ICIC Express Letters
Volume **10**, Number **7**, July **2016**

ICIC International ©2016 ISSN 1881-803X
pp. 1529–1534

GLOBAL ASYMPTOTIC STABILIZATION FOR A CLASS OF SWITCHED HIGH-ORDER PLANAR SYSTEMS VIA OUTPUT FEEDBACK

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Received November 2015; accepted February 2016

Abstract. *Based on the common Lyapunov function design method, this paper investigates the global asymptotic output feedback stabilization problem for a class of switched high-order planar systems under arbitrary switchings. By adopting the adding a power integrator technique and designing an implementable observer, an output feedback controller is constructed to ensure that the closed-loop system is globally asymptotically stable and the output can be regulated to the origin. The numerical example is provided to demonstrate the effectiveness of the proposed design scheme.*
Keywords: Switched high-order planar systems, Adding a power integrator technique, Global asymptotic stabilization, Common Lyapunov function

1. **Introduction.** Switched systems including some continuous dynamic subsystems and a switching rule are a special class of nonlinear hybrid systems. By using different design methods and taking different Lyapunov functions, the stabilization problems of switched nonlinear systems have been received much attention over the past few decades, see, e.g., [1-3] and the references therein. The motivation for the research on switched systems mainly comes from two reasons. One is that a lot of practical engineering systems are inherently multi-model and can be modeled by switched systems, such as networked systems [4] and air traffic control [5]. Another is that the switching control strategy has been extensively employed in many advanced controls, such as [6-8].

In recent years, with the aid of adding a power integrator technique, [9] systematicly studied the control problems of high-order nonlinear systems. However, for an extensive form of high-order systems, switched high-order nonlinear systems have gained little consideration. In the existing literature, [10] investigated the H_∞ control problem of switched nonlinear systems in p-normal form by multiple Lyapunov functions method and the adding a power integrator technique. [11] dealt with the state-constrained problem for a class of switched high-order nonlinear systems. It should be pointed out that the aforementioned results only considered the state feedback control, and there are no relevant results on the output feedback stabilization of switched high-order systems until now. And in most instances, the system states are unmeasurable, the state feedback control may not satisfy the system control requirements. Therefore, it is meaningful and necessary to study the output feedback control of switched high-order nonlinear systems.

The purpose of this paper is to solve the global asymptotic output feedback stabilization problem for a class of switched high-order nonlinear systems under arbitrary switchings. On the basis of common Lyapunov function design method, by introducing the adding a power integrator technique and designing an implementable observer, an output feedback controller is constructed to guarantee that the closed-loop system is globally asymptotically stable.

The remainder of this paper is organized as follows. Section 2 provides the problem formulation and some useful lemmas. Section 3 gives the output feedback controller design procedure. The stability analysis is given in Section 4, following a simulation example in Section 5. Section 6 concludes this paper.

2. **Problem Formulation and Some Useful Lemmas.**

This paper considers the following switched high-order planar system described by

$$\begin{aligned} \dot{x}_1 &= h_1^{\sigma(t)} x_2^p + f_1^{\sigma(t)}(x_1), \\ \dot{x}_2 &= h_2^{\sigma(t)} u^p + f_2^{\sigma(t)}(x_1, x_2), \\ y &= x_1, \end{aligned} \tag{1}$$

where $x = (x_1, x_2)^T \in R^2$, $u \in R$ and $y \in R$ are the system state, control input and output, respectively, and x_2 is unmeasurable. $p \in R^* =: \{q \in R_+\colon q \geq 1$ is a ratio of odd integers$\}$. $\sigma(t) : [0, +\infty) \to I = \{1, \cdots, m\}$ is the switching signal. For $\forall i = 1, 2$ and $\forall k \in I$, the mapping $f_i^k : R^i \to R$ is continuous differentiable (i.e., $\mathcal{C}^1$) with $f_i^k(0) = 0$, and h_i^k is a positive constant.

