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FAULT DETECTION FOR DISCRETE-TIME INTERVAL TYPE-2 FUZZY SYSTEMS

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Abstract. *This paper studies the problem of fault detection filter design for a class of nonlinear discrete-time systems in presence of parameter uncertainties based on the interval type-2 (IT2) T-S fuzzy model. On the basis of the IT2 T-S fuzzy model, the parameter uncertainty is tackled via membership functions with lower and upper bounds. A novel IT2 fault detection filter is constructed to ensure the residual system to be asymptotically stable and satisfy the predefined H_∞ performance. It is worth noting that the filter to be designed does not share the same premise variables, number of fuzzy rules and membership functions with the fuzzy model, which will lead to more flexible design. Finally, a practical example is given to illustrate the effectiveness of the method proposed in this paper.*

Keywords: Nonlinear systems, Parameter uncertainties, IT2 T-S fuzzy model, Fault detection filter

1. **Introduction.** In recent few decades, nonlinear systems have drawn considerable attention. Due to the limited capability of traditional linear system theory, various nonlinear control theories have been proposed, such as fuzzy control [1], sliding model control [2, 3] and adaptive control [4]. Compared with other nonlinear control strategies, Takagi-Sugeno (T-S) fuzzy model [5] can employ the priori and approximate the nonlinearity to any accuracy with so-called "IF-THEN" rules, which has been applied successfully in [6].

On the other hand, fault detection has been an active research field since the high safety, reliability and performance are demanded in industrial applications [7]. Using T-S fuzzy model, considerable results about fault detection for nonlinear systems have been reported. The authors in [8] proposed a filter-based fault detection scheme for a category of uncertain nonlinear systems. The problem of robust fault detection for a class of nonlinear systems with sensor faults and unknown bounded disturbances was investigated in [9]. The authors in [10] proposed a filter scheme for nonlinear networked systems with mixed delays and successive packet dropouts. An observer-based fault detection scheme was provided for nonlinear systems with sensor fault in [11]. Considering networked measurements, the authors in [12] focused on the problem of fault detection for nonlinear systems based on the fuzzy-model-based approach.

However, it should be mentioned that the results mentioned above are on the basis of type-1 T-S fuzzy model, in which membership functions are determined. Using such membership functions, there exist some conservativeness when uncertainties appear in systems. The interval type-2 fuzzy set was proposed in [13]. Membership functions of [13] are interval rather than determined and in this case, the uncertainty can be captured and expressed via membership functions with lower and upper bounds. Recently, some remarkable results have been scattered in the literature, see for example, [14, 15, 16, 17] and references therein. As an extension, the authors in [18] combined the IT-2 fuzzy set theory and T-S fuzzy model and constructed the IT2 T-S fuzzy model. With such model, the stability and stabilization problems for nonlinear systems subject to parameter uncertainties were investigated [18]. To consider more uncertain information, the footprint of uncertainty was taken into consideration in [19]. Considering unmatched membership functions, the authors in [20, 21] provided state-feedback-based and output-feedback-based control strategies and filter design for uncertain systems in the framework of IT2 T-S fuzzy model. In [22, 23], the IT2 T-S model was applied to nonlinear uncertain networked systems. However, there exist few fault detection results concerning nonlinear uncertain systems on the basis of IT2 T-S fuzzy model, which motivates this work.

This paper investigates the problem of fault detection filter (FDF) design for a class of nonlinear discrete-time systems subject to parameter uncertainties. The main contributions can be summarized as follows. 1) The problem of uncertainties existing in systems is handled through IT2 T-S fuzzy model, which can facilitate capturing and expressing uncertainties and reduce design conservativeness. 2) The problem of design flexibility is improved via adopting independent premise variables, membership functions and number of fuzzy rules for the FDF to be designed. 3) The problem of design conservativeness is taking into consideration by considering the footprint of uncertainty. Finally, simulation results are given to demonstrate the effectiveness of the method proposed in this paper.

The remainder of the paper is organized as follows. Section 2 describes the problem of fault detection subject to parameter uncertainties. The conditions of designing the FDF are formulated in Section 3. An practical example is provided to verify the usefulness of the proposed method in Section 4. Finally, Section 5 concludes the paper.

Notation: Notations without explicit statement in this paper are as same as existing relative papers.

