

DOUBLE IIR FILTER FOR ADAPTIVE FEED-FORWARD CONTROL SYSTEMS WITH STOCHASTIC DISTURBANCE EXCITATION

HYUN CHEOL CHO

School of Electrical and Electronic Engineering
Ulsan College
57 Daehak-ro, Nam-gu, Ulsan 44610, Korea
hcjo@uc.ac.kr

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ABSTRACT. *An infinite impulse response (IIR) filter is useful in modeling and control of dynamic systems for scientific and engineering applications. This paper presents a novel double-IIR (D-IIR) model integrating two IIR filters in series for adaptive feed-forward control (AFC). We first derive a parameter estimation algorithm for the D-IIR filter model using statistical optimization method. In addition, an adaptive D-IIR filter is devised for improving control performance of AFC systems subject to an uncertain stochastic disturbance. Finally, a numerical example is provided to test superiority and reliability of the proposed D-IIR filter based AFC system by comparing a conventional AFC method.*

Keywords: Double IIR filter, Adaptive feed-forward control, Stochastic disturbance

1. **Introduction.** A typical IIR filter is generally applied for modeling and control of stochastic dynamic systems in various engineering disciplines [1]. In recent years, various researches about IIR filter have been addressed in many applications. Bruton et al. constructed a 2-D IIR filter for broadband beamforming and designed its analog filter circuits for practical realization in [2]. Algreer et al. applied a recursive IIR filter technique to power conversion systems in which a parameter estimation algorithm is derived for online system identification of it in [3]. Kozacky and Ogunfunmi designed a cascade IIR-FIR filter model for adaptive noise control systems including narrowband periodic and uncorrelated wideband components in [4]. In addition, there are several investigations about particular design methodologies of IIR filters according to given engineering problems. Wang et al. proposed a new IIR filter design using evolutionary computation for optimization of systems with multi-objective functions in [5] and Botts et al. utilized Bayesian theory in IIR filter design especially for its parameter estimation in [6]. Mitiche et al. developed a 2-D IIR filter model along with a model reduction method using Prony theorem in [7], Wang et al. applied two-stage ensemble memetic algorithm to parameter estimation of IIR filters for its more effective convergence in [8], and Quelhas et al. employed a pole-zero mapping method for reducing complexity of optimal parameter estimation for IIR filter model in [9].

We present a novel D-IIR filter in this paper. The proposed D-IIR filter model is simply constructed by integrating two typical IIR filters in series and applied to AFC [10] of dynamic systems with unknown stochastic disturbance. We define a statistical objective function with respect to stochastic deviation between the output signals of plant and D-IIR filter. Then a parameter estimation algorithm is analytically derived based on a defined objective function and adjustment rules for its parameter vectors are expressed using statistical optimization method. Moreover, stability analysis for a D-IIR model is studied from stability theory about linear discrete-time system. Finally, we propose an adaptive D-IIR filter model in which a part of its parameters (zeros in characteristic equation)

is modified via online learning algorithm for improving control performance. Numerical simulation is carried out to demonstrate superiority and reliability of the proposed D-IIR filter based AFC system against stochastic disturbance excitation.

This paper is organized as follows. Section 2 presents the proposed D-IIR filter model for AFC systems and Section 3 derives a parameter estimation approach for the D-IIR filter systems. We develop adaptive D-IIR filter based AFC methodology in Section 4 and carry out numerical simulations to demonstrate reliability and superiority of the proposed AFC control system in Section 5. Lastly, conclusions and future works are respectively provided in Section 6.

2. DIIR Filter Model for AFC Systems. Generally, an objective of AFC is generating an active control input to decrease control error yielded in plants due to external disturbance excitation usually in active noise control and vibration control applications. In this paper, we deal with a generic AFC system including a stochastic disturbance with unknown statistics. Block diagram of a typical AFC system model with discrete-time index k is shown in Figure 1. Here, $x \in R$ is the scalar control input of the AFC system, $d \in R$ is a scalar external disturbance, $y \in R$ is a scalar system output, and $e = x - y$ is the tracking error of the plant and AFC system. The control objective of this AFC system is to make the output approximate zero in the steady state in the presence of an unknown stochastic disturbance. In other words, a control input is needed to cancel the plant output due to the disturbance excitation.

