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A MINIMAX ABSOLUTE AND RELATIVE DISPARITY APPROACH TO OBTAINING THE OWA OPERATOR WEIGHTS

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Abstract. *Determining the ordered weighted averaging (OWA) operator weights is important in decision making applications. Several approaches have been proposed in literature to obtain the associated weights. This paper provides a minimax relative disparity model (RDM) and a minimax relative and absolute disparity model (RADM) for obtaining OWA operator weights. The proposed model generates the OWA operator weights by minimizing the maximum absolute and relative difference between any two adjacent weights under a given level of ones. A numerical example is examined using OWA operator weights to show its applications.*
Keywords: OWA operator weights, Absolute and relative difference, Minimax disparity approach

1. **Introduction.** The ordered weighted averaging (OWA) operator defined by Yager [1] provides a general class of parametric aggregation operators that includes the min, max and average, and has shown to be useful for modeling many different kinds of aggregation problems. The OWA operator has been used in a wide range of applications such as neural networks [2,3], fuzzy logic controllers [4,5], decision making [6,7] and data mining [8,9]. To apply the OWA operator, a very crucial issue is the determination of the weights of the operator.

A number of techniques have been suggested for generating the weights. O'Hagan [10] first determined the OWA operator weights and suggested a maximum entropy method. Fullér and Majlender [11] showed that the maximum entropy model could be transformed into a polynomial equation that can be solved analytically. Fullér and Majlender [12] also suggested a minimum variance method to obtain the minimal variability OWA operator weights. Liu [13,14] presented the minimal variability OWA operator generating method with the equidifferent OWA operator. Wang and Parkan [15] proposed a linear programming model with a minimax disparity approach to obtain the OWA operator. Majlender [16] extended the maximum entropy method to Rényi entropy and proposed a maximum Rényi entropy OWA operator.

It is observed that most of the above mentioned methods produce regular weight distributions, which vary either in the form of exponential (geometric progression) or in the form of arithmetical progression. For example, the maximum entropy weights vary in the form of exponential, while the minimal variability weights and the minimax disparity weights vary in the form of equidistance. Although regular weight distributions make sense, there is no reason to believe that OWA operator weights can vary regularly. So, Wang et al. [17] proposed chi-square methods to determine the OWA operator weights. In this paper we will develop two new improved chi-square models to determine the OWA operator weights that can vary in a rational yet general rather than regular way.

2. The OWA Operator and Its Weight Generation Methods. An OWA operator of dimension n is a mapping F: $\Re \rightarrow \Re$ that has an associated weight vector $W = (w_1, \ldots, w_n)$ of having the properties $\sum_{i=1}^{n} w_i = 1$, $0 \leq w_i \leq 1$, and such that

$$F(x_1, \ldots, x_n) = \sum_{i=1}^{n} w_i b_i \tag{1}$$

where b_i is the ith largest of the collection of the aggregated objects $\{x_1, \ldots, x_n\}$. In the latter, we will denote the expression as $F_W(x)$. The degree of "orness" associated with this operator is defined as

$$\text{orness}(W) = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i = \alpha \tag{2}$$

For vector $x = (x_1, \ldots, x_n)$ and weights $W = (w_1, \ldots, w_n)$, if $\forall i, j$, $x_i \leq x_j$, implying $w_i \leq (\geq) w_j$, then we call W is monotone. If $w_i \leq w_j$ we call the OWA operator is increasing; on the other hand if $w_i \geq w_j$, we call it is decreasing.

To determine OWA operator weights, Fullér and Majlender [12] proposed a minimum variance method (MVM), which minimizes the variance of OWA operator weights under a given level of orness. Their method requires the solution of the following mathematical programming model:

$$\begin{aligned} &\text{Min } D^2(W) = \tfrac{1}{n} \sum_{i=1}^{n} \left(w_i - \tfrac{1}{n}\right)^2 \\ &\text{s.t. orness}(W) = \sum_{i=1}^{n} \tfrac{n-i}{n-1} \cdot w_i = \alpha, \quad 0 \leq \alpha \leq 1 \\ &\sum_{i=1}^{n} w_i = 1, \quad 0 \leq w_i \leq 1, \quad i = 1, \ldots, n \end{aligned} \tag{3}$$