2. **Problem Statement and Preliminaries.**

Consider the following IT-2 T-S model for a class of nonlinear systems with parameter uncertainties.

Plant Rule i: IF $f_1(\sigma(k))$ is M_{i1}, and $f_2(\sigma(k))$ is M_{i2} and, $\ldots$, and $f_\theta(\sigma(k))$ is $M_{i\theta}$, THEN

$$\begin{aligned} x(k+1) &= A_i x(k) + B_i u(k) + E_{1i} w(k) + E_{2i} f(k), \\ y(k) &= C_i x(k) + D_i u(k) + F_{1i} w(k) + F_{2i} f(k),\ i = 1, 2, \ldots, r, \end{aligned} \tag{1}$$

where M_{ij} denotes the fuzzy set, and $f(\sigma(k)) = [f_1(\sigma(k)), f_2(\sigma(k)), \ldots, f_\theta(\sigma(k))]^T$ stands for the premise variable. $x(k) \in R^{n_x}$ represents the state; $y(k) \in R^{n_y}$ stands for the measured output; $u(k) \in R^{n_u}$ denotes the control input; $w(k) \in R^{n_w}$ and $f(k) \in R^{n_f}$ stand for the external disturbance and the fault vector to be detected, respectively, which belong to $l_2[0, \infty)$. A_i, B_i, E_{1i}, E_{2i}, C_i, D_i, F_{1i} and F_{2i} are system matrices with appropriate dimensions. The following interval sets present the firing strength of the ith rule:

$$W_i(\sigma(k)) = [\underline{m}_i(\sigma(k)), \overline{m}_i(\sigma(k))],$$

where

$$\underline{m}_i(\sigma(k)) = \prod_{p=1}^{\theta} \underline{u}_{M_{ip}}(f_p(\sigma(k))) \geq 0,\ \overline{m}_i(\sigma(k)) = \prod_{p=1}^{\theta} \overline{u}_{M_{ip}}(f_p(\sigma(k))) \geq 0, \tag{2}$$

$$\overline{u}_{M_{ip}}(f_p(\sigma(k))) \geq \underline{u}_{M_{ip}}(f_p(\sigma(k))) \geq 0,\ \overline{m}_i(\sigma(k)) \geq \underline{m}_i(\sigma(k)) \geq 0, \tag{3}$$

$\underline{u}_{M_{ip}}(f_p(\sigma(k)))$, $\overline{u}_{M_{ip}}(f_p(\sigma(k)))$, $\underline{m}_i(\sigma(k))$ and $\overline{m}_i(\sigma(k))$ denote lower and upper membership functions, lower and upper grade of membership, respectively.

The overall dynamics of the IT2 T-S model (1) can be represented as follows:

$$x(k+1) = \sum_{i=1}^{r} m_i(\sigma(k)) [A_i x(k) + B_i u(k) + E_{1i} w(k) + E_{2i} f(k)],$$

$$y\left(k\right)=\sum_{i=1}^{r}m_i\left(\sigma(k)\right)\left[C_ix\left(k\right)+D_iu\left(k\right)+F_{1i}w(k)+F_{2i}f(k)\right], \quad (4)$$

where $m_i\left(\sigma(k)\right)=\frac{\underline{a}_i(\sigma(k))\underline{m}_i(\sigma(k))+\overline{a}_i(\sigma(k))\overline{m}_i(\sigma(k))}{\sum_{i=1}^{r}(\underline{a}_i(\sigma(k))\underline{m}_i(\sigma(k))+\overline{a}_i(\sigma(k))\overline{m}_i(\sigma(k)))}\geq 0$, $\sum_{i=1}^{r}m_i\left(\sigma(k)\right)=1$, $0\leq\underline{a}_i(\sigma(k))\leq 1$, $0\leq\overline{a}_i(\sigma(k))\leq 1$, $\underline{a}_i(\sigma(k))+\overline{a}_i(\sigma(k))=1$ with $\underline{a}_i(\sigma(k))$ and $\overline{a}_i(\sigma(k))$ being nonlinear weighting functions and $m_i\left(\sigma(k)\right)$ regarded as the grades of membership.

