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ROBUST FAULT DETECTION OF NONLINEAR MARKOVIAN JUMP SYSTEMS WITH PARTLY UNKNOWN TRANSITION PROBABILITIES

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Abstract. *A robust fault detection observer (RFDO) is designed to solve the robust fault detection problem of the nonlinear Markovian jump systems (NMJSs) with partly unknown transition probabilities. With the method of T-S fuzzy linearization, the original NMJSs are described as a set of local linear models. On this basis, free-connection weighting matrices are introduced to RFDO. A series of linear matrix inequalities which ensure the stochastic asymptotic stability of the system are obtained by using the constructed Lyapunov function. Furthermore, the design problem is formulated as a two-objective optimization algorithm. A simulation example is given to show that the designed RFDO can not only detect the fault sensitively, but have the robustness to unknown disturbances.*
Keywords: Robust fault detection, Nonlinear Markovian jump systems (NMJSs), Partly unknown transition probabilities, Free-connection weighting matrices, T-S fuzzy

1. **Introduction.** In the past 20 years, many scholars have studied the issues of fault detection. The main purpose of fault detection is to construct a residual signal, and then a residual evaluation function is obtained to compare with a predefined threshold. However, the inevitable modeling error and external disturbances may seriously affect the fault detection system. So the methods based on model have attracted much attention [1-3]. The main idea is to set two performance indexes: H_- has sensitivity to the faults; and the other one is H_∞, which has robustness to unknown disturbances. The problem of the robust fault detection is converted to optimization of H_-/H_∞ with the linear matrix inequalities (LMIs) approach.

On the other hand, MJSs have been widely investigated and the existing results cover a large variety of problems such as stochastic stability and stabilization [4], and filtering [5]. In spite of these developments, many results are under the assumption that one can access to the transition probabilities completely. However, in practical systems, rates of the stochastic jumps may not be measurable exactly or only part of the transition probabilities is available. [6] made some attempts for partly unknown transition probabilities, but it just considers the robustness to disturbances, not the sensitivity to the faults.

For NMJSs [7-9], the establishment of mathematical models is far more difficult than the one for linear systems. So far, there are no mature and general methods to analyze NMJSs. The T-S provides a new way for the analysis and control of nonlinear systems [10]. Firstly, a nonlinear system is regarded as fuzzy approximation of lots of local linear models, and then the control of the system is regarded as fuzzy approximation of many local linear systems.

In this paper, for NMJSs with partly unknown transition probabilities, we will study robust fault detection system and design the optimal RFDO. We will also use T-S fuzzy linearization for NMJSs and the relationship between the elements of transition probabilities in jump systems, and study the robust fault detection by considering free-connection weighting matrix W_i, which deeply reduces conservative of the optimal solution. On this basis, we will use LMI toolbox to get the optimal solution of the RFDO.

The rest of this paper is organized as follows. The system description and correlation definition is introduced in Section 2. In Section 3, the design and optimization of RFDO are addressed. A numerical simulation is given in Section 4. Finally, the concluding remarks are given in the last section.

2. System Description and Correlation Definition.

The following time-delay NMJSs are considered

$$\begin{cases} \dot{x}(t) = f_1(x(t), x(t-\tau), d(t), f(t), r_t) + g(x(t), x(t-\tau), d(t), f(t), i) \\ y(t) = f_2(x(t), x(t-\tau), d(t), f(t), r_t) + h(x(t), x(t-\tau), d(t), f(t), i) \\ x(t) = \eta(t),\ r_t = r_0,\ t \in [-\tau, 0] \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state, $x(t-\tau)$ is the time-delay state, $f(t) \in R^q$ is the fault, $y(t) \in R^l$ is output, $\tau > 0$ is the delay constant, $d(t) \in L_2^q[0, \infty]$ is the external disturbance and r_0 is the initial mode. $\eta(t)$ is defined as the initial state function on $[-\tau, 0]$.

Given a probability space (Ω, F, P) where Ω is the sample space, F is the event's algebra and P is the probability measure on F. Transition probability is defined as:

$$P_r\left\{r_{t+\Delta t} = j \,|\, r_t = i\right\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \neq j \\ 1 + \pi_{ij}\Delta t + o(\Delta t), & i = j \end{cases} \tag{2}$$

where $\Delta t > 0$ and $\lim_{\Delta t \downarrow 0} o(\Delta t)/\Delta t \to 0$. $\pi_{ij} \geq 0$ is the transition probability from the model i at time t to mode j $(i \neq j)$ at time $t + \Delta t$ and $\sum_{j=1, j\neq i}^{N} \pi_{ij} = -\pi_{ii}$.

