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STABILIZATION OF NETWORKED CONTROL SYSTEMS UNDER ACTUATOR SATURATION

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Abstract. *This paper presents a new stabilization method for networked control systems with actuator saturation. Time delay in networked control system with actuator saturation is considered. The stabilizing controller of the system is designed. The relationship between the time delay and the domain of attraction is also presented. Numerical result is provided to show the effectiveness of our method.*
Keywords: Networked control systems, Actuator saturation, Time delay, Stabilization

1. **Introduction.** With the development of communication and computer technology, Networked Control Systems (NCSs) develop rapidly in the last decade. And they offer many benefits and advantages over the traditional control systems [1, 2, 3]. However, with the introduction of networks into control systems, NCSs also bring issues and challenges to deal with, such as time delay, package losses, communication noise, bandwidth scheduling, and signal quantization [4, 5]. Time delays and package losses are the most important issues which can reduce the system performance. They have received a lot of attention both in continuous-time and discrete-time. There are two categorized models in the related works: one is the deterministic bound method, which places bounds on time delays and package losses; the other one is the stochastic method, with which time delay and package losses are molded as certain probability distributions [6]. The stabilization problem of NCSs with time delay and package losses is presented in many papers [1]. There is some room to improve the system performance in NCSs.

In control systems, the physical actuator is subject to saturation owing to its maximum and minimum limits. Therefore, the saturation nonlinearities are nonnegligible in NCSs. When we design a close-loop system, the actuator saturation needs to be considered; otherwise, the system may deteriorate, and even lose the stability. Methods for estimating the domain of attraction are studied in [8]. The analysis and design for delta operator system with actuator saturation are presented in [9]. In NCSs, the stabilization is studied with actuator saturation [10]. In this reference, the estimating of the attraction domain under a pre-designed saturated linear feedback is given. The performance is guaranteed when the time delay is within a sampling step.

In our paper, considering actuator saturation in NCSs with time delay is presented, we give the estimating of the attraction domain in NCSs under actuator saturation. And numerical simulation is also presented.

In Section 2, problem formulation is presented. In Section 3, our main result is given. The simulation result is given in Section 4, and conclusions are drawn in Section 5.

2. Problem Formulation.

Consider an NCS with actuator saturation described as follows:

$$\begin{cases} x(k+1) = Ax(k) + B\mathrm{sat}(u(k-\tau)), & k > 0 \\ x(s) = \varphi(s), & s \in [-\tau, 0] \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the plant state, $u(k-\tau) \in \mathbb{R}^m$ is the control input with constant time delay τ, and A and B are matrices appropriate dimensions. $\varphi(s)$ is the initial state in $[-\tau, 0]$. The function "sat" is the standard saturation function of appropriate dimension. Moreover, sat: $\mathbb{R}^m \to \mathbb{R}^m$, and $\mathrm{sat}(u) = [\mathrm{sat}(u_1), \mathrm{sat}(u_2), \cdots, \mathrm{sat}(u_m)]^T$, where $\mathrm{sat}(u_i) = \mathrm{sgn}(u_i)\min\{1, |u_i|\}$. τ is the constant input time-delay of control signal acting on the plant.

The networked controller takes the following form

$$u(k-\tau) = Fx(k-\tau) \tag{2}$$

By controller (2), system (1) can be rewritten as follows:

$$x(k+1) = Ax(k) + B\mathrm{sat}(Fx(k-\tau)) \tag{3}$$

where $F \in \mathbb{R}^{m\times n}$ is the feedback gain, denote the ith row of F as f_i and define

$$\mathcal{L}(F) := \{x \in \mathbb{R}^n : |f_i x| \le 1, i = 1, 2, \cdots, m\} \tag{4}$$

where $\mathcal{L}(F)$ is the linear region of saturation. That is, $\mathcal{L}(F)$ is the region where system (3) is linear in $x(k-\tau)$ if F is the feedback matrix.

Definition 2.1. *Given an initial state $x_0 = \varphi \in C^1[-d, 0]$, denote the state trajectory of system (3) that starts from x_0 as $x(k, x_0)$ at time step k, and the domain of attraction of the origin is given as*

$$\mathcal{S} := \left\{\varphi \in C^1[-\tau, 0] : \lim_{k\to\infty} x(k, x_0) = 0\right\} \tag{5}$$

Since it is not possible to obtain the domain of attraction of system (3) analytically, our objective of this paper is to obtain estimates of the domain of attraction. The main problems considered in this paper are summarized as follows.