The filter-based fault detection scheme is adopted in this paper. To construct more flexible IT2 filter, the FDF to be designed does not need to share the same premise variables, number of fuzzy rules and membership functions with the plant. And the IT2 FDF with s-rule is described as follows:

Filter Rule j: IF $g_1\left(\sigma(k)\right)$ is N_{j1}, and $g_2\left(\sigma(k)\right)$ is N_{j2} and, …, and $g_{\bar{\Psi}}\left(\sigma(k)\right)$ is $N_{j\bar{\Psi}}$, THEN

$$\begin{aligned}x_f\left(k+1\right)&=A_{fj}x_f\left(k\right)+B_{fj}y_f(k),\\ r(k)&=C_{fj}x_f\left(k\right)+D_{fj}y_f(k),\ j=1,2,\ldots,s,\end{aligned} \quad (5)$$

where $x_f\left(k\right)\in R^{n_x}$ denotes the state of the FDF; $r\left(k\right)\in R^{n_r}$ is the "residual" that is compatible with $f(k)$ and $A_{fj},\ B_{fj},\ C_{fj}$ and D_{fj} are FDF matrices to be determined. The following interval sets describe the firing strength of the jth rule:

$$\Omega_j(\sigma(k))=\left[\underline{\omega}_j(\sigma(k)),\ \overline{\omega}_j(\sigma(k))\right],$$

where $\underline{\omega}_i(\sigma(k))=\prod_{q=1}^{\bar{\Psi}}\underline{u}_{N_{jq}}\left(g_q(\sigma(k))\right)\geq 0$, $\overline{\omega}_i(\sigma(k))=\prod_{q=1}^{\bar{\Psi}}\overline{u}_{N_{jq}}(g_q(\sigma(k)))\geq 0$, $\overline{u}_{N_{jq}}(g_q(\sigma(k)))\geq\underline{u}_{N_{jq}}(g_q(\sigma(k)))\geq 0$, $\overline{\omega}_i(\sigma(k))\geq\underline{\omega}_i(\sigma(k))\geq 0$, $\underline{u}_{N_{jq}}(g_p(\sigma(k)))$, $\overline{u}_{N_{jq}}(g_p(\sigma(k)))$, $\underline{\omega}_i(\sigma(k))$ and $\overline{\omega}_i(\sigma(k))$ denote lower and upper membership functions, lower and upper grade of membership, respectively.

Then, the overall fuzzy FDF is represented as follows:

$$\begin{aligned}x_f\left(k+1\right)&=\sum_{j=1}^{s}\omega_j\left(\sigma(k)\right)\left[A_{fj}x_f\left(k\right)+B_{fj}y_f(k)\right],\\ r(k)&=\sum_{j=1}^{s}\omega_j\left(\sigma(k)\right)\left[C_{fj}x_f\left(k\right)+D_{fj}y_f(k)\right],\end{aligned} \quad (6)$$

where $\omega_j\left(x(k)\right)=\frac{\underline{b}_j(\sigma(k))\underline{\omega}_j(\sigma(k))+\bar{b}_j(\sigma(k))\overline{\omega}_j(\sigma(k))}{\sum_{j=1}^{s}\left(\underline{b}_j(\sigma(k))\underline{\omega}_j(\sigma(k))+\bar{b}_j(\sigma(k))\overline{\omega}_j(\sigma(k))\right)}\geq 0$, $\sum_{j=1}^{s}\omega_j\left(\sigma(k)\right)=1$, $\underline{b}_j(\sigma(k))\in\left[\begin{array}{cc}0&1\end{array}\right]$, $\underline{b}_j(\sigma(k))\in\left[\begin{array}{cc}0&1\end{array}\right]$, $\underline{b}_j(\sigma(k))+\bar{b}_j(\sigma(k))=1$, $\underline{b}_i(\sigma(k))$ and $\bar{b}_j(\sigma(k))$ are predefined nonlinear functions and $\omega_j\left(\sigma(k)\right)$ is regarded as the grade of membership.