The transition probability matrix is partly unknown, which is described as follows:

$$\prod = \begin{bmatrix} \pi_{11} & ? & \cdots & \pi_{1N} \\ \pi_{21} & ? & \cdots & \pi_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ ? & \pi_{N2} & \cdots & ? \end{bmatrix} \tag{3}$$

where "?" represents the unaccessible elements. For notation clarity, $\forall i \in S$, denote:

$$T_k^i \triangleq \{j : \pi_{ij} \text{ is known}\}, \quad T_{uk}^i \triangleq \{j : \pi_{ij} \text{ is unknown}\}$$

If $\mathrm{T}_k^i \neq \emptyset$, it is further described as: $\mathrm{T}_k^i = \{k_1^i, k_2^i, \cdots, k_m^i\}$, $1 \leq m \leq N$.

Assumption 2.1. *The time-varying but norm-bounded uncertainties $g(\cdot)$ and $h(\cdot)$ satisfy:*

$$\begin{cases} \quad g(x(t), x(t-\tau), d(t), f(t), i) \\ = \Delta A_i(l)x(t) + \Delta A_{di}(l)x(t-\tau) + \Delta B_{di}(l)d(t) + \Delta B_{fi}(l)f(t) \\ \quad h(x(t), x(t-\tau), d(t), f(t), i) \\ = \Delta C_i(l)x(t) + \Delta C_{di}(l)x(t-\tau) + \Delta D_{di}(l)d(t) + \Delta D_{fi}(l)f(t) \end{cases} \tag{4}$$

with

$$\begin{aligned} &\begin{bmatrix} \Delta A_i(l) & \Delta A_{di}(l) & \Delta B_{di}(l) & \Delta B_{fi}(l) \\ \Delta C_i(l) & \Delta C_{di}(l) & \Delta D_{di}(l) & \Delta D_{fi}(l) \end{bmatrix} \\ &= \begin{bmatrix} M_i(l) \\ M_{yi}(l) \end{bmatrix} \Gamma_i(t,l) \begin{bmatrix} N_i(l) & N_{\tau i}(l) & N_{di}(l) & N_{fi}(l) \end{bmatrix} \end{aligned}$$

where $M_i(l)$, $M_{yi}(l)$, $N_i(l)$, $N_{\tau i}(l)$, $N_{di}(l)$ and $N_{fi}(l)$ are constant matrices. They reflect the structural information of uncertainty. $\Gamma_i(t,l)$ is the time-varying unknown matrix with Lebesgue measurable elements satisfying $\Gamma_i^T(t,l)\Gamma_i(t,l) \leq I$.

Assumption 2.2. *$\begin{bmatrix} C_i(l) & A_i(l) \end{bmatrix}$ is observable, $D_{fi}(l)$ is a full rank matrix and $p \leq m$.*

By using the method of single point fuzzy, product inference and weighted center-average defuzzifier, the global fuzzy system can be expressed as:

$$\begin{cases} \dot{x}(t) = \sum_{l=1}^{s} h_l(\mu(t)) \{[A_i(l) + \Delta A_i(l)]x(t) + [A_{di}(l) + \Delta A_{di}(l)]x(t-\tau) \\ \qquad +[B_{di}(l) + \Delta B_{di}(l)]d(t) + [B_{fi}(l) + \Delta B_{fi}(l)]f(t)\} \\ y(t) = \sum_{l=1}^{s} h_l(\mu(t)) \{[C_i(l) + \Delta C_i(l)]x(t) + [C_{di}(l) + \Delta C_{di}(l)]x(t-\tau) \\ \qquad +[D_{di}(l) + \Delta D_{di}(l)]d(t) + [D_{fi}(l) + \Delta D_{fi}(l)]f(t)\} \\ x(t) = \eta(t),\ r_t = r_0,\ t \in [-\tau, 0],\ l = 1, 2, \cdots, S \end{cases} \tag{5}$$