Remark 2.1. *The switching signal $\sigma(t)$ has finite number of switching during any finite time interval in this paper. This is a common assumption in the switched nonlinear literature, i.e., the Zeno and impulsive phenomena are excluded consideration here.*

Remark 2.2. *It should be pointed out that when the switching law is arbitrary, one powerful way to deal with the stabilization problems of switched systems is to find a common Lyapunov function for all subsystems. This paper aims to use the common Lyapunov function design method to address the global output feedback control for the switched high-order planar system (1) under arbitrary switchings. To achieve this objective, we need the following assumption and lemmas.*

Assumption 2.1. *For $\forall i = 1, 2$ and $\forall k \in I$, there exist $\mathcal{C}^1$ nonnegative functions $\varphi_1^k(x_1)$ and $\psi_1^k(x_1)$ such that*

$$|f_1^k(x_1)| \leq |x_1|^p \varphi_1^k(x_1), \quad |f_2^k(x_1, x_2)| \leq (|x_1|^p + |x_2|^p)\psi_1^k(x_1).$$

The adding a power integrator technique is based on the following three lemmas, which plays an important role in proving the main result of this paper.

Lemma 2.1. [12] *Suppose $p \in R^*$ and x, y be real-valued functions. For a constant $c > 0$, then $|x^p - y^p| \leq c|x - y||(x - y)^{p-1} + y^{p-1}|$.*

Lemma 2.2. [12] *Let m, n be positive constants. For any positive number γ, one has $x^m y^n \leq \frac{m}{m+n}\gamma|x|^{m+n} + \frac{n}{m+n}\gamma^{-\frac{m}{n}}|y|^{m+n}$.*

Lemma 2.3. [12] *For any $x, y \in R$ and $p \in R^*$, then $-(x-y)(x^p - y^p) \leq -\frac{1}{2^{p-1}}(x-y)^{p+1}$.*

3. **Output Feedback Controller Design.**

In what follows, we give the output feedback controller design procedure by using the adding a power integrator technique.

3.1. **State feedback control.** Firstly, choosing $\eta_1 = x_1$ and $V_1(x_1) = \frac{\lambda_1}{2}\eta_1^2$ ($\lambda_1 > 0$), for $\forall k \in I$, by Assumption 2.1, it can be deduced that

$$\begin{aligned} \dot{V}_1(x_1) &\le \lambda_1 h_1^k \eta_1 x_2^p + \lambda_1 \eta_1^{1+p}\varphi_1^k(x_1) \\ &\le \lambda_1 h_1^k \eta_1 (x_2^p - x_2^{*p}) + \lambda_1 h_1^k \eta_1 x_2^{*p} + \lambda_1 \eta_1^{1+p}\varphi_1(x_1), \end{aligned} \tag{2}$$

where $\varphi_1(x_1) \ge \max_{\forall k\in I}\{\varphi_1^k(x_1)\}$ is a $\mathcal{C}^1$ nonnegative function. The first virtual controller

$$x_2^* = -\left(\frac{c_{11} + \lambda_1\varphi_1(x_1)}{\lambda_1 h_1^{\min}}\right)^{\frac{1}{p}} \eta_1 =: -\gamma_1(x_1)\eta_1,\ c_{11} > 0,\ h_1^{\min} = \min_{\forall k\in I}\{h_1^k\} \tag{3}$$

leads to

$$\dot{V}_1(x_1) \le -c_{11}\eta_1^{1+p} + \lambda_1 h_1^k \eta_1 (x_2^p - x_2^{*p}). \tag{4}$$

Next, define $\eta_2 = x_2 - x_2^*$ and $V_2(x_1, x_2) = V_1(x_1) + \frac{\lambda_2}{2}\eta_2^2$ ($\lambda_2 > 0$). For $\forall k \in I$, from (4), it follows that

$$\begin{aligned} \dot{V}_2(x_1, x_2) \le{}& -c_{11}\eta_1^{1+p} + \lambda_2 h_2^k \eta_2 u^p + \lambda_1 h_1^k \eta_1 (x_2^p - x_2^{*p}) \\ &+\lambda_2\eta_2\left(f_2^k(x_1, x_2) - \frac{\partial x_2^*}{\partial x_1}\left(h_1^k x_2^p + f_1^k(x_1)\right)\right). \end{aligned} \tag{5}$$