In order to identify the fault timely when it arises, a residual reference model is introduced in the form of $\bar{f}(z)=W(z)f(z)$. A minimal realization of $\bar{f}(z)=W(z)f(z)$ is as follows:

$$\begin{aligned}\hat{x}(k+1)&=A_w\hat{x}(k)+B_wf(k),\\ \bar{f}(k)&=C_w\hat{x}(k)+D_wf(k),\end{aligned} \quad (7)$$

This section gives the evaluation function $\|r\|$ and threshold J_{th} to determine whether a systems is suffering fault or not. The details are as follows:

$$\begin{cases}\|r\|>J_{th}\rightarrow\text{with faults}\rightarrow\text{alarm},\\ \|r\|\leq J_{th}\rightarrow\text{no faults},\end{cases}$$

where

$$\|r\| \triangleq \frac{1}{L}\sqrt{\sum_{k=t_1}^{t_2} r(k)^T r(k)},\ L = t_2 - t_1 + 1,\ J_{th} \triangleq \sup_{w(k)\in l_2,\ f(k)=0} \|r\|.$$

The residual system is represented as follows:

$$\begin{aligned}\tilde{x}(k+1) &= \sum_{i=1}^{r}\sum_{j=1}^{s} m_i(\sigma(k))\,\omega_j(\sigma(k))\left[\bar{A}_{1ij}\tilde{x}(k) + \bar{B}_{1ij}\tilde{u}(k)\right],\\ \bar{z}(k) &= \sum_{i=1}^{r}\sum_{j=1}^{s} m_i(\sigma(k))\,\omega_j(\sigma(k))\left[\bar{C}_{1ij}\tilde{x}(k) + \bar{D}_{1ij}\tilde{u}(k)\right],\end{aligned} \tag{8}$$

where

$$\tilde{x}(k) = \begin{bmatrix} x^T(k) & x_f^T(k) & \hat{x}^T(k)\end{bmatrix}^T,\ \tilde{u}(k) = \begin{bmatrix} u^T(k) & w^T(k) & f^T(k)\end{bmatrix}^T,$$

$$\bar{A}_{1ij} = \begin{bmatrix} A_i & 0 & 0\\ B_{fj}C_i & A_{fj} & 0\\ 0 & 0 & A_w\end{bmatrix},\ \bar{B}_{1ij} = \begin{bmatrix} B_i & E_{1i} & E_{2i}\\ B_{fj}D_i & B_{fj}F_{1i} & B_{fj}F_{2i}\\ 0 & 0 & B_w\end{bmatrix},\ \bar{z}(k) = r(k) - \bar{f}(k),$$

$$\bar{C}_{1ij} = \begin{bmatrix} D_{fj}C_i & C_{fj} & -C_w\end{bmatrix},\ \bar{D}_{1ij} = \begin{bmatrix} D_{fj}D_i & D_{fj}F_{1i} & D_{fj}F_{2i} - D_w\end{bmatrix}.$$

To consider more information of uncertainty and facilitate stability analysis of system (8), following the idea in [19], the state space of interest H consists of q connected sub-state spaces H_z $(z = 1, 2, \ldots, q)$ with $H = \cup_{z=1}^{q} H_z$. Then we divide the FOU into $\varsigma + 1$ sub-FOUs. For $l = 1, 2, \ldots, \varsigma+1$. Expressions of lower and upper membership functions in the l-th sub-FOU are rewritten as $\underline{h}_{ijl}(\sigma(k)) = \sum_{z=1}^{q}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_{n_x}=1}^{2}\prod_{\tau=1}^{n_x} \upsilon_{\tau i_\tau zl}(\sigma_\tau(k))\,\underline{\vartheta}_{iji_1i_2\ldots i_nzl}$, $\overline{h}_{ijl}(\sigma(k)) = \sum_{z=1}^{q}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_{n_x}=1}^{2}\prod_{\tau=1}^{n_x} \upsilon_{\tau i_\tau zl}(\sigma_\tau(k))\,\overline{\vartheta}_{iji_1i_2\ldots i_nzl}$, where $0 \le \underline{h}_{ijl}(\sigma(k)) \le \overline{h}_{ijl}(\sigma(k)) \le 1$, $\underline{\vartheta}_{iji_1i_2\ldots i_nzl} \le \overline{\vartheta}_{iji_1i_2\ldots i_nzl}$. $\underline{\vartheta}_{iji_1i_2\ldots i_nzl}$ and $\overline{\vartheta}_{iji_1i_2\ldots i_nzl}$ are constant scalars to be determined; $0 \le \upsilon_{\tau i_s zl}(x_\tau(k)) \le 1$ with the property $\upsilon_{\tau 1zl}(\sigma_\tau(k)) + \upsilon_{\tau 2zl}(\sigma_\tau(k)) = 1$ for τ, $s = 1, 2, \ldots, n_x$; $l = 1, 2, \ldots, \varsigma + 1$; $i_\tau = 1, 2$; $x(k) \in H_z$; and $\upsilon_{\tau i_s zl}(\sigma_\tau(k)) = 0$ if otherwise. Thus, $\sum_{z=1}^{q}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_{n_x}=1}^{2}\prod_{\tau=1}^{n_x} \upsilon_{\tau i_\tau zl}(\sigma_\tau(k)) = 1$ for all l is achieved, which is used to analyze the system stability. For simplisity, the variable $\sigma(k)$ is omitted.

