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FRACTAL IMAGE CODING BASED ON AN IMPROVED GRAY-LEVEL TRANSFORMATION

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Abstract. *In order to improve the quality of decoded images in fractal image coding, a fractal coding method based on an improved gray-level transformation is proposed in this paper. According to the collage theorem, the upper limit of decoded image quality is determined by the collage errors. By theoretical derivations, we can see that collage errors are mainly determined by the variances of range blocks. Thus, an improved transformation aiming to suppress range blocks' variances is proposed to reduce their corresponding collage errors. Experiments show that compared with the conventional fractal coding method, the proposed method can provide better decoded image quality at the same encoding time and compression ratio.*

Keywords: Fractal image coding, Collage error, Quadtree

1. **Introduction.** Fractal image coding is firstly proposed as an image compression method [1]. Because of its novel idea, as time goes by, more and more researchers focus their interest on it and it has successfully become an useful tool in many other image processing applications such as image magnification [2,3], image denoising [4,5], image retrieval [6] and image watermarking [7]. On the one hand, fractal image coding has the advantages of potential high compression ratio, fast decoding and novel idea. On the other hand, its main drawback is the high computational complexity in the encoding process. Thus, the researchers worldwide attempt to either overcome its weaknesses or exploit its advantages [8-10]. Since fractal image coding is a lossy image compression method, how to improve the quality of decoded images is critical for its practical applications. By theoretical derivations, we find that for the range blocks with large variances, satisfying block matching cannot be achieved and this will lead to large reconstructing errors. In order to solve this problem, we first assign a constant value which will be divided in the encoder, and then the block matching is performed. Since the range blocks with large variances have been contracted to the ones whose variances are small, this operation can effectively reduce the matching errors of the range blocks with large variances. In the decoder, the same constant value will be multiplied. Thus, extra bits will not be needed to store the constant value. In the experiments, we first determine the constant value which can provide best performance for fractal coding method. Then, combined with quadtree method, the proposed method can provide higher PSNR at the same Bpp and higher PSNR at the same encoding time.

This paper mainly consists of the following parts. In Section 2, the conventional fractal coding and related theorem are described. The proposed gray-level transformation and related derivations are represented in Section 3. Performance comparison between the conventional and proposed methods is performed in Section 4. Finally, the conclusions are given in Section 5.

FIGURE 1. Eight isometric transformations for partial Barbara

2. **Review of Conventional Fractal Image Coding.** Based on the collage theorem, the fractal encoding process is to construct a transformation T whose fixed point approximates the original image well. The collage error in the encoding process provides an upper limit for the decoded image. In the practical fractal encoding process, firstly, the input image is partitioned into a set of non-overlapping range blocks whose size is $B \times B$. Then, the domain blocks can be also obtained by sliding a window over the input image with a search step δ. Furthermore, after contracting all the domain blocks into the same size of range blocks, a domain block pool can be established by performing eight isometric transformations on the contracted domain blocks. For partial Barbara, eight isometric transformations are illustrated in Figure 1. For each range block, search for the best matched domain block which can minimize the following function

$$E = \min_{s,o} \| \boldsymbol{R} - s\boldsymbol{D} - o\boldsymbol{I} \|^2 \tag{1}$$

where $\boldsymbol{I}$ denotes a matrix whose elements are all ones. E is the corresponding matching error. s and o denote the scaling coefficient and offset coefficient of the affine transformation, respectively. By setting the derivatives of (1) with respect to s and o to zeros, respectively, we can get

$$s = \langle \boldsymbol{R} - \bar{r}\boldsymbol{I}, \boldsymbol{D} - \bar{d}\boldsymbol{I} \rangle / \| \boldsymbol{D} - \bar{d}\boldsymbol{I} \|^2, \quad o = \bar{r} - s\bar{d} \tag{2}$$

where $\langle \bullet, \bullet \rangle$ denotes the inner product. $\bar{d}$ and $\bar{r}$ are the mean values of the contracted domain block and the range block, respectively. Finally, the fractal image encoding can be completed by storing all the matching information as fractal codes.

Correspondingly, the decoding process is similar to the encoding procedures. Generally, a blank image is selected as the initial image and then the same transformations stored in fractal codes are applied on it recursively. After several iterations, the reconstructed image will be obtained and for partial Barbara, Figure 2 illustrates the first eight iteration images.