where $\mu(t) = \begin{bmatrix} \mu_1(t) & \mu_2(t) & \cdots & \mu_S(t) \end{bmatrix}$, and for $\forall l = 1, 2, \cdots, S$

$$h_l(\mu(t)) = \mu_l(\mu(t)) \Big/ \sum\nolimits_{l=1}^{s} h_l(\mu(t)), \quad u_l(\mu(t)) = \prod\nolimits_{m=1}^{g} F_m^l(\mu_m(t)) \tag{6}$$

where $F_m^l(\mu_m(t))$ is the grade of membership of $\mu_m(t)$ in the fuzzy set F_m^l. $\mu_l(\mu(t))$ is the degree of membership for rule l. Here, $\mu_l(\mu(t)) \geq 0$ and $\sum_{l=1}^{s} h_l(u(t)) > 0$, then:

$$\sum\nolimits_{l=1}^{s} h_l(u(t)) = 1,\ 0 \leq h_l(\mu(t)) \leq 1,\ l = 1, 2, \cdots, S \tag{7}$$

Definition 2.1. *When $d(t) = 0$, $f(t) = 0$ in the NMJSs (1), for any initial state $x(t) = \eta(t)$ and initial mode r_0, the system (1) is asymptotically stable if condition (8) holds:*

$$\lim_{T \to \infty} E\left\{ \int_0^T \|x(t, \eta(t), r_0)\|^2 dt | r_0, x(t) = \eta(t), t \in \begin{bmatrix} -h & 0 \end{bmatrix} \right\} < \infty \tag{8}$$

Definition 2.2. *Let $V(x(t), r_t, t > 0) = V(x, i)$ be the stochastic Lyapunov-Krasovskii function. Define its weak infinitesimal operator as:*

$$\Gamma V(x, i) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \{E[V(x(t + \Delta t), r_{t+\Delta t}, t + \Delta t) | x(t) = x, r_t = i] - V(x(t), i, t)\} \tag{9}$$

3. Design and Optimization of the Robust Fault Detection Observer (RFDO).

3.1. Designing the robust fault detection observer (RFDO).

The global RFDO for the system (5) is constructed as:

$$\begin{cases} \dot{\bar{x}}(t) = \sum_{l=1}^{s} h_m(\mu(t)) \{A_i(l)\bar{x}(t) + A_{di}(l)\bar{x}(t-\tau) + H_i(l)(y(t) - \bar{y}(t))\} \\ \bar{y}(t) = \sum_{l=1}^{s} h_m(\mu(t)) \{C_i(l)\bar{x}(t) + C_{di}(l)\bar{x}(t-\tau)\} \\ \bar{y}(t) = \zeta(t),\ r_t = r_0,\ t \in \begin{bmatrix} -\tau & 0 \end{bmatrix},\ l = 1, 2, \cdots, S \end{cases} \tag{10}$$

where $\bar{x}(t) \in R^n$ is estimated state; $\bar{y}(t) \in R^l$ is the estimated output; $\zeta(t)$ is the estimated initial state function defined on $[-\tau\ 0]$. $H_i(l)$ is the observer parameters for the stay. Define estimated state error for system as $e(t) = x(t) - \bar{x}(t)$, output estimated error as $r(t) = y(t) - \bar{y}(t)$ and $\hat{x}(t) = \begin{bmatrix} x^T(t) & e^T(t) \end{bmatrix}^T$. From (5) and (10), the following fuzzy fault detection system can be obtained:

$$\begin{cases} \dot{\hat{x}}(t) = \hat{A}_i(l,m)\hat{x}(t) + \hat{A}_{di}(l,m)\hat{x}(t-\tau) + \hat{B}_{di}(l,m)d(t) + \hat{B}_{fi}(l,m)f(t) \\ r(t) = \hat{C}_i(l,m)\hat{x}(t) + \hat{C}_{di}(l,m)\hat{x}(t-\tau) + \hat{D}_{di}(l)d(t) + \hat{D}_{fi}(l)f(t) \end{cases} \tag{11}$$

where

$$\hat{A}_i(l,m) = \sum\nolimits_{l=1}^{s} h_l(u(t)) \sum\nolimits_{m=1}^{s} h_m(u(t)) \begin{bmatrix} A_i(l) + \Delta A_i(l) & 0 \\ \Delta A_i(l) - H_i(l)\Delta C_i(m) & A_i(l) - H_i(l)\Delta C_i(m) \end{bmatrix}$$

