26) with (27) results in

$$\begin{aligned} \dot{V}_3 \le & -a_1(k_1+1)z_1^2 - (a_3 - k_1 a_1) z_2^2 - k_2 g_m z_3^2 - \frac{\lambda_1}{2\gamma_1}\tilde{p}^2 + \frac{\lambda_1}{2\gamma_1} p^2 \\ & + \frac{1}{2}a^2 + \frac{1}{2}\varepsilon^2 + \frac{g_m}{\gamma_2}\tilde{\theta}\left(\frac{\gamma_2}{2a^2} z_3^2 S^T(Z) S(Z) - \dot{\hat{\theta}}\right). \end{aligned} \tag{29}$$

Choose an adaption law as

$$\dot{\hat{\theta}} = -\lambda_2 \hat{\theta} + \frac{\gamma_2}{2a^2} z_3^2 S^T(Z) S(Z). \tag{30}$$

Then, substituting (30) into (29) gives

$$\dot{V}_3 \le -a_1(k_1+1)z_1^2 - (a_3 - k_1 a_1) z_2^2 - k_2 g_m z_3^2 - \frac{\lambda_1}{2\gamma_1}\tilde{p}^2 + \frac{\lambda_1}{2\gamma_1} p^2$$

$$+\frac{1}{2}a^2 + \frac{1}{2}\varepsilon^2 + \frac{\lambda_2}{\gamma_2} g_m \tilde{\theta}\hat{\theta}. \tag{31}$$

By using the following inequality

$$\frac{g_m \lambda_2}{\gamma_2} \tilde{\theta}\hat{\theta} \le -\frac{g_m \lambda_2}{2\gamma_2} \tilde{\theta}^2 + \frac{g_m \lambda_2}{2\gamma_2} \theta^2,$$

(29) can be rewritten as

$$\dot{V}_3 \le -a_1(k_1+1)z_1^2 - (a_3 - k_1 a_1)z_2^2 - k_2 g_m z_3^2 - \frac{\lambda_1}{2\gamma_1}\tilde{p}^2 - \frac{\lambda_2 g_m}{2\gamma_2}\tilde{\theta}^2$$
$$+\frac{\lambda_2 g_m}{2\gamma_2}\theta^2 + \frac{\lambda_1}{2\gamma_1}p^2 + \frac{1}{2}a^2 + \frac{1}{2}\varepsilon^2. \tag{32}$$

Now, the main result will be summarized as the following theorem.

Theorem 3.1. *Consider the chaotic system (1), the controller (27) and adaptive law (30) under Assumption 2.1. Then, for bounded initial conditions, all signals in the closed loop system are uniformly ultimately bounded.*

Proof: Choose a Lyapunov function candidate as $V = V_n$. Then, its time derivative is

$$\dot{V} \le -a_0 V + b_0, \quad t \ge 0, \tag{33}$$

where $a_0 = \min\{2a_1(k_1+1), (a_3 - k_1 a_1), 2k_2 g_m, \lambda_i, i = 1, 2.\}$ and $b_0 = \frac{\lambda_2 g_m}{2\gamma_2}\theta^2 + \frac{\lambda_1}{2\gamma_1}p^2 + \frac{1}{2}a^2 + \frac{1}{2}\varepsilon^2$.

Therefore, we can conclude that all signals in the closed-loop system are uniformly ultimately bounded. Then, v is also bounded.

4. **Numerical Example.** In this section, the simulation is run for $\beta = 0.8$, which denotes Lü system. The input saturation limit is chosen as u_M =100. Based on Theorem 3.1, construct neural-based adaptive control input signals α_1 in (13), α_2 in (18) and v in (27) with adaption laws $\hat{p}$ in (19) and $\hat{\theta}$ in (30). The design parameters are chosen as follows: $k_1 = 0.05$, $k_2 = 1$, $a = 3$, $\lambda_1 = 0.1$, $\lambda_2 = 0.5$ and $\gamma_1 = \gamma_2 = 1$. Moreover, the initial conditions are given by $[s_1(0), s_2(0), s_3(0)]^T = [10, 10, 10]^T$, and $\left[\hat{p}(0), \hat{\theta}(0)\right]^T = [0, 0]^T$.