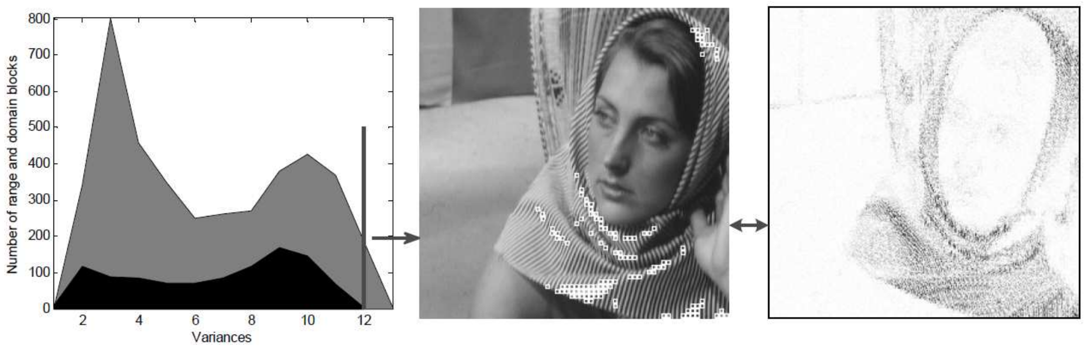
3. **The Proposed Method.** In order to facilitate the expression, the matching errors between the range and domain blocks are rewritten as follows

$$E = \| \boldsymbol{R} - s\boldsymbol{D} - o\boldsymbol{I} \|^2 \tag{3}$$

Substituting (2) into (3) yields

$$E = \| \boldsymbol{R} - s\boldsymbol{D} - o\boldsymbol{I} \|^2 = \| \boldsymbol{R} - \bar{r}\boldsymbol{I} \|^2 - s^2 \| \boldsymbol{D} - \bar{d}\boldsymbol{I} \|^2 \tag{4}$$

FIGURE 2. The first eight iterations for partial image



(a) (b) (c)

FIGURE 3. Illustration of the range blocks with larger variances and their corresponding reconstructed errors. (a) Histogram of variances of the range and domain blocks. The gray and black areas are for the range and domain blocks, respectively. (b) The white boxes illustrate the range blocks at the right side of the vertical line in (a). (c) Reconstructed image. Darker brightness represents larger reconstructed error.

In theory, in order to guarantee the contractivity of fractal decoding process, the requirement of $| s |< 1$ should be satisfied. Thus, from (4), we know that small matching error can be only obtained for the range blocks whose variances are close to $s^2 \parallel \boldsymbol{D} - \bar{d}\boldsymbol{I} \parallel^2$. For the range blocks whose variances are larger than all the domain blocks, their corresponding $\parallel \boldsymbol{R} - \bar{r}\boldsymbol{I} \parallel^2$s will be much larger than $s^2 \parallel \boldsymbol{D} - \bar{d}\boldsymbol{I} \parallel^2$s. Thus, small matching errors cannot be obtained for all the domain blocks. For partial Barbara, the gray and black regions in Figure 3(a) represent the distributions of variances of range and domain blocks, respectively. Since the contracted domain blocks are obtained by averaging four range blocks at the same position, we know that the variances' maximum of range blocks is larger than that of domain blocks. The white boxes in Figure 3(b) show the range blocks whose variances are always larger than those of all the domain blocks. Moreover, from Figure 3(c), we can observe that the range blocks in white boxes have larger reconstructed errors. Actually, 3.03% of total range blocks in the white boxes contributes to 25.69% of total collage errors and 25.26% of total reconstructed errors, respectively.

From the above analysis, we know that large variances of range blocks may lead to large matching errors and reconstructed errors. In order to suppress the variances of the range blocks with large variances, we introduce an operator $\gamma(\bullet)$ as follows

$$\gamma(\boldsymbol{R}) = \frac{\boldsymbol{R} - \bar{r}\boldsymbol{I}}{c} + \bar{r}\boldsymbol{I} \tag{5}$$

where $\bar{r}$ is the mean value of $\boldsymbol{R}$ and $\boldsymbol{I}$ is a matrix whose elements are all ones. From (5), we can see that the variances of range blocks can be reduced by dividing a constant c. Similar to (4), the corresponding matching function can be derived as follows

$$E' = \| \gamma(\boldsymbol{R}) - s\boldsymbol{D} - o\boldsymbol{I} \|^2 = \frac{\| \boldsymbol{R} - \bar{r}\boldsymbol{I} \|^2 - s^2 \| \boldsymbol{D} - \bar{d}\boldsymbol{I} \|^2}{c^2} \tag{6}$$