$$\hat{A}_{di}(l,m) = \sum_{l=1}^{s} h_l(u(t)) \sum_{m=1}^{s} h_m(u(t)) \begin{bmatrix} A_{di}(l) + \Delta A_{di}(l) & 0 \\ \Delta A_{di}(l) - H_i(l)\Delta C_{di}(m) & A_{di}(l) - H_i(l)\Delta C_{di}(m) \end{bmatrix}$$

$$\hat{B}_{di}(l,m) = \sum_{l=1}^{s} h_l(u(t)) \sum_{m=1}^{s} h_m(u(t)) \begin{bmatrix} B_{di}(l) + \Delta B_{di}(l) \\ B_{di}(l) + \Delta B_{di}(l) - H_i(l)[D_{di}(m) + \Delta D_{di}(m)] \end{bmatrix}$$

$$\hat{C}_i(l,m) = \sum_{l=1}^{s} h_l(u(t)) \sum_{m=1}^{s} h_m(u(t)) \begin{bmatrix} C_i(l) + \Delta C_i(l) - C_i(m) & C_i(m) \end{bmatrix}$$

$$\hat{C}_{di}(l,m) = \sum_{l=1}^{s} h_l(u(t)) \sum_{m=1}^{s} h_m(u(t)) \begin{bmatrix} C_{di}(l) + \Delta C_{di}(l) - C_{di}(m) & C_{di}(m) \end{bmatrix}$$

$$\hat{D}_{di}(l) = \sum_{l=1}^{s} h_l(u(t)) \sum_{m=1}^{s} h_m(u(t)) \left[D_{di}(l) + \Delta D_{di}(l)\right];$$

$$\hat{D}_{fi}(l) = \sum_{l=1}^{s} h_l(u(t)) \sum_{m=1}^{s} h_m(u(t)) \left[D_{fi}(l) + \Delta D_{fi}(l)\right]$$

Lemma 3.1. [11] *(Dynkin's formula) Suppose $z(x(t)) \in C_0^2([0,t], R^n)$, $x(0) = x_0$, then*

$$E\left\{z(x(t))\right\} = E\left\{z(x_0)\right\} + E\left\{\int_0^t \nabla z(x(s))ds\right\} \tag{12}$$

Theorem 3.1. *The error dynamic system (11) with $d(t) = 0$ and $f(t) = 0$ is stochastically stable, if there exists a set of positive-definite symmetric matrices P_i and Q, symmetric matrix W_i, satisfying the following matrix inequalities for all $i \in M$ and $1 \le l \le m \le S$:*

$$\Xi_i(l,m) = \begin{bmatrix} \Pi_i(l,m) & P_i\left[\hat{A}_{hi}(l,m) + \hat{A}_{hi}(m,l)\right] \\ * & -2Q \end{bmatrix} < 0 \tag{13}$$

$$P_j - W_i \le 0,\ j \in T_{uk}^i,\ j \ne i \tag{14}$$

$$P_j - W_i \ge 0,\ j \in T_{uk}^i,\ j = i \tag{15}$$

where $\Pi_i(l,m) = P_i\left[\hat{A}_i(l,m) + \hat{A}_i(m,l)\right] + \left[\hat{A}_i(m,l) + \hat{A}_i(l,m)\right]^T P_i + 2\sum_{j\in T_K^i} \pi_{ij}(P_j - W_i) + 2Q$.

Remark 3.1. *The main objective of this paper is to design the RFDO, making the final reconstruction system (11) stochastically stable; meanwhile, the residual signal $r(t)$ has good robustness to unknown disturbance $d(t)$ and high sensitivity to the fault signal $f(t)$.*

To decrease the effects of disturbances to residual, allow the residual generator has strong robustness to the unknown disturbance signal $d(t)$, require that the system (11) satisfies the following performance index γ of H_∞:

$$E\left\{\int_0^\infty \mathrm{r}_d^T \mathrm{r}_d dt\right\} \le \gamma^2 E\left\{\int_0^\infty \mathrm{d}^T \mathrm{d} dt\right\}_{\mathrm{f}=0} \tag{16}$$

