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THE PERFORMANCE EVALUATION OF DHT PRECODED OFDM SYSTEMS WITH CARRIER FREQUENCY OFFSET IN TIME-SELECTIVE FADING CHANNEL

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Abstract. *Orthogonal frequency division multiplexing (OFDM) is a promising technique combating frequency-selective fading, but it is vulnerable to time-selective fading. The channel time variation brings inter-carrier inference (ICI) that degrades the bit error rate (BER) performance of the systems. Discrete Hartley transform (DHT) precoded OFDM is studied for peak to average power ratio (PAPR) reduction; however, the BER performance is not in-depth research. In this paper, the performance of DHT precoded OFDM (DHT-OFDM) systems with carrier frequency offset (CFO) in time-selective fading channel is evaluated. We show that the DHT-OFDM systems are transformed into the single carrier systems with cyclic prefix (SC-CP), which are less sensitive to CFO compared to the OFDM systems. Furthermore, DHT-OFDM systems with zero forcing (ZF) equalization provide no diversity gain compared to SC-CP. The simulation results are coincided with the theoretical derivation.*
Keywords: OFDM, CFO, DHT-OFDM, ZF, Time-selective fading channel

1. **Introduction.** Orthogonal frequency division multiplexing has emerged as the leading modulation technology for the next communication system, owing to the immunity to frequency-selective channel and the high spectral efficiency. OFDM divides the available spectrum into a number of overlapping orthogonal narrowband sub-channels, and hence converts a frequency selective channel into a non-frequency selective channel. This is the reason why OFDM is a promising technique combating frequency-selective fading. However, the increased symbol duration makes it much more time-selective, which is sensitive to the frequency offset that caused by the Doppler shift or a time-varying channel. The transmitted signals are not independent on each sub-carrier, because the time variation of fading channel over an OFDM block period causes sub-channel orthogonality loss. As demonstrated in [1], the carrier frequency offset degrades the performance of OFDM systems.

Several researchers and the references have addressed the inter-carrier inference in OFDM systems. The main concerns are on CFO estimation [2], but most of them require pilot symbols or training sequences, and the rapid time variation of the channel makes its estimation more complicated. In [3], the ICI self-cancellation schemes have been proposed to compress ICI level of OFDM systems significantly at the cost of frequency efficiency loss. To improve performance, several equalizers [4] at the receiver for interference suppression schemes have been proposed, while they usually require channel information. In a recent work, a discrete Hartley transform precoded OFDM is attracting people's attention, which has been proposed in [5] for reducing the PAPR of OFDM systems, while their focus is on PAPR performance. In [6], a low complexity discrete Hartley transform precoded OFDM systems have been proposed. It is a pity that their focus is on frequency selective fading and the issue of time selective fading has not been explicitly mentioned. It is necessary to support mobile communications at high speed in 802.16e standard, thus the channel may change much time-selective. In this case, how to improve the performance of OFDM under time-varying channel becomes more urgent.

In this paper, the performance of DHT-OFDM systems with CFO in time-selective fading channel is evaluated. According to the property of DHT and inverse fast Fourier transform (IFFT), the DHT-OFDM systems are transformed into the SC-CP which are less sensitive to CFO. As the single carrier systems, DHT-OFDM systems provide no diversity gain compared to SC-CP. Simulation results also show that the systems have a substantial improvement in bit error performance over conventional OFDM systems which coincide with the theoretical derivation.

The rest of this paper is organized as follows. The systems model is presented in Section 2. In Section 3, the performance of DHT-OFDM systems is analyzed. Section 4 gives some simulation results, and a conclusion is given in Section 5.

2. Systems Model.

In conventional OFDM system, each stream would be split into N parallel flows and carried over at different center frequencies. The transmitted signals are modulated on N sub-channels, and moreover, this time-domain multiplexing can be efficiently performed by using IFFT. In contrast to the conventional OFDM systems, each symbol of a block in DHT-OFDM systems is firstly transformed by means of DHT before multiplexed into the sub-channels.

$$x_p(i) = \sum_{m=0}^{N-1} x(m) cas\left(\frac{2\pi}{N} im\right), \quad 0 \le i \le N-1, \tag{1}$$

where $cas(2\pi im/N)$ is equal to $[\cos(2\pi im/N) + \sin(2\pi im/N)]$ and m is the mth transmission signal. A typical discrete-time systems model of DHT-OFDM transceiver is depicted in Figure 1.