By Lemmas 2.1-2.2, one deduces that

$$\begin{aligned} \lambda_1 h_1^k \eta_1 (x_2^p - x_2^{*p}) &\le \lambda_1 h_1^{\max} c|\eta_1||\eta_2|\left(x_2^{p-1} + x_2^{*p-1}\right) \\ &\le 2^{p-1} c\lambda_1 h_1^{\max}\left(|\eta_1||\eta_2|^p + \gamma_1^{p-1}(x_1)|\eta_1|^p|\eta_2|\right) \\ &\le l_{21}\eta_1^{1+p} + \alpha_{21}(x_1)\eta_2^{1+p}, \end{aligned} \tag{6}$$

where $h_1^{\max} = \max_{\forall k\in I}\{h_1^k\}$, l_{21} is a positive constant, and $\alpha_{21}(x_1)$ is a $\mathcal{C}^1$ nonnegative function.

According to Assumption 2.1 and Lemma 2.2, for $\forall k \in I$, there exist positive constant l_{22}, $\mathcal{C}^1$ nonnegative functions $\psi_1(x_1) \ge \max_{\forall k\in I}\{\psi_1^k(x_1)\}$, $\bar{\varphi}_1(x_1) \ge |\frac{\partial x_2^*}{\partial x_1}|(h_1^{\max} + \varphi_1(x_1)) + \psi_1(x_1)$ and $\alpha_{22}(x_1)$ such that

$$\begin{aligned} &\lambda_2\eta_2\left(f_2^k(x_1, x_2) - \frac{\partial x_2^*}{\partial x_1}(h_1^k x_2^p + f_1^k(x_1))\right) \\ &\le \lambda_2|\eta_2|(|x_1|^p + |x_2|^p)\bar{\varphi}_1(x_1) \\ &\le \lambda_2|\eta_2|((1 + 2^{p-1}\gamma_1^p(x_1))|\eta_1|^p + 2^{p-1}|\eta_2|^p)\bar{\varphi}_1(x_1) \\ &\le l_{22}\eta_1^{1+p} + \alpha_{22}(x_1)\eta_2^{1+p}. \end{aligned} \tag{7}$$

Taking

$$\begin{aligned} &c_{21} = c_{11} - l_{21} - l_{22} > 0,\ c_{22} > 0,\ h_2^{\min} = \min_{\forall k\in I}\{h_2^k\}, \\ &x_3^* = -\left(\frac{c_{22} + \alpha_{21}(x_1) + \alpha_{22}(x_1)}{\lambda_2 h_2^{\min}}\right)^{\frac{1}{p}} \eta_2 =: -\gamma_2(x_1)\eta_2, \end{aligned} \tag{8}$$

and substituting (6)-(8) into (5), we have

$$\dot{V}_2(x_1, x_2) \le -c_{21}\eta_1^{1+p} - c_{22}\eta_2^{1+p} + \lambda_2 h_2^k \eta_2 (u^p - x_3^{*p}). \tag{9}$$

3.2. **Observer design.** Since x_2 in system (1) is unmeasurable, we introduce the following observer

$$\dot{\hat{z}} = h_2^{\sigma(t)} u^p - \frac{\partial\Gamma(x_1)}{\partial x_1}\left(h_1^{\sigma(t)}\hat{x}_2^p + f_1^{\sigma(t)}(x_1)\right),\quad \hat{x}_2 = \hat{z} + \Gamma(x_1), \tag{10}$$

where $\hat{x}_2$ is the estimation of x_2, $\Gamma(x_1)$ is a nonlinear gain function with $\Gamma(0) = 0$ and $\frac{\partial\Gamma(x_1)}{\partial x_1} > 0$ to be determined. By (8) and (10), one can obtain the implementable controller

$$u = -\gamma_2(x_1)(\hat{x}_2 + \gamma_1(x_1)x_1) = -\gamma_2(x_1)(\hat{z} + \Gamma(x_1) + \gamma_1(x_1)x_1). \tag{11}$$