Considering system (3)

1) Find an estimate of the domain of attraction which is as large as possible with respect to some given bounded and convex reference set $\mathcal{X}_R \subset \mathbb{R}^n$.

2) Design the feedback gain F in system (3) such that an estimate of the domain of attraction is as large as possible with respect to some given bounded and convex reference set $\mathcal{X}_R \subset \mathbb{R}^n$.

A set is said to be (contractively) invariant if all the trajectories starting from it will remain in it forever (and converge to zero as k approaches infinity). For the NCS (3), as it is time-invariant, that is, for time step k, no matter updating step or holding step, $x(k, x_0)$ must be inside this set.

Since it is not possible to obtain the domain of attraction of system (3) analytically, our objective of this paper is to design a state feedback controller (2) such that the closed-loop system is asymptotically stable. And we are also interested in obtaining an estimate of the domain of attraction $\mathcal{X}_r \subset \mathcal{S}$, where $\mathcal{X}_r := \{\varphi \in C^1[-d, 0] : \max|\varphi| \le \gamma\}$ with scalars $\gamma > 0$, that will be maximized in what follows. Let $\mathcal{D}$ be the set of $m \times m$ diagonal matrices whose diagonal elements are either 1 or 0. There are 2^m elements in $\mathcal{D}$. Suppose that each element of $\mathcal{D}$ is labeled as D_i, $i = 1, 2, \cdots, 2^m$. Then

$$\mathcal{D} = \{D_i : i \in [1, 2^m]\}$$

Denote $D_i^- = I - D_i$. It is obtained that D_i^- is also an element of $\mathcal{D}$ if $D_i \in \mathcal{D}$. Given two vectors $u, v \in \mathbb{R}^m$, we have that

$$\left\{D_i u + D_i^- v : i \in [1, 2^m]\right\}$$

is the set of vectors formed by choosing some elements from u and the rest from v. Given two matrices $F, H \in \mathbb{R}^{m\times n}$, it is obtained that

$$\left\{D_i F + D_i^- H : i \in [1, 2^m]\right\}$$

is the set of matrices formed by choosing some rows from F and the rest from H.

Before ending this section, we recall the following lemma which will be used in sequel to drive our main results in this paper.

Lemma 2.1. *Let $u, v \in \mathbb{R}^m$, and*

$$u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}, \quad v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix}.$$

Suppose that $|v_i| \le 1$ for all $i \in [1, m]$, then

$$\mathrm{sat}(u) \in co\left\{D_i u + D_i^- v : i \in [1, 2^m]\right\}$$

where $co\{\cdot\}$ is the convex hull of a group of sets. For a group of sets $u^1, u^2, \cdots, u^\ell$, the convex hull of these sets is defined as

$$co\left\{u^i : i \in [1, \ell]\right\} := \left\{\sum_{i=1}^{\ell} \alpha_i u^i : \sum_{i=1}^{\ell} \alpha_i = 1, \alpha_i \ge 0\right\}.$$

3. Main Results.

Theorem 3.1. *For the closed-loop NCS with actuator saturation described as (3), given scalars $\hat{\delta}$, τ $(0 < \tau)$, if there exist positive definite matrix $P \in \mathbb{R}^{n\times n}$, $H \in \mathbb{R}^{m\times n}$, $Q > 0$, such that the following inequalities are feasible:*

$$\begin{bmatrix} -P + Q & 0 & A^T \\ * & -Q & [B(D_i + D_i^- H)]^T \\ * & * & -P \end{bmatrix} < 0, \ i = 1, 2, \ldots, 2^m \tag{6}$$

and $\mathcal{E}(P, 1) \subset \mathcal{L}(H)$, i.e.,

$$\begin{bmatrix} 1 & h_i \\ h_i^T & P \end{bmatrix} \ge 0, \ i = 1, 2, \ldots, m \tag{7}$$

then system (3) is locally asymptotically stable. An estimate of the domain of attraction is given by

$$\mathcal{T}(\gamma) \le 1 \tag{8}$$

with

$$\mathcal{T}(\gamma) = \gamma^2 \left[\lambda_{\max}(P) + \tau \lambda_{\max}(Q)\right]$$