Wang and Parkan [15] proposed a minimax disparity approach (MDA) for generating OWA operator weights. They presented the following model for minimizing the maximum disparity:

$$\begin{aligned} &\text{Min } \left\{ \underset{i=1,\ldots,n-1}{\text{Max}} |w_i - w_{i+1}| \right\} \\ &\text{s.t. orness}(W) = \sum_{i=1}^{n} \tfrac{n-i}{n-1} \cdot w_i = \alpha, \quad 0 \leq \alpha \leq 1 \\ &\sum_{i=1}^{n} w_i = 1, \quad 0 \leq w_i \leq 1, \quad i = 1, \ldots, n \end{aligned} \tag{4}$$

There is a common characteristic for above mentioned models. That is, the OWA operator weights should be made as equally important as possible. So, based on such a characteristic, Wang et al. [17] proposed the chi-square model (CSM) for obtaining OWA operator weights.

$$\begin{aligned} &\text{Min } J(W) = \sum_{i=1}^{n-1} \left(\tfrac{w_i}{w_{i+1}} + \tfrac{w_{i+1}}{w_i} - 2 \right) \\ &\text{s.t. orness}(W) = \sum_{i=1}^{n} \tfrac{n-i}{n-1} \cdot w_i = \alpha, \quad 0 \leq \alpha \leq 1 \\ &\sum_{i=1}^{n} w_i = 1, \quad 0 \leq w_i \leq 1, \quad i = 1, \ldots, n \end{aligned} \tag{5}$$

The following section provides two new disparity OWA weight determination models as an extension to the above models.

3. **The Minimax Absolute and Relative Disparity Method for Obtaining OWA Operator Weights.** As Wang et al. [17] mentioned to determine the OWA operator weights, when the orness constraint is taken into consideration, models (3)-(5) could be understood as making all the weights as close to each other as possible under a given degree of orness. In this paper, two proportional difference models between two adjacent weights are made as small as possible. We propose the following model for minimizing the maximum disparity:

$$\begin{aligned}
&\text{Min } J_1(W) = \min\left\{\max_{i=1,\ldots,n-1}\left|\tfrac{w_i}{w_{i+1}}-1\right|\right\}\\
&\text{s.t. orness}(W) = \sum_{i=1}^{n}\tfrac{n-i}{n-1}\cdot w_i = \alpha, \quad 0 \le \alpha \le 1\\
&\sum_{i=1}^{n} w_i = 1, \quad 0 \le w_i \le 1, \quad i = 1,\ldots,n
\end{aligned} \tag{6}$$

and

$$\begin{aligned}
&\text{Min } J_2(W) = \min\left\{\max_{i=1,\ldots,n-1}\left|\tfrac{w_i}{w_{i+1}}-1\right| + \max_{i=1,\ldots,n-1}|w_i - w_{i+1}|\right\}\\
&\text{s.t. orness}(W) = \sum_{i=1}^{n}\tfrac{n-i}{n-1}\cdot w_i = \alpha, \quad 0 \le \alpha \le 1\\
&\sum_{i=1}^{n} w_i = 1, \quad 0 \le w_i \le 1, \quad i = 1,\ldots,n
\end{aligned} \tag{7}$$

The two new models both produce as equally important OWA operator weights as possible for a given orness degree. For convenience, we refer to model (6) as the relative difference model (RDM) for determining the OWA operator weights and model (7) as relative and absolute difference model (RADM). The RDM (6) and RADM (7) can be reconfigured as the following models:

$$\begin{aligned}
&\text{Min } \rho\\
&\text{s.t. } 1-\rho \le \tfrac{w_j}{w_{j+1}} \le 1+\rho, \quad j = 1,\ldots,n-1\\
&\text{orness}(W) = \sum_{i=1}^{n}\tfrac{n-i}{n-1}\cdot w_i = \alpha, \quad 0 \le \alpha \le 1\\
&\sum_{i=1}^{n} w_i = 1, \quad 0 \le w_i \le 1, \quad i = 1,\ldots,n
\end{aligned} \tag{8}$$