Then, system (8) can be rewritten as follows:

$$\begin{aligned}\tilde{x}(k+1) &= \sum_{i=1}^{r}\sum_{j=1}^{s} h_{ij}\left[\bar{A}_{1ij}\tilde{x}(k) + \bar{B}_{1ij}\tilde{u}(k)\right],\\ \bar{z}(k) &= \sum_{i=1}^{r}\sum_{j=1}^{s} h_{ij}\left[\bar{C}_{1ij}\tilde{x}(k) + \bar{D}_{1ij}\tilde{u}(k)\right],\end{aligned} \tag{9}$$

where

$$h_{ij} = m_i\omega_j = \sum_{l=1}^{\varsigma+1} \rho_{ijl}\left(\underline{\zeta}_{ijl}\underline{h}_{ijl} + \overline{\zeta}_{ijl}\overline{h}_{ijl}\right),$$

$$\rho_{ijl} = \begin{cases} 1, & h_{ijl} \in \text{sub-FOU } l,\\ 0, & \text{else,}\end{cases}\quad \sum_{i=1}^{r}\sum_{j=1}^{s} h_{ij} = 1,\ 0 \le \underline{\zeta}_{ijl} \le \overline{\zeta}_{ijl} \le 1,$$

where the functions $\underline{\zeta}_{ijl}$ and $\overline{\zeta}_{ijl}$ are not required to be known. $\underline{\zeta}_{ijl} + \overline{\zeta}_{ijl} = 1$ for i, j and l.

The purpose of the paper is to design a FDF on the basis of IT-2 framework such that the following conditions are satisfied simultaneously.

1) The residual system in (9) is asymptotically stable;
2) Under zero initial condition, with a given positive scalar γ, the inequality $\sqrt{\sum_{k=0}^{\infty} \bar{z}^T(k)\bar{z}(k)} \leq \gamma \left\| \sqrt{\sum_{k=0}^{\infty} \tilde{u}(k)^T \tilde{u}(k)} \right\|_2$ should be satisfied.

3. **Main Results.** In this section, for given FDF gain matrices A_{fj}, B_{fj}, C_{fj}, D_{fj} ($j = 1, 2, \ldots, s$), the sufficient criteria are given to ensure residual system (9) to be asymptotically stable and satisfy the predefined H_∞ performance.

Theorem 3.1. *Considering fuzzy system in (9), for a positive scalar γ, system in (9) is asymptotically stable and satisfies a given disturbance attenuation index, if there exist symmetric matrices $H_1 > 0$, $V > 0$, $\bar{W} > 0$, $\bar{W}_{ijl} > 0$, and matrices $\bar{A}_{fj}$, $\bar{B}_{fj}$, $\bar{C}_{fj}$, $\bar{D}_{fj}$, $\bar{M}$, where*

$$\bar{W}_{ijl} = \begin{bmatrix} \bar{W}_{1ijl} & \bar{W}_{2ijl} & \cdots & \bar{W}_{6ijl} \\ * & \bar{W}_{7ijl} & \cdots & \bar{W}_{11ijl} \\ * & * & \ddots & \vdots \\ * & * & * & \bar{W}_{21ijl} \end{bmatrix}, \quad \bar{M} = \begin{bmatrix} \bar{M}_1 & \bar{M}_2 & \cdots & \bar{M}_6 \\ * & \bar{M}_7 & \cdots & \bar{M}_{11} \\ * & * & \ddots & \vdots \\ * & * & * & \bar{M}_{21} \end{bmatrix},$$