From (6), we know that if c satisfies $c > 1$, even if $\boldsymbol{R}$ has a large variance, the corresponding matching error can be also reduced effectively. The corresponding minimum matching error function can be modified as

$$E = \min_{s,o} \| \gamma(\boldsymbol{R}) - s\boldsymbol{D} - o\boldsymbol{I} \|^2 \tag{7}$$

4. **Experiments.** Commonly used 512×512 image of Barbara is used in our experiment. The quality of decoded images is measured by Peak Signal to Noise Ratio (PSNR) as follows:

$$PSNR = 10 \log_{10} \left(255^2 \Big/ \left(\frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} (\boldsymbol{I}_{ij} - \boldsymbol{I}^*_{ij})^2 \right) \right) \tag{8}$$

where M and N are the height and width of the input image. $\boldsymbol{I}$ and $\boldsymbol{I}^*$ represent the original image and decoded image, respectively. The fractal codes, s and o, are quantized by 5 and 7 bits, respectively. This section mainly consists of the following two parts.

A. Determination of parameter c.

In (5), selecting appropriate c is critical for our proposed transformation in (6). The image of Barbara is encoded and decoded with different cs and the corresponding PSNRs of decoded images are illustrated in Figure 4. We can see that when c is set to be 3, we can get the highest PSNR.

B. Performance assessment for the proposed method.

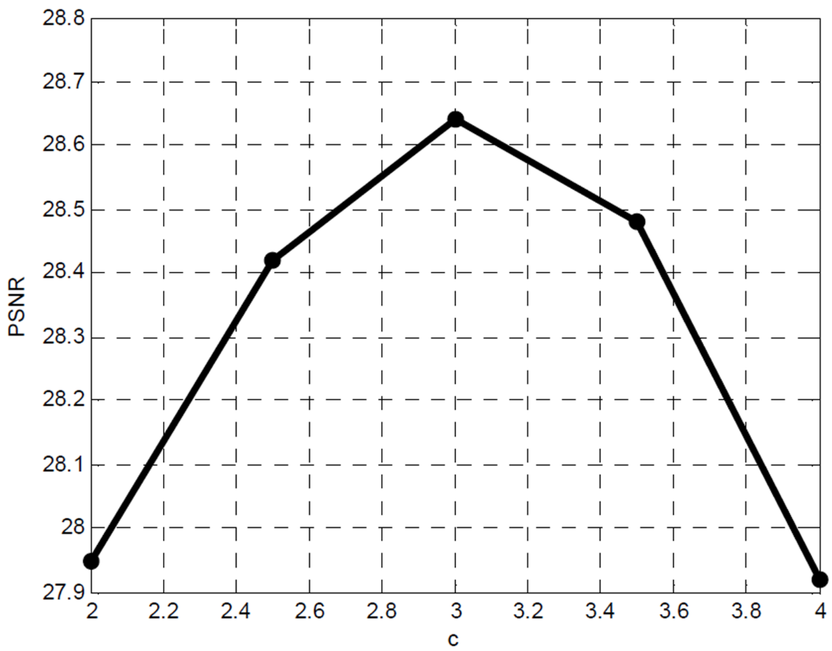
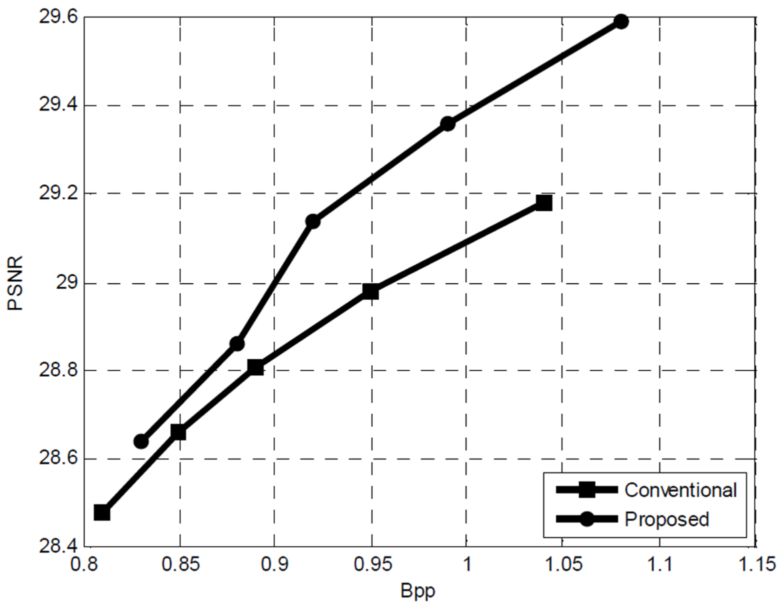