Theorem 3.2. *For a given $\gamma > 0$, the dynamic error system is stochastically stable and satisfies (16), if there exist mode-dependent symmetric positive-definite matrices P_{1i}, P_{2i}, Q_{11}, Q_{22}, matrix Q_{12}, mode-dependent matrix $\bar{H}_i(l)$ and scalars $\alpha_i(l,m) > 0$, symmetric matrices of appropriate dimensions $W_{1i} = W_{1i}^T$, $W_{2i} = W_{2i}^T$ satisfying the following LMIs for all $i \in M$ and $1 \le l \le m \le S$:*

$$\Lambda_i(l,m) = \begin{bmatrix} \Lambda_{1i}(l,m) & \Lambda_{2i}(l,m) & \Lambda_{3i}(l,m) & \Lambda_{4i}(l,m) & \Lambda_{5i}(l,m) \\ * & \Lambda_{6i}(l,m) & \Lambda_{7i}(l,m) & \Lambda_{8i}(l,m) & 0 \\ * & * & \Lambda_{9i}(l,m) & \Lambda_{10i}(l,m) & 0 \\ * & * & * & -2I & M_{yi}(l) + M_{yi}(m) \\ * & * & * & * & -[\alpha_i(l,m) + \alpha_i(m,l)]I \end{bmatrix} < 0 \tag{17}$$

$$P_{nj} - W_{ni} \le 0,\ j \in T_{uk}^i,\ j \ne i,\ n = 1,2 \tag{18}$$

$$P_{nj} - W_{ni} \geq 0,\ j \in T^i_{uk},\ j = i,\ n = 1, 2 \tag{19}$$

where

$$\Lambda_{1i}(l,m) = \begin{bmatrix} \Lambda_{11i}(l,m) + \Lambda_{11i}(m,l) & Q_{12} \\ * & \Lambda_{12i}(l,m) + \Lambda_{12i}(m,l) \end{bmatrix};$$

$$\Lambda_{11i}(l,m) = P_{1i}A_i(l) + A_i^T(l)P_{1i} + \sum\nolimits_{j\in T^i_K} \pi_{ij}(P_{1j} - W_{1i}) + Q_{11} + \alpha_i(l,m)N_i^T(l)N_i(l);$$

$$\Lambda_{12i}(l,m) = P_{2i}A_i(l) + A_i^T(l)P_{2i} - \bar{H}_i(l)C_i(m) - C_i^T(m)\bar{H}_i^T(l) + \sum\nolimits_{j\in T^i_K} \pi_{ij}(P_{2j} - W_{2i}) + Q_{22};$$

$$\Lambda_{2i}(l,m) = \begin{bmatrix} \Lambda_{21i}(l,m) & 0 \\ 0 & \Lambda_{22i}(l,m) \end{bmatrix};\ \Lambda_{4i}(l,m) = \begin{bmatrix} 0 \\ [C_i(l) + C_i(m)]^T \end{bmatrix};$$

$$\Lambda_{21i}(l,m) = -2\gamma^2 I + \alpha_i(l,m)N_{di}^T(l)N_{di}(l) + \alpha_i(l,m)N_{di}^T(m)N_{di}(m);$$

$$\Lambda_{22i}(l,m) = P_{2i}[A_{di}(l) + A_{di}(m)] - \bar{H}_i(l)C_{hi}(m) - \bar{H}_i(m)C_{hi}(l);$$

$$\Lambda_{3i}(l,m) = \begin{bmatrix} P_{1i}[B_{di}(l) + B_{di}(m)] + \alpha_i(l,m)N_i^T(l)N_{di}(l) + \alpha_i(m,l)N_i^T(m)N_{di}(m) \\ P_{2i}[B_{di}(l) + B_{di}(m)] - \bar{H}_i(l)D_{di}(m) - \bar{H}_i(m)D_{di}(l) \end{bmatrix};$$

$$\Lambda_{5i}(l,m) = \begin{bmatrix} P_{1i}[M_i(l) + M_i(m)] \\ P_{2i}[M_i(l) + M_i(m)] - \bar{H}_i(l)M_{yi}(m) - \bar{H}_i(m)M_{yi}(l) \end{bmatrix};$$