satisfying the following conditions:

$$\begin{bmatrix} \hat{P} & \varpi\breve{\Pi}_{ij} \\ * & \tilde{\Phi}_{ijl} \end{bmatrix} < 0, \quad \begin{bmatrix} \hat{P} & \breve{\Pi}_{ij} \\ * & \tilde{\Omega} \end{bmatrix} < 0, \tag{10}$$

where

$$\breve{\Pi}_{ij} = \begin{bmatrix} \Delta_{1ij} & \bar{A}_{fj} & 0 & \Delta_{3ij} & \Delta_{4ij} & \Delta_{5ij} \\ \Delta_{2ij} & \bar{A}_{fj} & 0 & \Delta_{6ij} & \Delta_{7ij} & \Delta_{8ij} \\ 0 & 0 & VA_w & 0 & 0 & VB_w \\ \bar{D}_{fj}C_i & \bar{C}_{fj} & -C_w & \bar{D}_{fj}D_i & \bar{D}_{fj}F_{1i} & \bar{D}_{fj}F_{2i} - D_w \end{bmatrix},$$

$$Q = \begin{bmatrix} -H_1 & -\bar{W} & 0 \\ * & -\bar{W} & 0 \\ * & * & -V \end{bmatrix},$$

$$\tilde{\Phi}_{ijl} = \begin{bmatrix} \bar{\Phi}_{1ijl} & \bar{\Phi}_{2ijl} & \cdots & \bar{\Phi}_{6ijl} \\ * & \bar{\Phi}_{7ijl} & \cdots & \bar{\Phi}_{11ijl} \\ * & * & \ddots & \vdots \\ * & * & * & \bar{\Phi}_{21ijl} \end{bmatrix}, \quad \tilde{\Omega} = \begin{bmatrix} \bar{\Omega}_1 & \bar{\Omega}_2 & \cdots & \bar{\Omega}_6 \\ * & \bar{\Omega}_7 & \cdots & \bar{\Omega}_{11} \\ * & * & \ddots & \vdots \\ * & * & * & \bar{\Omega}_{21} \end{bmatrix},$$

$$\begin{aligned}
\hat{P} &= diag\{Q, -I\}, \ \Delta_{1ij} = H_1A_i + \bar{B}_{fj}C_i, \ \Delta_{2ij} = \bar{W}^TA_i + \bar{B}_{fj}C_i, \\
\Delta_{3ij} &= H_1B_i + \bar{B}_{fj}D_i, \ \Delta_{4ij} = H_1E_{1i} + \bar{B}_{fj}F_{1i}, \ \Delta_{5ij} = H_1E_{2i} + \bar{B}_{fj}F_{2i}, \\
\Delta_{6ij} &= \bar{W}^TB_i + \bar{B}_{fj}D_i, \ \Delta_{7ij} = \bar{W}^TE_{1i} + \bar{B}_{fj}F_{1i}, \ \Delta_{8ij} = \bar{W}^TE_{2i} + \bar{B}_{fj}F_{2i}, \\
\bar{\Phi}_{1ijl} &= -\underline{\vartheta}_{iji_1i_2\ldots i_nzl}H_1 - \left(\underline{\vartheta}_{iji_1i_2\ldots i_nzl} - \overline{\vartheta}_{iji_1i_2\ldots i_nzl}\right)\bar{W}_{1ijl} + \left(\underline{\vartheta}_{iji_1i_2\ldots i_nzl} - \frac{1}{rs}\right)\bar{M}_1, \\
\bar{\Phi}_{\iota_0ijl} &= -\underline{\vartheta}_{iji_1i_2\ldots i_nzl}\bar{W} - \left(\underline{\vartheta}_{iji_1i_2\ldots i_nzl} - \overline{\vartheta}_{iji_1i_2\ldots i_nzl}\right)\bar{W}_{\iota_0ijl} \\
&\quad + \left(\underline{\vartheta}_{iji_1i_2\ldots i_nzl} - \frac{1}{rs}\right)\bar{M}_{\iota_0}, \ \iota_0 = 2, 7, \\
\bar{\Phi}_{\iota_1ijl} &= -\left(\underline{\vartheta}_{iji_1i_2\ldots i_nzl} - \overline{\vartheta}_{iji_1i_2\ldots i_nzl}\right)\bar{W}_{\iota_1ijl} \\
&\quad + \left(\underline{\vartheta}_{iji_1i_2\ldots i_nzl} - \frac{1}{rs}\right)\bar{M}_{\iota_1}, \ \iota_1 = 8, \ldots, 11, \\
\bar{\Phi}_{12ijl} &= -\underline{\vartheta}_{iji_1i_2\ldots i_nzl}V - \left(\underline{\vartheta}_{iji_1i_2\ldots i_nzl} - \overline{\vartheta}_{iji_1i_2\ldots i_nzl}\right)\bar{W}_{12ijl} + \left(\underline{\vartheta}_{iji_1i_2\ldots i_nzl} - \frac{1}{rs}\right)\bar{M}_{12},
\end{aligned}$$