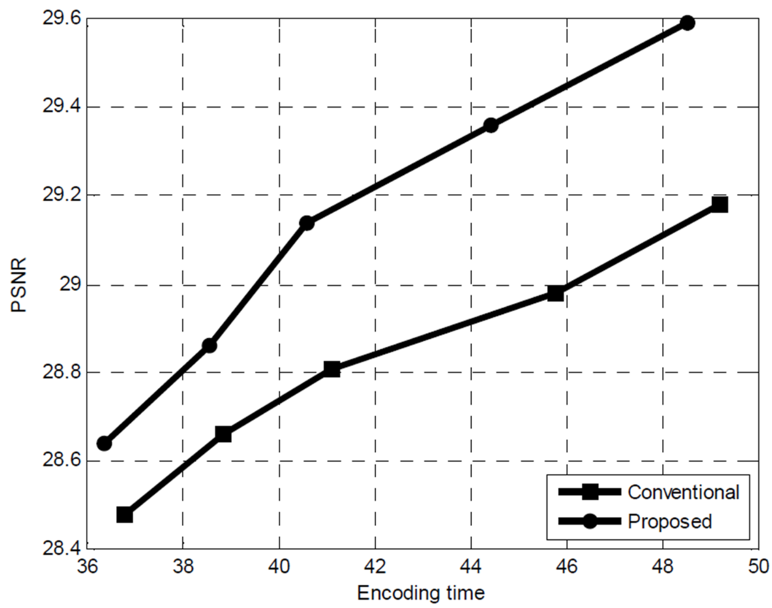
FIGURE 4. PSNR versus c

Table 1. Performance of the conventional and proposed methods with different T_1s

	Image	Barbara				
	T_1	20	30	40	50	60
PSNR (dB)	Conventional	29.18	28.98	28.81	28.66	28.48
	Proposed	29.59	29.36	29.14	28.86	28.64
Bpp	Conventional	1.04	0.95	0.89	0.85	0.81
	Proposed	1.08	0.99	0.92	0.88	0.83
Time (s)	Conventional	49.22	45.78	41.11	38.86	36.80
	Proposed	48.53	44.45	40.61	38.55	36.37



(a)



(b)

Figure 5. Comparison between the conventional and proposed methods: (a) PSNR versus Bpp, (b) PSNR versus encoding time

In the following part, the fractal coding algorithm is combined with the quadtree method. The quadtree structure consists of three levels and the block sizes are 16×16, 8×8 and 4×4, respectively. The partitioning threshold changes by $T_{i+1} = 2 \times T_i$, $i = 1, 2, 3$, as the quadtree level increases. We compare the conventional method with our proposed algorithm by PSNR, Bpp and encoding time. Since many new methods originate from the conventional method [1], the proposed method can also provide a new way to improve the other existing methods. With different T_1s, Table 1 illustrates the corresponding experimental results for Barbara. Figures 5(a) and 5(b) show the PSNR versus Bpp and PSNR versus encoding time, respectively. It can be obviously seen that the proposed method can provide higher PSNR at different Bpps and higher PSNR at different encoding times.

5. **Conclusions.** In this paper, an improved fractal image coding method is proposed. By analyzing the block matching process in fractal encoding, we discover that before block matching operations, dividing a constant value for range blocks can reduce the matching errors between the range and the contracted domain blocks. Correspondingly, the quality of decoded images can be also improved. Experiments show that combined with the quadtree method, the proposed method can provide better performance than the conventional method. In the future research, we will continue to explore other methods to improve the quality of decoded images in fractal coding.

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