After serial-to-parallel (S/P) conversion, the input symbols are grouped into N parallel symbols. Such symbols $X_p = [x_p(0), x_p(1), \ldots, x_p(N-1)]^T$ are created. These symbols

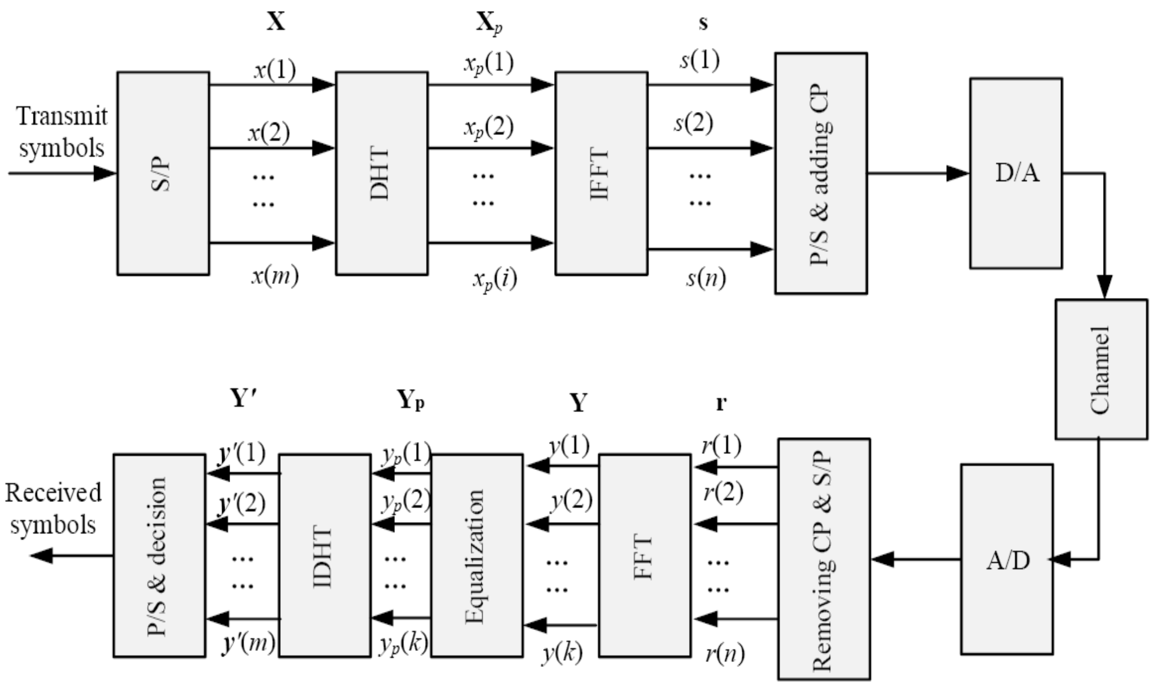

Figure 1. The systems model of DHT-OFDM

are transformed by IFFT, and then,

$$s(n) = \frac{1}{N} \sum_{i=0}^{N-1} x_p(i) e^{j\frac{2\pi}{N}in}, \quad 0 \le n \le N-1, \tag{2}$$

where $s(n)$ is the time domain signal and N is the number of carriers. Then the parallel symbols are converted to serial symbols. In order to minimize the inter-symbol interference (ISI), a guard interval usually referred to as cyclic prefix (CP) is inserted, which is a cyclic extension of the IFFT output sequence. The length of CP is chosen to be larger than the maximum delay spread. The digital-to-analog (D/A) converter then creates an analog time-domain signal which is transmitted through the channel.

In mobile radio environment, several replicas of the transmitted signal are received from different directions with different time delays. The time variant impulse response model of the multipath channel is considered in [7]

$$h(t) = \sum_{l=0}^{M-1} h_l(t) \delta(t - \tau_l), \tag{3}$$

where M is the number of propagation paths, τ_l is the delay time of the lth path, and $h_l(t)$ is the time-variant path gain of the lth path.

The CFO destroys the subcarrier orthogonality when the symbols pass the multipath channel, because the channel experiences a longer time variation. At the receiver, CP is removed and the signals are converted to the frequency domain by fast Fourier transform (FFT). Received signals at the sampling time nts in presence of CFO can be written as [8]

$$y(k) = \frac{1}{N} \sum_{n=0}^{N-1} h(k) \sum_{i=0}^{N-1} \sum_{m=0}^{N-1} x(m) cas\left(\frac{2\pi}{N} im\right) e^{j\frac{2\pi}{N}in} e^{-j\frac{2\pi}{N}kn} e^{j\frac{2\pi}{N}\varepsilon n} + Z(k), \quad 0 \le k \le N-1, \tag{4}$$

where $Z(k)$ is the frequency complex white Gaussian noise. From the properties of the IFFT and DHT, Formula (4) can be expressed by

$$y(k) = \frac{\sqrt{2}}{2} e^{j\frac{\pi}{4}} \frac{1}{N} \sum_{n=0}^{N-1} h(k)[x(m) - jx(N-m)] e^{-j\frac{2\pi}{N}kn} e^{j\frac{2\pi}{N}\varepsilon n} + Z(k), \quad 0 \le k \le N-1. \tag{5}$$