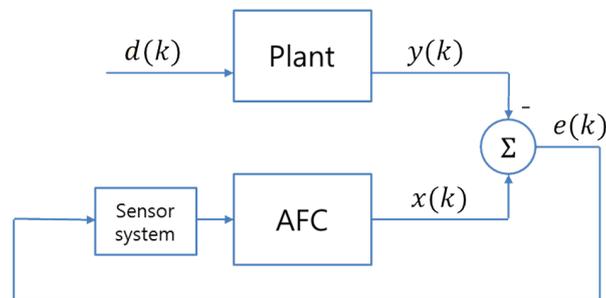


FIGURE 1. A typical AFC system model

We propose a D-IIR filter model for constructing AFC system to realize particular control objectives. Its framework is depicted in Figure 2 in which two generic IIR filters are simply connected in series. Here, the output variables γ and x are expressed mathematically as

$$\gamma(k) = \mathbf{a}^T \mathbf{\Gamma}_1(k) = \mathbf{b}^T \mathbf{\Delta}(k) \quad (1)$$

$$x(k) = \mathbf{\alpha}^T \mathbf{X}(k) + \mathbf{\beta}^T \mathbf{\Gamma}_2(k) \quad (2)$$

where $\mathbf{a} \in R^{n_1}$, $\mathbf{b} \in R^{m_1}$, $\mathbf{\alpha} \in R^{n_2}$, and $\mathbf{\beta} \in R^{m_2}$ are filter parameter vectors, and the matrixes $\mathbf{\Delta} \in R^{m_1}$ and $\mathbf{\Gamma}_1 \in R^{n_1}$ contain current and past input signals for e , and past output signals for γ , and similarly the matrixes $\mathbf{\Gamma}_2 \in R^{m_2}$ and $\mathbf{X} \in R^{n_2}$ include current and past input signals for γ , and past output signals for x respectively.

3. Parameter Estimation of D-IIR Filter Model. The filter parameter vectors in (1) and (2) should be optimally selected by an optimization technique to achieve the desired control performance of AFC system. This task is carried out to derive adjustment rules for them numerically. We first define an objective function composed of an error scalar e in Figure 1. Since disturbance scalar d is considered as a random variable, the