$$\begin{aligned}
\bar{\Phi}_{\iota_2 ijl} &= -\left(\underline{\vartheta}_{iji_1i_2\ldots i_nzl} - \overline{\vartheta}_{iji_1i_2\ldots i_nzl}\right)\bar{W}_{\iota_2 ijl} \\
&\quad + \left(\underline{\vartheta}_{iji_1i_2\ldots i_nzl} - \frac{1}{rs}\right)\bar{M}_{\iota_2},\ \iota_2 = 3,\ldots,6,13,14,15,17,18,20, \\
\bar{\Phi}_{\iota_3 ijl} &= -\underline{\vartheta}_{iji_1i_2\ldots i_nzl}\gamma^2 I - \left(\underline{\vartheta}_{iji_1i_2\ldots i_nzl} - \overline{\vartheta}_{iji_1i_2\ldots i_nzl}\right)\bar{W}_{\iota_3 ijl} \\
&\quad + \left(\underline{\vartheta}_{iji_1i_2\ldots i_nzl} - \frac{1}{rs}\right)\bar{M}_{\iota_3},\ \iota_3 = 16,19,21, \\
\bar{\Omega}_1 &= -H_1 - \bar{W}_{1ijl} + \bar{M}_1,\ \bar{\Omega}_2 = -\bar{W} - \bar{W}_{2ijl} + \bar{M}_2,\ \bar{\Omega}_7 = -\bar{W} - \bar{W}_{7ijl} + \bar{M}_7, \\
\bar{\Omega}_{\iota_4} &= -\bar{W}_{\iota_4 ijl} + \bar{M}_{\iota_4},\ \iota_4 = 3,\ldots,6,13,14,15,17,18,20, \\
\bar{\Omega}_{\iota_5} &= -\bar{W}_{\iota_5 ijl} + \bar{M}_{\iota_4},\ \iota_5 = 8,9,10,11,\ \bar{\Omega}_{12} = -V - \bar{W}_{12ijl} + \bar{M}_{12}, \\
\bar{\Omega}_{\iota_6} &= -\gamma^2 I - \bar{W}_{\iota_6 ijl} + \bar{M}_{\iota_5},\ \iota_6 = 16,19,21.
\end{aligned}$$

Furthermore, if the aforementioned conditions hold, the FDF gain matrices in the form of (5) can be computed by $\begin{bmatrix} A_{fj} & B_{fj} \\ C_{fj} & D_{fj} \end{bmatrix} = \begin{bmatrix} \bar{W}^{-1} & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \bar{A}_{fj} & \bar{B}_{fj} \\ \bar{C}_{fj} & \bar{D}_{fj} \end{bmatrix}$.

4. **Numerical Example.** Consider the tunnel diode circuit governed by the dynamic equation: $i_D(k) = 0.002v_D(k) + \partial v_D^3(k)$, where ∂ is an uncertain parameter and $\partial \in [0.01, 0.03]$.

Define $x_1(k) = v_C(k)$, $x_2(k) = i_L(k)$ and $\bar{f} = 0.002 + \partial v_D^2(k)$. Then, one can get

$$\begin{aligned}
Cx_1(k) &= -\bar{f}x_1(k) + x_2(k), \\
Lx_2(k) &= -x_1(k) - Rx_2(k) + w(k),
\end{aligned}$$

where $C = 20$ mF, $L = 1000$ mH and $R = 10\ \Omega$.