Zero forcing equalization is used in the frequency on the kth subchannel, and the receiver has perfect channel state information. Then the impact of time-variant response can be removed. Based on (4) and (5), the desired symbols are obtained by DHT, and then

$$y'(m) = \frac{\sqrt{2}}{2} e^{j\frac{\pi}{4}} \frac{1}{N} \sum_{k=0}^{N-1} \sum_{n=0}^{N-1} [x(m) - jx(N-m)] e^{-j\frac{2\pi}{N}kn} cas\left(\frac{2\pi}{N} km\right) e^{j\frac{2\pi}{N}\varepsilon n} + Z(m), \quad 0 \le m \le N-1, \tag{6}$$

where $Z(m)$ is the DHT of $Z(k)/h(k)$. Based on the properties of FFT and IDHT, finally, Formula (6) can be rewritten as

$$y'(m) = \frac{1}{N} e^{j\frac{2\pi}{N}\varepsilon n} x(m) + Z(m). \tag{7}$$

Here, the demodulation of $y'(m)$ only depends on itself and the additive noise. $e^{j2\pi\varepsilon n/N}$ is characterized by a phase rotation, which produces ISI. The DHT-OFDM systems are transformed into the single carrier systems with cyclic prefix, while the slightly difference is the additive noise. By the analysis of Section 3, the additive noise is equivalent. Compared to OFDM systems, the single-carrier systems are less sensitive to CFO. This result coincides with the analysis in [9,10] that OFDM is orders of magnitude more sensitive to frequency offset.

3. **Performance Analysis.** For QPSK modulation, the BER of the kth subchannel is

$$P(k) = Q\left(\sqrt{\frac{\varepsilon_s}{\delta^2(k)}}\right), \tag{8}$$

where ε_s is the symbol energy, and $\delta^2(k)$ is noise variance. The average BER is

$$P = \frac{1}{N}\sum_{k=0}^{N-1} E\left[Q\left(\sqrt{\frac{\varepsilon_s}{\delta^2(k)}}\right)\right], \tag{9}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{+\infty} e^{-\frac{t^2}{2}}dt$.

Assume that the frequency noise $Z(k)$ is complex white Gaussian noise with variance N_0. The modulation scheme is QPSK, and modulation symbols have symbol energy ε_s. $Z(k)$ is given in the following form,

$$Z(k) = \frac{1}{N}\sum_{n=0}^{N-1} Z(n)e^{-j\frac{2\pi}{N}kn}, \quad k = 0, 1, \ldots, N-1. \tag{10}$$

The kth element of the noise vector in DHT-OFDM systems has variance given by

$$\sigma^2(k) = N_0 \sum_{k=0}^{N-1} \frac{|w_{k,i}|^2}{|h(k)|^2}, \tag{11}$$

where $w_{k,i}$ denotes the (k, i)th element of Hartley matrix. Considering $\sum\limits_{k=0}^{N-1} |w_{k,i}|^2 = 1$, the average mean square error (MSE) can be written as

$$\sigma^2_{MSE} = \frac{1}{N}\sum_{k=0}^{N-1} \frac{N_0}{|h(k)|^2}. \tag{12}$$

For DHT-OFDM systems, the noise variances in all the subchannels are the same. Combining (8) and (9), the BER of the DHT-OFDM systems can be given as

$$P_{DHT\_OFDM} = Q\left(\sqrt{\frac{\gamma}{\frac{1}{N}\sum\limits_{k=0}^{N-1}\frac{1}{|h(k)|^2}}}\right), \tag{13}$$

where γ is the signal-to-noise ratio (SNR) ε_s/N_0. The BER of the SC-CP is given by

$$P_{SC\_CP} = Q\left(\sqrt{\frac{\gamma}{\frac{1}{N}\sum\limits_{k=0}^{N-1}\frac{1}{|h(k)|^2}}}\right). \tag{14}$$

Based on Equations (13) and (14), we observe that the SNR is equal,

$$P_{DHT\_OFDM} = P_{SC\_CP}. \tag{15}$$

From Formula (15), we can know that DHT-OFDM systems are transformed into the SC-CP. In other words, DHT-OFDM systems with ZF equalization provide no diversity gain compared to SC-CP.