Defining $e_2 = x_2 - \hat{x}_2 = x_2 - \hat{z} - \Gamma(x_1)$, using (1) and (10), one has

$$\dot{e}_2 = f_2^{\sigma(t)}(x_1, x_2) - \frac{\partial \Gamma(x_1)}{\partial x_1} h_1^{\sigma(t)}(x_2^p - \hat{x}_2^p). \tag{12}$$

Considering $W_2(e_2) = \frac{\lambda}{2} e_2^2$ ($\lambda > 0$), for $\forall k \in I$, from (12), it follows that

$$\dot{W}_2(e_2) = \lambda e_2 \left(-\frac{\partial \Gamma(x_1)}{\partial x_1} h_1^k (x_2^p - \hat{x}_2^p) + f_2^k(x_1, x_2) \right). \tag{13}$$

By Lemma 2.3 and $\frac{\partial \Gamma(x_1)}{\partial x_1} > 0$, one can obtain

$$\begin{aligned} -\lambda h_1^k \frac{\partial \Gamma(x_1)}{\partial x_1} e_2 (x_2^p - \hat{x}_2^p) &= -(x_2 - \hat{x}_2)(x_2^p - \hat{x}_2^p) \frac{\partial \Gamma(x_1)}{\partial x_1} \lambda h_1^k \\ &\le -\frac{\partial \Gamma(x_1)}{\partial x_1} \frac{\lambda h_1^{\min}}{2^{p-1}} e_2^{1+p}. \end{aligned} \tag{14}$$

Applying Assumption 2.1 and Lemma 2.2, for $\forall k \in I$, there are positive constants l_{31} and l_{32}, a $\mathcal{C}^1$ nonnegative function $\beta_{11}(x_1)$ such that

$$\begin{aligned} \lambda e_2 f_2^k(x_1, x_2) &\le \lambda |e_2| \left(|x_1|^p + |x_2|^p \right) \psi_1^k(x_1) \\ &\le \lambda |e_2| \left((1 + 2^{p-1} \gamma_1^p(x_1)) |\eta_1|^p + 2^{p-1} |\eta_2|^p \right) \psi_1(x_1) \\ &\le l_{31} \eta_1^{1+p} + l_{32} \eta_2^{1+p} + \beta_{11}(x_1) e_2^{1+p}. \end{aligned} \tag{15}$$

Substituting (14)-(15) into (13) yields

$$\dot{W}_2(e_2) \le l_{31} \eta_1^{1+p} + l_{32} \eta_2^{1+p} - \left(\frac{\partial \Gamma(x_1)}{\partial x_1} \frac{\lambda h_1^{\min}}{2^{p-1}} - \beta_{11}(x_1) \right) e_2^{1+p}. \tag{16}$$

4. **Stability Analysis.** We state the main result of this paper as follows.

Theorem 4.1. *If Assumption 2.1 holds for the switched high-order planar system (1), under the output feedback controllers (10) and (11), then the closed-loop system is globally asymptotically stable and the output can be regulated to the origin.*

Proof: Introducing the entire Lyapunov function $V(x_1, x_2, e_2) = V_2(x_1, x_2) + W_2(e_2)$, from (9) and (16), one leads to

$$\begin{aligned} \dot{V}(x_1, x_2, e_2) \le& -(c_{21} - l_{31}) \eta_1^{1+p} - (c_{22} - l_{32}) \eta_2^{1+p} \\ &- \left(\frac{\partial \Gamma(x_1)}{\partial x_1} \frac{\lambda h_1^{\min}}{2^{p-1}} - \beta_{11}(x_1) \right) e_2^{1+p} + \lambda_2 h_2^k \eta_2 \left(u^p - x_3^{*p} \right). \end{aligned} \tag{17}$$