$$\Lambda_{6i}(l,m) = \begin{bmatrix} -2Q_{11} + \alpha_i(l,m)N_{\tau i}^T(l)N_{\tau i}(l) + \alpha_i(m,l)N_{\tau i}^T(m)N_{\tau i}(m) & -2Q_{12} \\ * & -2Q_{22} \end{bmatrix};$$

$$\Lambda_{7i}(l,m) = \begin{bmatrix} \alpha_i(l,m)N_{\tau i}^T(l)N_{\tau i}(l) + \alpha_i(m,l)N_{\tau i}^T(m)N_{\tau i}(m) \\ 0 \end{bmatrix};$$

$$\Lambda_{8i}(l,m) = \begin{bmatrix} 0 \\ [C_{di}(l) + C_{di}(m)]^T \end{bmatrix};$$

$$\Lambda_{9i}(l,m) = -2\gamma^2 I + \alpha_i(l,m)N_{di}^T(l)N_{di}(l) + \alpha_i(l,m)N_{di}^T(m)N_{di}(m);$$

$$\Lambda_{10i}(l,m) = D_{di}^T(l) + D_{di}^T(m).$$

The corresponding fault detection observer is $H_i(l) = P_{2i}^{-1}\bar{H}_i(l)$.

Similarly, in order to ensure the residual generator has high sensitivity to the fault, require that the system (11) satisfies the following performance index β of H_-:

$$E\left\{\int_0^\infty \mathrm{r}_f^T \mathrm{r}_f dt\right\} \geq \beta^2 E\left\{\int_0^\infty \mathrm{f}^T \mathrm{f} dt\right\}_{\mathrm{d}=0} \tag{20}$$

Theorem 3.3. *For a given positive $\beta > 0$, the dynamic error system is stochastically stable and satisfies (20), if there exist mode-dependent symmetric positive-definite matrices P_{1i}, P_{2i}, Q_{11}, Q_{22}, matrix Q_{12}, mode-dependent matrix $\bar{H}_i(l)$ and vectors $\beta_i(l,m)$, symmetric matrix of appropriate dimensions $W_i = W_i^T$ satisfying the following LMIs for all $i \in M$ and $1 \leq l \leq m \leq S$:*

$$\Delta_i(l,m) = \begin{bmatrix} \Delta_{1i}(l,m) & \Delta_{2i}(l,m) & \Delta_{3i}(l,m) & \Delta_{5i}(l,m) \\ * & \Delta_{4i}(l,m) & \Delta_{5i}(l,m) & 0 \\ * & * & \Delta_{6i}(l,m) & -M_{yi}(l) - M_{yi}(m) \\ * & * & * & -[\beta_i(l,m) + \beta_i(m,l)]I \end{bmatrix} < 0 \tag{21}$$

$$P_{nj} - W_{ni} \leq 0,\ j \in T^i_{uk},\ j \neq i,\ n = 1, 2 \tag{22}$$

$$P_{nj} - W_{ni} \geq 0,\ j \in T^i_{uk},\ j = i,\ n = 1, 2 \tag{23}$$

$$\Delta_{1i}(l,m) = \begin{bmatrix} \Delta_{11i}(l,m) + \Delta_{11i}(m,l) & Q_{12} \\ * & \Lambda_{12i}(l,m) + \Lambda_{12i}(m,l) \end{bmatrix};$$

$$\Delta_{2i}(l,m) = \begin{bmatrix} \Delta_{21i}(l,m) & 0 \\ 0 & \Delta_{22i}(l,m) \end{bmatrix}$$

$$\Delta_{11i}(l,m) = P_{1i}A_i(l) + A_i^T(l)P_{1i} + \sum\nolimits_{j\in T_K^i} \pi_{ij}(P_{1j} - W_{1i}) + Q_{11} + \beta_i(l,m)N_i^T(l)N_i(l);$$

$$\Delta_{21i}(l,m) = P_{1i}[A_{di}(l) + A_{di}(m)] + \beta_i(l,m)N_i^T(l)N_{\tau i}(l) + \beta_i(m,l)N_i^T(m)N_{\tau i}(m);$$

$$\Delta_{3i}(l,m) = \begin{bmatrix} P_{1i}[B_{fi}(l) + B_{fi}(m)] + \beta_i(l,m)N_i^T(l)N_{fi}(l) + \beta_i(m,l)N_i^T(m)N_{fi}(m) \\ P_{2i}[B_{fi}(l) + B_{fi}(m)] - \bar{H}_i(l)D_{fi}(m) - \bar{H}_i(m)D_{fi}(l) - [C_i(l) + C_i(m)]^T \end{bmatrix};$$