The simulation results indicate that the proposed controller guarantees the boundedness of all the signals in the closed-loop system. The details are shown in Figures 1-4.

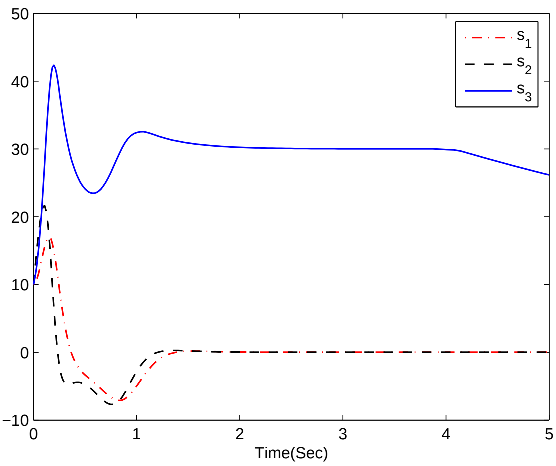

Figure 1. State variables s_1, s_2 and s_3

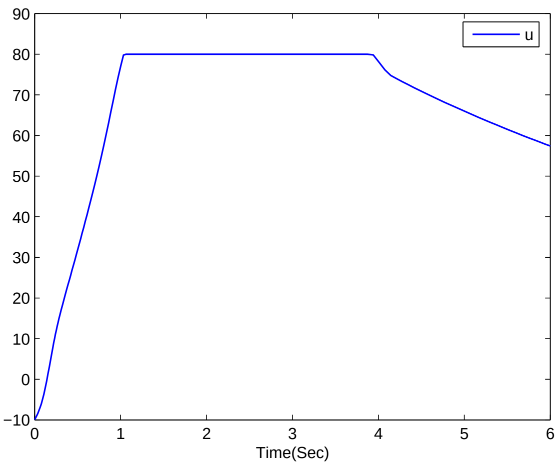

Figure 2. The true control input u

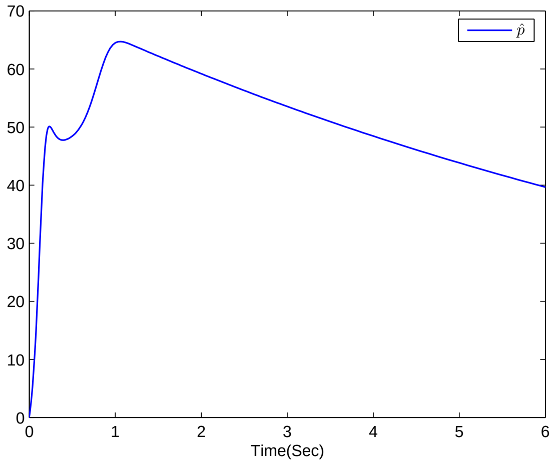

FIGURE 3. The adaptive parameter $\hat{p}$

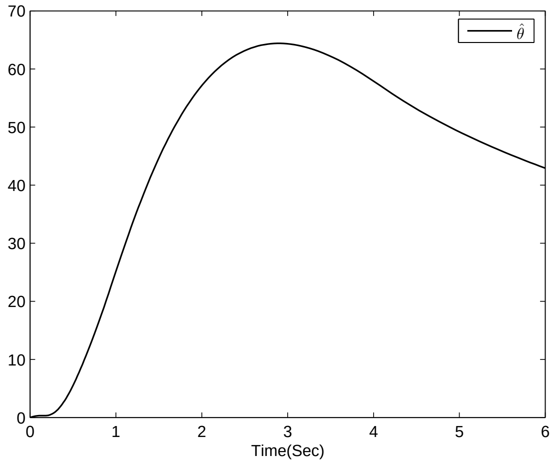

FIGURE 4. The adaptive parameter $\hat{\theta}$

5. **Conclusion.** This paper proposes an adaptive neural control strategy for uncertain unified chaotic systems with input saturation. The presented adaptive neural controller ensures that all signals in the closed-loop system are bounded. Simulation results further demonstrate the effectiveness of the proposed control scheme. Our future research will mainly focus on the output-feedback control for the original system (1) based on the result in this paper.

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