and

$$\begin{aligned}
&\text{Min } \{\rho+\xi\}\\
&\text{s.t. } 1-\rho \le \tfrac{w_j}{w_{j+1}} \le 1+\rho, \quad j = 1,\ldots,n-1\\
&1-\xi \le w_j - w_{j+1} \le 1+\xi, \quad j = 1,\ldots,n-1\\
&\text{orness}(W) = \sum_{i=1}^{n}\tfrac{n-i}{n-1}\cdot w_i = \alpha, \quad 0 \le \alpha \le 1\\
&\sum_{i=1}^{n} w_i = 1, \quad 0 \le w_i \le 1, \quad i = 1,\ldots,n
\end{aligned} \tag{9}$$

Models (8) and (9) are both nonlinear and can be solved by using LINGO or MATLAB software package. Note that models (8) and (9) are not applicable in $\alpha = 0$ and $\alpha = 1$, which are two extreme degrees of orness. We will use $\alpha = 0.0001$ and $\alpha = 0.9999$ to represent the two extreme cases in the next section.

4. **Numerical Example.** In this section, we examine the two new models with a numerical example and verify their applicability in determining OWA operator weights. Suppose $n = 5$ and it is needed to determine the OWA operator weights satisfying different degrees of orness: $\alpha = 0, 0.1, \ldots, 0.9, 1$, which are provided by the decision maker.

Tables 1 and 2 show the OWA operator weights determined by models (6) and (7), respectively, which are also depicted in Figures 1 and 2. The models are solved by using MATLAB software package.

TABLE 1. The OWA operator weights determined by the RDM

W	orness$(W) = \alpha$										
	0.9999	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0001
w_1	0.9998	0.7376	0.5368	0.3582	0.2714	0.20	0.1106	0.0340	0.0002	0.0012	0.3870×10^{-4}
w_2	0.4087×10^{-4}	0.1629	0.2153	0.2789	0.2437	0.20	0.1653	0.1272	0.0886	0.0356	0.4087×10^{-4}
w_3	0.4087×10^{-4}	0.0627	0.1591	0.2018	0.2089	0.20	0.2089	0.2018	0.1591	0.0627	0.4087×10^{-4}
w_4	0.4087×10^{-4}	0.0356	0.0886	0.1272	0.1653	0.20	0.2437	0.2789	0.2153	0.1629	0.4087×10^{-4}
w_5	0.3870×10^{-4}	0.0012	0.0002	0.0340	0.1106	0.20	0.2714	0.3582	0.5368	0.7376	0.9998

TABLE 2. The OWA operator weights determined by the RADM

W	orness$(W) = \alpha$										
	0.9999	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0001
w_1	0.9996	0.7330	0.4667	0.36	0.28	0.20	0.12	0.04	0.0000	0.0003	0.0000×10^{-4}
w_2	0.3999×10^{-3}	0.1685	0.3167	0.28	0.24	0.20	0.16	0.12	0.0500	0.0340	0.0000×10^{-4}
w_3	0.0000×10^{-4}	0.0642	0.1667	0.20	0.20	0.20	0.20	0.20	0.1667	0.0642	0.0000×10^{-4}
w_4	0.0000×10^{-4}	0.0340	0.0500	0.12	0.16	0.20	0.24	0.28	0.3167	0.1685	0.3999×10^{-3}
w_5	0.0000×10^{-4}	0.0003	0.0000	0.04	0.12	0.20	0.28	0.36	0.4667	0.7330	0.9996