To estimate the last term on the right-hand side of (17), by (8), (11) and Lemmas 2.1-2.2, one gets

$$\begin{aligned} \lambda_2 h_2^k \eta_2 \left(u^p - x_3^{*p} \right) &\le c \lambda_2 h_2^{\max} \gamma_2^p(x_1) |\eta_2| |e_2| ((x_2 - x_2^*)^{p-1} + (x_2 - x_2^* - e_2)^{p-1}) \\ &\le 2^{p-1} c \lambda_2 h_2^{\max} \gamma_2^p(x_1) \left(|\eta_2|^p |e_2| + |\eta_2| |e_2|^p \right) \\ &\le l_{33} \eta_2^{1+p} + \beta_{12}(x_1) e_2^{1+p}, \end{aligned} \tag{18}$$

where $h_2^{\max} = \max_{\forall k \in I} \{h_2^k\}$, $l_{33} > 0$ is a constant, and $\beta_{12}(x_1)$ is a $\mathcal{C}^1$ nonnegative function.

Combining (17) with (18), we have

$$\begin{aligned} \dot{V}(x_1, x_2, e_2) \le& -(c_{21} - l_{31}) \eta_1^{1+p} - (c_{22} - l_{32} - l_{33}) \eta_2^{1+p} \\ &- \left(\frac{\partial \Gamma(x_1)}{\partial x_1} \frac{\lambda h_1^{\min}}{2^{p-1}} - \beta_{11}(x_1) - \beta_{12}(x_1) \right) e_2^{1+p}. \end{aligned} \tag{19}$$

By selecting

$$c_{31} = c_{21} - l_{31} > 0, \ c_{32} = c_{22} - l_{32} - l_{33} > 0, \ c_{33} > 0,$$

$$\Gamma(x_1) = \frac{2^{p-1}}{\lambda h_1^{\min}} \left(c_{33}x_1 + \int_0^{x_1} (\beta_{11}(s) + \beta_{12}(s))\, ds \right), \tag{20}$$

(19) changes into

$$\dot{V}(x_1, x_2, e_2) \le -c_{31}\eta_1^{1+p} - c_{32}\eta_2^{1+p} - c_{33}e_2^{1+p}. \tag{21}$$

By (21), one can conclude that the closed-loop system consisting of (1), (10) and (11) is globally asymptotically stable and the output can be regulated to the origin.

5. **A Simulation Example.** Consider the following switched nonlinear system

$$\begin{aligned} \dot{x}_1 &= h_1^{\sigma(t)} x_2^{\frac{5}{3}} + f_1^{\sigma(t)}(x_1), \\ \dot{x}_2 &= h_2^{\sigma(t)} u^{\frac{5}{3}} + f_2^{\sigma(t)}(x_1, x_2), \\ y &= x_1, \end{aligned} \tag{22}$$

where $\sigma(t) : [0, +\infty) \to \{1, 2\}$, $h_1^1 = \frac{6}{5}$, $h_1^2 = 1$, $h_2^1 = \frac{1}{2}$, $h_2^2 = \frac{2}{5}$, $f_1^1 = \frac{1}{5}x_1^{\frac{5}{3}}$, $f_1^2 = \frac{1}{6}x_1 \sin x_1$, $f_2^1 = \frac{1}{4}x_1^{\frac{5}{3}} \sin x_2$, $f_2^2 = \frac{1}{5}x_1^{\frac{5}{3}} \sin x_1x_2$. Following the design procedure in Section 3 and selecting $c_{11} = 2$, $c_{22} = 1$, $c_{33} = 0.05$, $\lambda = 5$, $\lambda_1 = 0.1$, $\lambda_2 = 0.001$, $l_{21} = 1.65$, $l_{22} = 0.206$, $l_{31} = 0.14$ and $l_{33} = 0.95$, one can get the output feedback controller

$$\begin{aligned} u &= -129.9019\left(\hat{z} + \Gamma(x_1) + 6.0703x_1\right), \ \Gamma(x_1) = 12.6073x_1, \\ \dot{\hat{z}} &= h_2^{\sigma(t)} u^{\frac{5}{3}} - \frac{\partial \Gamma(x_1)}{\partial x_1} \left(h_1^{\sigma(t)} \left(\hat{z} + \Gamma(x_1)\right)^{\frac{5}{3}} + f_1^{\sigma(t)}(x_1) \right). \end{aligned} \tag{23}$$