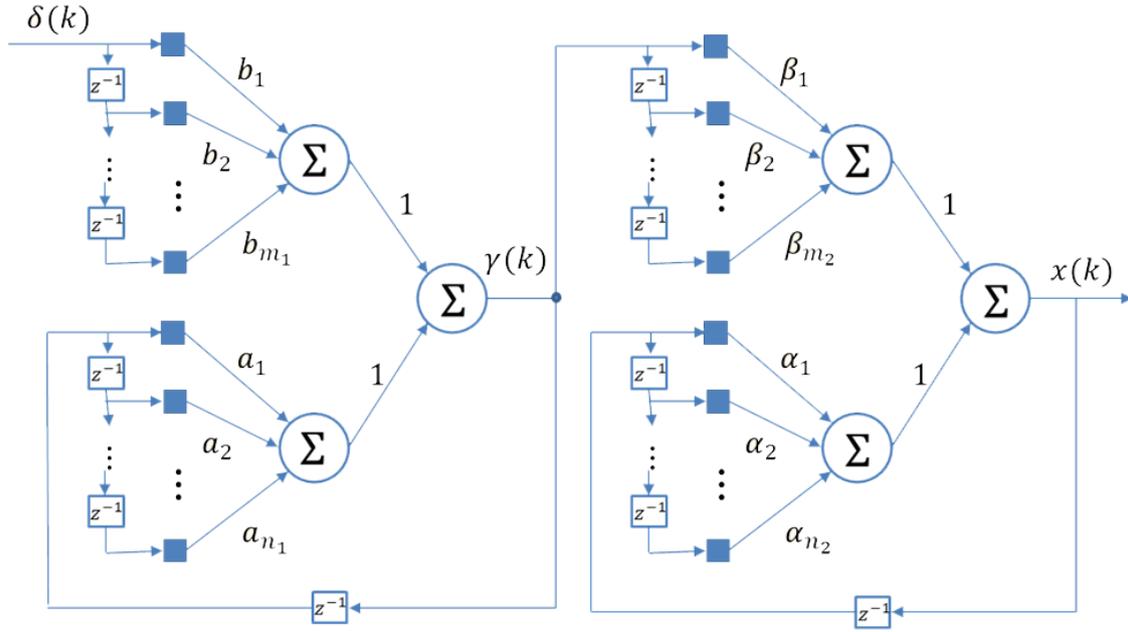


FIGURE 2. A proposed D-IIR filter model

plant output variable y becomes also stochastic. We employ the statistical expectation E for defining an objective function as

$$J = \frac{1}{2}E[e^2] = \frac{1}{2}E[(x - y)^2] = \frac{1}{2}\{E[x^2] - 2E[x]E[y] + E[y^2]\} \quad (3)$$

Note that variables x and y are statistically independent in (3). By substituting (2) to $E[x]$ and $E[x^2]$ in (3) respectively, we have

$$E[x] = E[\alpha^T \mathbf{X} + \beta^T \Gamma_2] = \alpha^T E[\mathbf{X}] + \beta^T E[\Gamma_2] \quad (4)$$

and

$$E[x^2] = E[(\alpha^T \mathbf{X} + \beta^T \Gamma_2)^2] = \alpha^T E[\mathbf{X} \mathbf{X}^T] \alpha + 2\alpha^T E[\mathbf{X}] \beta^T E[\Gamma_2] + \beta^T E[\Gamma_2 \Gamma_2^T] \beta \quad (5)$$

We substitute the last terms of (4) and (5) to (3) and finally obtain an expectation of an objective function as

$$J = \frac{1}{2} \{ \alpha^T \mathbf{C}_{\mathbf{X}\mathbf{X}} \alpha + 2\alpha^T \mathbf{m}_{\mathbf{X}} \beta^T \mathbf{m}_{\Gamma_2} + \beta^T \mathbf{C}_{\Gamma_2 \Gamma_2} \beta - 2\alpha^T \mathbf{m}_{\mathbf{X}} m_y - 2\beta^T \mathbf{m}_{\Gamma_2} m_y + m_{y^2} \} \quad (6)$$

with $\mathbf{C}_{\mathbf{X}\mathbf{X}} = E[\mathbf{X}(k) \mathbf{X}(k)^T]$, $\mathbf{m}_{\mathbf{X}} = E[\mathbf{X}(k)]$, $\mathbf{m}_{\Gamma_2} = E[\Gamma_2(k)]$, $\mathbf{C}_{\Gamma_2 \Gamma_2} = E[\Gamma_2(k) \Gamma_2(k)^T]$, and $m_y = E[y(k)]$, $m_{y^2} = E[y^2(k)]$. Adjustment rules for the filter parameter vectors α , β , \mathbf{a} , and \mathbf{b} in (1) and (2) are derived for minimizing a defined objective function in (6) using optimization method. First, for the parameter vector α , we carry out the partial differentiation against an objective function in (6) with respect to α and then let the result be equal to zero as

$$\frac{\partial J}{\partial \alpha} = \mathbf{M}_{\mathbf{X}\mathbf{X}} \alpha + \mathbf{m}_{\mathbf{X}} (\beta^T \mathbf{m}_{\Gamma_2}) - \mathbf{m}_{\mathbf{X}} m_y = 0 \quad (7)$$