$$\Delta_{4i}(l,m) = \begin{bmatrix} -2Q_{11} + \beta_i(l,m)N_{\tau i}^T(l)N_{\tau i}(l) + \beta_i(m,l)N_{\tau i}^T(m)N_{\tau i}(m) & -2Q_{12} \\ * & -2Q_{22} \end{bmatrix};$$

$$\Delta_{5i}(l,m) = \begin{bmatrix} \beta_i(l,m)N_{\tau i}^T(l)N_{\tau i}(l) + \beta_i(m,l)N_{\tau i}^T(m)N_{\tau i}(m) \\ [C_{hi}(l) + C_{hi}(m)]^T \end{bmatrix};$$

$$\begin{aligned}\Delta_{6i}(l,m) &= 4\beta^2 I - \left[D_{fi}^T(l) + D_{fi}^T(m)\right] - [D_{fi}(l) + D_{fi}(m)] \\ &\quad + \beta_i(l,m)\left[N_{fi}^T(l)N_{fi}(l) + N_{fi}^T(m)N_{fi}(m)\right]\end{aligned}$$

The corresponding fault detection observer is $H_i(l) = P_{2i}^{-1}\bar{H}_i(l)$.

3.2. **Optimizing the robust fault detection observer (RFDO).** According to the design of RFDO (11), the next task is to find the optimal observer gain matrix H_i, in order to design the optimal fault detection observer.

Remark 3.2. *There are many parameters of observer which satisfy Theorems 3.2 and 3.3, to ensure system (11) is asymptotically stable, and the observer can sensitively detect the fault and is robust to the unknown disturbance at the same time. So choose $\beta > 0$ and $\gamma > 0$ which satisfy (13)-(15) and (17)-(19) to make the following performance index minimal*

$$J = \gamma/\beta \tag{24}$$

Then an RFDO is achieved.

On the other hand, to separate the unknown disturbance and fault signal well, we need to select an appropriate threshold J_{th} to achieve the aim of the fault detection and isolation.

$$J_{th} = \sup_{d\in L_2, f=0} E\left\{\int_0^\tau r^T r dt\right\} = \gamma^2 \Delta d \tag{25}$$

So, the fault detection is realized by

$$\begin{cases} f(r) = E\left\{\int_0^\tau r^T r dt\right\} > J_{th} \rightarrow \text{with fault} \rightarrow \text{alarm} \\ f(r) = E\left\{\int_0^\tau r^T r dt\right\} \le J_{th} \rightarrow \text{no alarm (fault} - \text{free)} \end{cases} \tag{26}$$

4. **Numerical Simulation.** Consider the following tunnel diode circuits in [13].

By LMIs (17)-(19) and (21)-(23), obtain $\gamma_{\min} = 0.12$ and $\beta_{\max} = 1.10$. Recalling the optimal algorithm formulated, the optimal values are $\gamma = 0.18$ and $\beta = 0.76$; thus, the mode-dependent robust fault detection gain matrices are as follows:

$$H_1(1) = \begin{bmatrix} 2.7354 \\ 3.3483 \end{bmatrix},\ H_1(2) = \begin{bmatrix} 2.6385 \\ 3.1542 \end{bmatrix},\ H_2(1) = \begin{bmatrix} 0.5350 \\ 1.3368 \end{bmatrix},\ H_2(2) = \begin{bmatrix} 0.5942 \\ 1.3215 \end{bmatrix}.$$

To illustrate the effectiveness, setting time-delay parameter as $\tau = 2$s, jump mode is shown in Figure 1. Assume the fault is the step one with amplitude of 1 which happened at 9s. The interference is white noise with the variance of 0.05, as shown in Figure 2. The residual response and residual evaluation function are as shown in Figures 3 and 4 respectively.