Proof: To prove Theorem 3.1, we first show that under the conditions of this theorem, it holds that

$$x_0 \in \mathcal{X}_r \Rightarrow x(k) \in \mathcal{E}(P,1), \tag{9}$$

and the Lyapunov function is given as

$$V(k) = V_1(k) + V_2(k) = x^T(k)Px(k) + \sum_{d=k-\tau}^{k-1} x^T(d)Qx(d)$$

where $P > 0$, $Q > 0$. Then, $\Delta V(k)$ yields

$$\Delta V(k) = \Delta V_1(k) + \Delta V_2(k) \tag{10}$$

and the next we get that:

$$\begin{aligned}\Delta V_1(k) &= V_1(k+1) - V_1(k)\\ &= [Ax(k) + B\text{sat}(Fx(k-\tau))]^T P[Ax(k) + B\text{sat}(Fx(k-\tau))] - x^T(k)Px(k)\end{aligned}$$

and

$$\begin{aligned}\Delta V_2(k) &= V_2(k+1) - V_2(k)\\ &= \sum_{d=k-\tau+1}^{k} x^T(d)Qx(d) - \sum_{d=k-\tau}^{k-1} x^T(d)Qx(d)\\ &= x^T(k)Qx(k) - x^T(k-\tau)Qx(k-\tau)\end{aligned}$$

So according to (10), $\Delta V(k)$ yields

$$\begin{aligned}&\Delta V(k)\\ &= \Delta V_1(k) + \Delta V_2(k)\\ &= [Ax(k) + B\text{sat}(Fx(k-\tau))]^T P[Ax(k) + B\text{sat}(Fx(k-\tau))] - x^T(k)Px(k)\\ &\quad + x^T(k)Qx(k) - x^T(k-\tau)Qx(k-\tau)\\ &= \begin{bmatrix} x(k) \\ x(k-\tau) \end{bmatrix}^T \begin{bmatrix} A^T \\ [B(D_iF + D_i^- H)]^T \end{bmatrix} P \begin{bmatrix} A^T \\ [B(D_iF + D_i^- H)]^T \end{bmatrix}^T \begin{bmatrix} x(k) \\ x(k-\tau) \end{bmatrix}\\ &\quad + \begin{bmatrix} x(k) \\ x(k-\tau) \end{bmatrix}^T \begin{bmatrix} -P+Q & 0 \\ 0 & -Q \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-\tau) \end{bmatrix}\\ &\le \begin{bmatrix} x(k) \\ x(k-\tau) \end{bmatrix}^T \left(\begin{bmatrix} A^T \\ [B(D_iF + D_i^- H)]^T \end{bmatrix} P \begin{bmatrix} A^T \\ [B(D_iF + D_i^- H)]^T \end{bmatrix}^T \right.\\ &\quad \left. + \begin{bmatrix} -P+Q & 0 \\ 0 & -Q \end{bmatrix} \right) \begin{bmatrix} x(k) \\ x(k-\tau) \end{bmatrix}\\ &= \eta^T \Upsilon_i \eta\end{aligned} \tag{11}$$

where $\eta = \begin{bmatrix} x(k) \\ x(k-\tau) \end{bmatrix}$ and

$$\Upsilon_i = \begin{bmatrix} A^T \\ [B(D_iF + D_i^- H)]^T \end{bmatrix} P \begin{bmatrix} A^T \\ [B(D_iF + D_i^- H)]^T \end{bmatrix}^T + \begin{bmatrix} -P+Q & 0 \\ 0 & -Q \end{bmatrix} < 0 \tag{12}$$

for all $i \in \mathbb{I}[1,m]$. If the system satisfies above (12) such as $\Upsilon_i < 0$, and then it will be asymptotically stable.