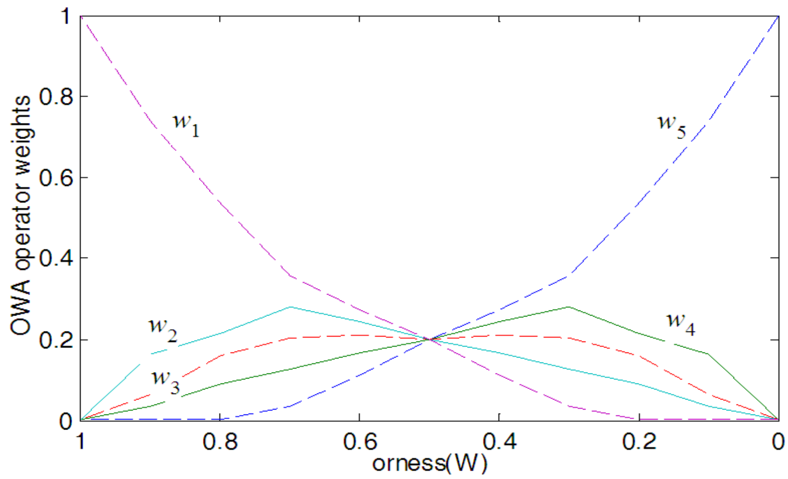

FIGURE 1. Variation of the relative difference OWA operator weights

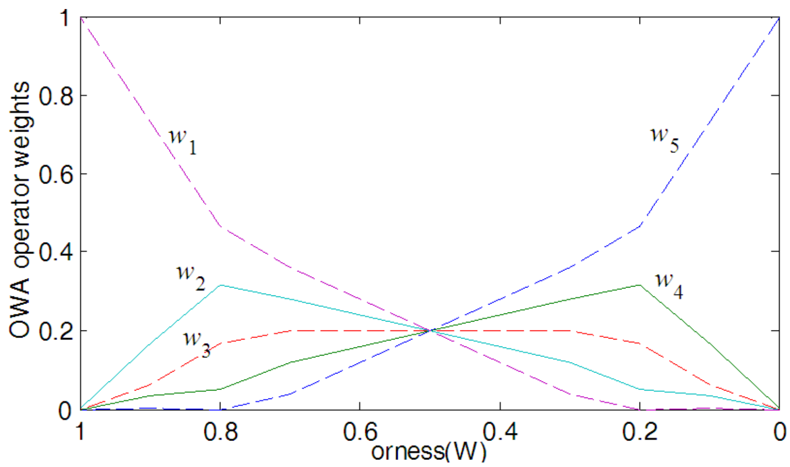

FIGURE 2. Variation of the relative and absolute difference OWA operator weights

To find the slight differences among the five methods, we consider the distributions of the OWA operator weights in Table 3. Take $\alpha = 0.8$ for example. Table 3 shows the distribution of the OWA operator weights determined by the five methods under the given

Table 3. The distribution of the OWA operator weights under $\alpha = 0.8$

Method	Difference of two adjacent weights				Ratio of two adjacent weights			
	$w_1 - w_2$	$w_2 - w_3$	$w_3 - w_4$	$w_4 - w_5$	w_1/w_2	w_2/w_3	w_3/w_4	w_4/w_5
RDM	0.3215	0.0562	0.0705	0.0884	2.4932	1.3532	1.7957	–
RADM	0.15	0.15	0.1167	0.05	1.4736	1.8998	3.334	–
CSM	0.3235	0.1280	0.0458	0.0158	2.3848	2.2121	1.7659	1.3591
MDA	0.14	0.14	0.14	0.14	1.4375	1.7778	4.5	–
MVM	0.14	0.14	0.14	0.14	1.4375	1.7778	4.5	–

degree of orness, from which it can be seen very clearly that the weights determined by the two new methods vary neither in the form of exponential nor in the form of arithmetical progression, but in a very general way. This is what we are expecting because there is no evidence to support the OWA operator weights to follow a very regular distribution [17]. However, if $\alpha = 0.6$ or 0.7, Table 2 shows the weight distributions of RADM in the form of equidistance, which is because of the absolute deviation dominant.

5. **Conclusions.** In this paper, we have proposed two new models for determining OWA operator weights. The two new models prove to be practical and effective and can produce the OWA operator weights that are very close to those obtained by existing methods. However, the weights are determined by considering the absolute deviation and relative deviation of two adjacent weights, and thus the two new models do not follow a regular distribution and therefore make more sense.

In future research, we will further investigate the related properties of the absolute deviation and relative deviation models through an analytic solution. A detailed application study of different models may also be a topic of interest.

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