For demonstration, the external disturbance and fault are given as:

$$w(k) = \begin{cases} \text{rand}[0,\ 1], & 20 < k < 70, \\ 0, & \text{else}, \end{cases},\ f(k) = \begin{cases} 1, & 30 < k < 60, \\ 0, & \text{else}, \end{cases}$$

Referring to the modeling process in [18] and computing the LMIs (10) with $\gamma = 1.05$, the effectiveness of Theorem 3.1 can be validated.

Figure 1 and Figure 2 show the fault detection without the disturbance and fault detection with the disturbance, respectively. From these figures, we can see that the designed FDF can distinguish the fault signal from the disturbance.

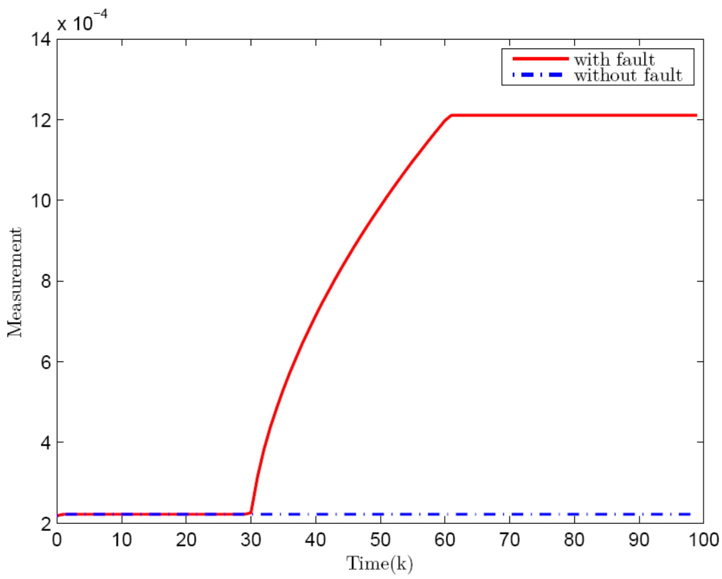

FIGURE 1. Fault detection without input $u(k)$ and disturbance $w(k)$

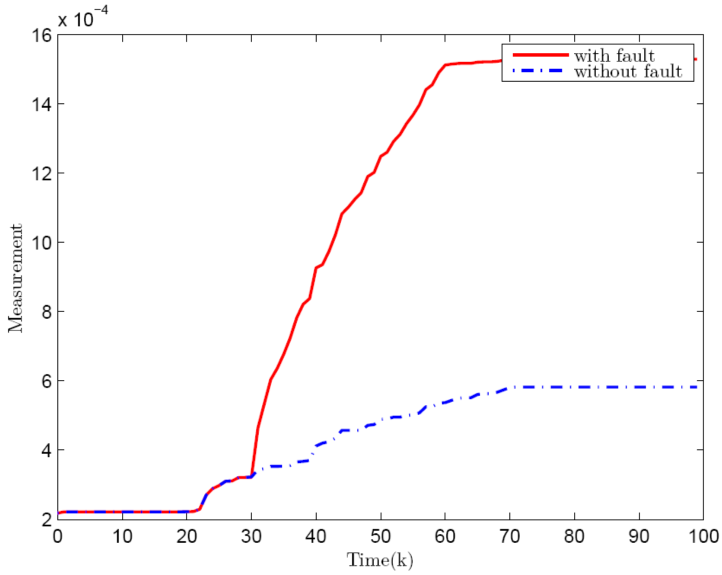

FIGURE 2. Fault detection with disturbance $w(k)$

5. **Conclusions.** In this paper, the problem of filter-based fault detection for a category of nonlinear systems subject to uncertainties has been studied. By using the IT2 T-S fuzzy model, the uncertain systems have been modeled and uncertainties have been captured and expressed through lower and upper membership functions. Employing independent premise variables, membership functions and number of fuzzy rules, the IT2 FDF designed in this paper increases the design flexibility. The footprint of uncertainty has been considered for more uncertain information and less conservative results. Based on these techniques, sufficient conditions have been derived to design the desired IT2 FDF and ensure the residual system is asymptotically stable and satisfies disturbance attenuation performance. Finally, simulation results have been provided to demonstrate the effectiveness of the methodology proposed in this paper. In future work, time delay will be considered to handle fault detection problem based on the IT2 T-S fuzzy model.

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