By the Schur complement, (12) is equivalent to

$$\begin{bmatrix} -P+Q & 0 & A^T \\ * & -Q & [B(D_iF + D_i^- H)]^T \\ * & * & -P \end{bmatrix} < 0 \tag{13}$$

Therefore, we can conclude that when inequalities (13) are feasible, we will have $\Delta V(k) < 0$ for any $x \neq 0$. And accordingly,

$$x^T(k)Px(k) \leq V(x(k)) < V(x_0) \leq \gamma^2 [\lambda_{\max}(P) + \tau\lambda_{\max}(Q)] = \mathcal{T}(\gamma) \tag{14}$$

Hence if $\mathcal{T}(\gamma) \leq 1$, we obtain that $x^T(k)Px(k) \leq 1$ and all the trajectories of $x(k)$ that start from $\mathcal{T}(\gamma) \leq 1$ will remain within $\mathcal{E}(P, 1)$, and thereby the control constraints $\|u(k)\| \leq 1$. To prove $x_0 \in \mathcal{X}_r \Rightarrow \lim_{t_k\to\infty} \|x(t_k)\| = 0$, from $\lim_{k\to\infty} V(k) = 0$, we have $\lim_{k\to\infty} V_1(k) = 0$, which implies $\lim_{k\to\infty} \|x(k)\|^2 = 0$.

Summarizing the above analysis, we can conclude that $\lim_{k\to\infty} \|x(k, x_0)\|^2 = 0$. Hence, the proof is completed. From Theorem 3.1, it is seen that an optimization procedure can be proposed to maximize the initial conditions, i.e., to obtain a maximized estimate of domain of attraction. As the method commonly adopted in the literature, we also select appropriate γ in (12), and an approximating optimization problem can be obtained as

$$\min \varsigma \quad \text{s.t.} \begin{cases} \text{Inequalities (13)} \\ P - \omega_1 I \leq 0 \\ Q - \omega_2 I \leq 0 \end{cases} \tag{15}$$

where $\varsigma = \omega_1 + \tau\omega_2$, and ω_i, $i = 1, 2$, are the introduced variables for weighting in the optimization procedure. Then, a maximized estimate of the domain of the attraction can be obtained by $\gamma_{\max} = 1/\sqrt{\Lambda}$, where

$$\Lambda = \lambda_{\max}(P) + \tau\lambda_{\max}(Q)$$

4. **Numerical Example.** We use an example of [4] to illustrate our results. The system is described by (3) with

$$A = \begin{bmatrix} 0.8 & 0.002 \\ 0 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}, \; F = \begin{bmatrix} 0.005 \\ -0.1304 \end{bmatrix}^T$$

For the above NCS with actuator saturation, when $\tau = 5$, according to (15), we can get that

$$P = \begin{bmatrix} 1.7429 & 0.0009 \\ 0.0009 & 1.9187 \end{bmatrix}, \; Q = \begin{bmatrix} -0.0508 & -0.0001 \\ -0.0001 & 0.0524 \end{bmatrix}, \; \omega_2 = -0.1619, \; \varsigma = 0.1219$$

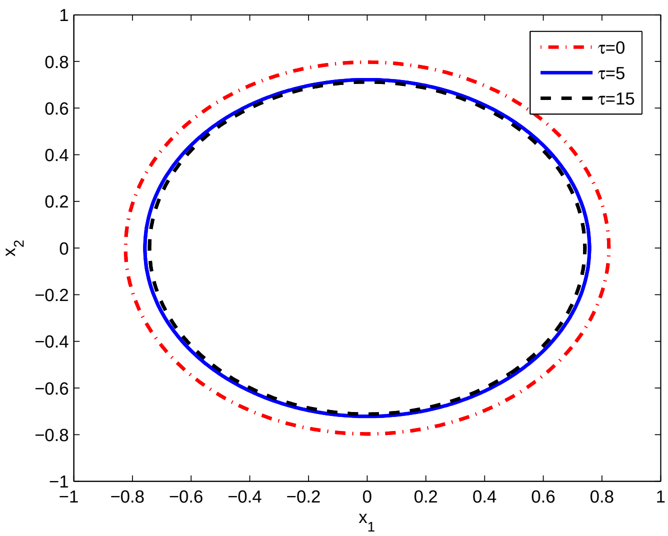

FIGURE 1. The domain of attraction in NCS system

When we change the constant time delay τ, the domain of attraction is changing as Figure 1.

5. **Conclusion.** This paper has presented a new approach to study the NCSs with actuator saturation. The stabilization condition has been given for the NCSs with time delay under actuator saturation. The relationship between the time delay and the domain of attraction is presented. Numerical result is included to demonstrate the effectiveness of our proposed method.

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