4. **Simulation Results.** In this section, simulations are performed to evaluate the advances of DHT-OFDM systems in time-selective fading. We consider the systems with 512 subcarriers which employ QPSK modulation, and the bandwidth is 5 MHz. The carrier frequency offset is calculated based on a carrier frequency of 2.4 GHz, and the time-selective fading channel have four resolvable multipath components with propagation delays and fading margins of [0 300 600 900] μs and [0 10 20 30] dB, respectively. ZF equalization is considered with perfect channel state information and the length of cyclic prefix L = 40.

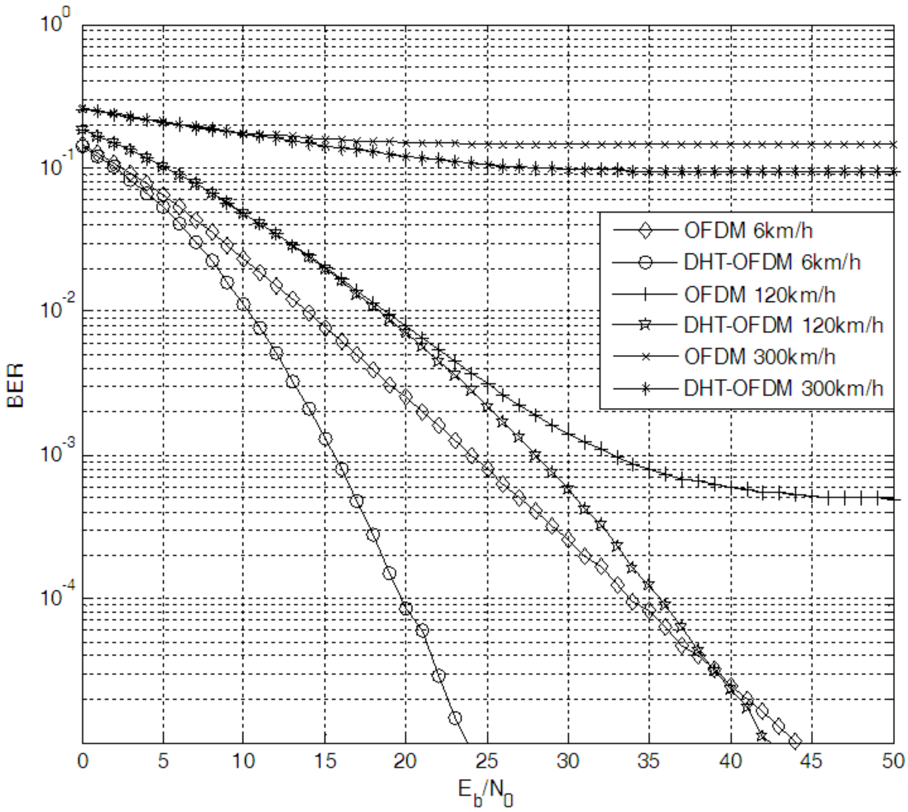

Figure 2. BER of the conventional OFDM and the DHT-OFDM under time selective channel with different CFO

Figure 2 shows the BER vs. E_b/N_0 performance in time-selective channel with different CFO, where E_b is the received bit energy. BER is compared by three different vehicle speeds chosen as 6 km/h, 120 km/h, and 300 km/h, which are typical for urban and suburban environments. Although OFDM systems are sensitive to frequency offset and phase noise, DHT-OFDM systems convert the multicarrier systems into the single carrier systems, which are less sensitive to frequency offset. Besides, it is clearly in Figure 2 that BER decreases as the SNR increases. DHT-OFDM systems with ZF equalizer achieve significant improvement compared with OFDM systems, and this result coincides with the analysis. Meanwhile, there is a floor effect in both OFDM systems and DHT-OFDM systems as the carrier offset effect increasing. With the mobile speeds increasing, the large CFO produces large orthogonality distortion and leads to larger inter-carrier inference, which lower the achievable SNR. For OFDM systems, the BER performance degradation is mainly determined by the ICI.

5. **Conclusion.** This paper has evaluated the performance of DHT-OFDM systems with carrier frequency offset in time-selective fading. It is shown that the BER performance of DHT-OFDM systems is better than conventional OFDM systems by transforming the multicarrier systems into the single carrier systems, which are robust to the inter carrier

interference caused by carrier frequency offset. Simulation results also coincide with the theoretical derivation. In the future, the performance of DHT-OFDM in MIMO systems can be further exploited.

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