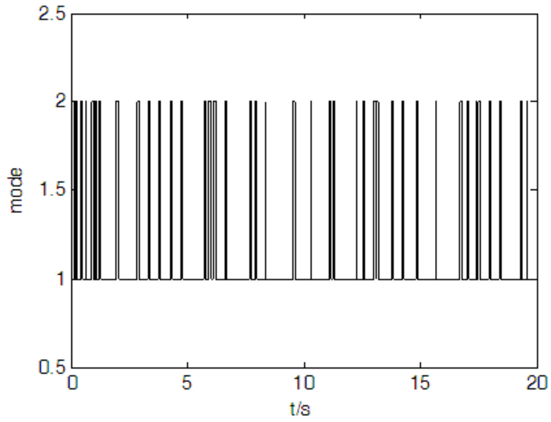

Figure 1. Jump mode

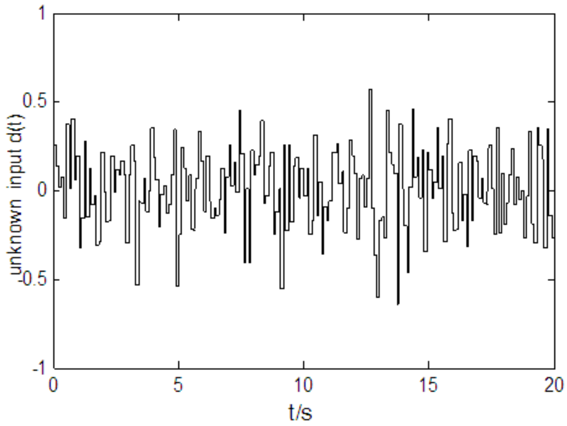

Figure 2. Disturbance signal

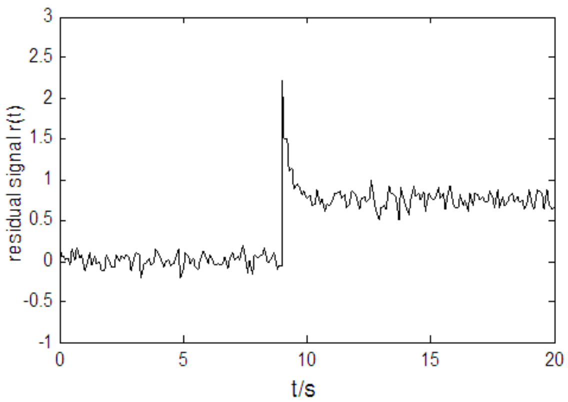

Figure 3. Residual signal

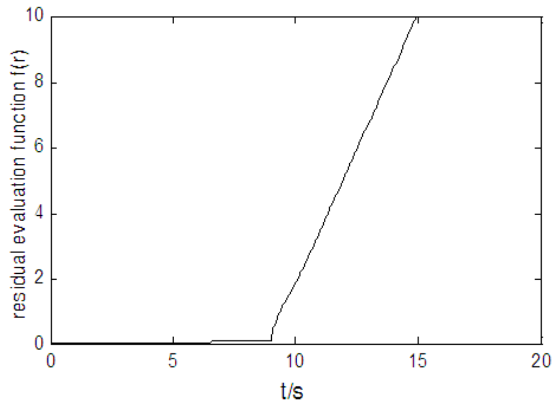

Figure 4. Residual evaluation function

With $J_{th} = \sup_{d(t)\in L_2, f(t)=0} E\left\{\int_0^{20} r^T(t)r(t)dt\right\} = 0.42$, $f(r) = E\left\{\int_0^{9.6} r^T(t)r(t)dt\right\} = 0.5 > J_{th}$ can be seen from Figure 4, the appearing fault will be detected within 0.7s after its occurrence.

Remark 4.1. *Compared with [12], the fault of which is detected in 1s after fault occurs, clearly, the proposed optimization design method in this paper can quickly detect the fault, anyway [12] is for NMJSs with completely known transition probabilities, so this method has more practicability and higher sensitivity.*

5. **Conclusions.** This paper is mainly to solve the robust fault detection with partly unknown transition probabilities for NMJSs. Based on LMIs, the free-connection weighting matrices are introduced into the original system and the reconstructed observer, the stability condition of the system is obtained. At the same time, sufficient conditions of the selected performance indexes are given and proved. A numerical simulation is given to show the effectiveness and advantages by addressing the free-connection matrices into the robust fault detection.

However, the stability condition, the robustness and sensitivity of RFDO system are just sufficient in this paper. There are some limitations. Next, it is more challenging to obtain the necessary and sufficient conditions for achieving the stability and the relevant performances of RFDO system.

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