In the simulation, we choose the initial values $x_1(0) = -0.1$, $x_2(0) = -1$, $\hat{z}(0) = 2$. Figure 1 demonstrates the effectiveness of the output feedback controller.

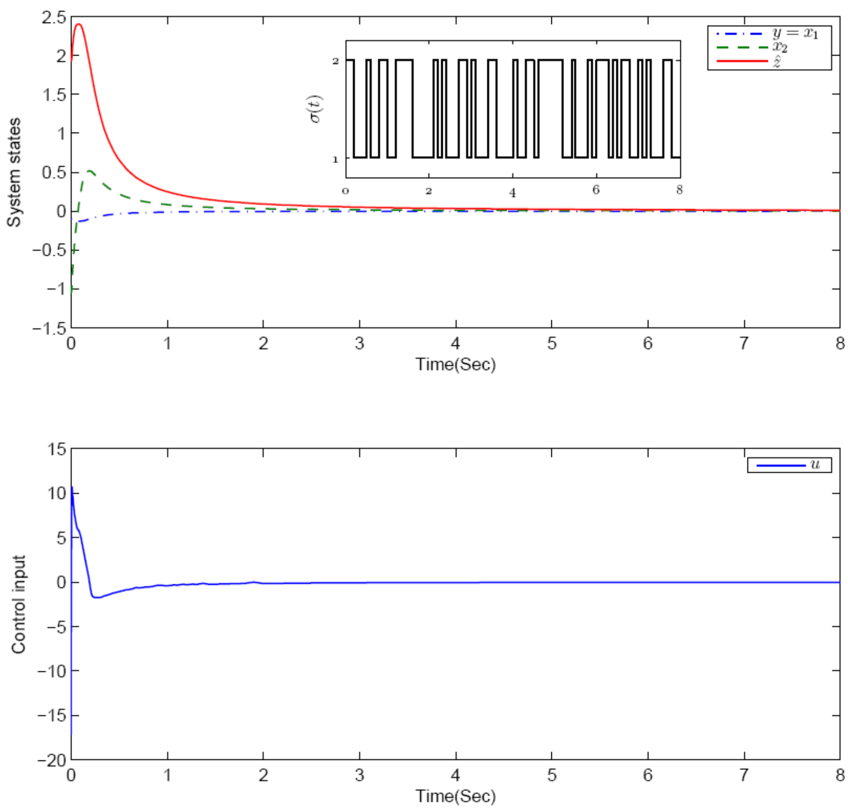

FIGURE 1. The responses of the closed-loop systems (22) and (23)

6. **Conclusion.** This paper deals with the global asymptotic output feedback stabilization problem for a class of switched high-order planar system (1) by introducing the adding a power integrator technique. Based on the common Lyapunov function design method and designing an implementable observer, the output feedback controller is explicitly constructed to ensure that the closed-loop system is globally asymptotically stable and the output can be regulated to the origin.

There also exist some problems to be investigated: one is to consider the output feedback control for system (1) with feedforward form as in [13]. Another is to solve the data-driven problem of system (1) discussed in [14].

Acknowledgment. The authors would like to express sincere gratitude to the editor and reviewers for their helpful suggestions in improving the quality of this paper. This work was supported by the National Natural Science Foundation of China (Nos. 61403041, 61503036 and 61573172), the Program for Liaoning Excellent Talents in University (No. LJQ2015001), the Project Funded by China Postdoctoral Science Foundation (No. 2015M580435), and the Program for Liaoning Innovative Research Team in University (No. LT2013023).

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