From the last term we obtain an adjustment rule of parameter $\alpha(k)$ as

$$\alpha(k) = \mathbf{C}_{\mathbf{X}\mathbf{X}}^{-1} \mathbf{m}_{\mathbf{X}} (m_y - \beta^T \mathbf{m}_{\Gamma_2}) \quad (8)$$

Second, in the same way, for parameter β , we have

$$\frac{\partial J}{\partial \beta} = \alpha^T \mathbf{m}_{\mathbf{X}} \mathbf{m}_{\Gamma_2} + \mathbf{C}_{\Gamma_2 \Gamma_2} \beta - \mathbf{m}_{\Gamma_2} m_y = 0 \quad (9)$$

And from (9) an adjustment rule of parameter $\beta(k)$ is given by

$$\beta(k) = \mathbf{C}_{\Gamma_2\Gamma_2}^{-1} \mathbf{m}_{\Gamma_2} (m_y - \alpha^T \mathbf{m}_X) \quad (10)$$

Next, we derive an adjustment rule for the parameter vector \mathbf{a} . Similarly, we have a partial differential equation against an objective function in (6) with respect to \mathbf{a} as

$$\frac{\partial J}{\partial \mathbf{a}} = \frac{1}{2} \left\{ \frac{\partial(\alpha^T \mathbf{m}_X \beta^T \mathbf{m}_{\Gamma_2} + \frac{1}{2} \beta^T \mathbf{C}_{\Gamma_2\Gamma_2} \beta - \beta^T \mathbf{m}_{\Gamma_2} m_y)}{\partial \mathbf{a}} \right\} \quad (11)$$

where the two terms $\beta^T \mathbf{m}_{\Gamma_2}$ and $\beta^T \mathbf{C}_{\Gamma_2\Gamma_2} \beta$ are expanded respectively as

$$\beta^T \mathbf{m}_{\Gamma_2} = \beta_1 \mathbf{a}^T E[\Gamma_1] + \beta_1 \mathbf{b}^T E[\Delta] + \sum_{i=2}^{m_2} \beta_i E[\gamma(k-i+1)] \quad (12)$$

and

$$\begin{aligned} \beta^T \mathbf{C}_{\Gamma_2\Gamma_2} \beta &= \beta_1^2 E[\gamma^2] + \sum_{i=2}^{m_2} \beta_i^2 E[\gamma^2(k-i+1)] \\ &= \beta_1^2 \{ \mathbf{a}^T \mathbf{C}_{\Gamma_1\Gamma_1} \mathbf{a} + 2(\mathbf{a}^T \mathbf{m}_{\Gamma_1})(\mathbf{b}^T \mathbf{m}_{\Delta}) + \mathbf{b}^T \mathbf{C}_{\Delta\Delta} \mathbf{b} \} \\ &\quad + \sum_{i=2}^{m_2} \beta_i^2(k) E[\gamma^2(k-i+1)] \end{aligned} \quad (13)$$

with $\mathbf{C}_{\Gamma_1\Gamma_1} = E[\Gamma_1\Gamma_1^T]$, $\mathbf{m}_{\Gamma_1} = E[\Gamma_1]$, $\mathbf{C}_{\Delta\Delta} = E[\Delta\Delta^T]$, and $\mathbf{m}_{\Delta} = E[\Delta]$. By substituting (12) and (13) to (11), we then calculate the partial differential equation as

$$\frac{\partial J}{\partial \mathbf{a}} = \alpha^T \mathbf{m}_X (\beta_1 \mathbf{m}_{\Gamma_1}) + \beta_1^2 \left(\frac{1}{2} \mathbf{C}_{\Gamma_1\Gamma_1} \mathbf{a} + \mathbf{m}_{\Gamma_1} (\mathbf{b}^T \mathbf{m}_{\Delta}) \right) - \beta_1 \mathbf{m}_{\Gamma_1} m_y = 0 \quad (14)$$

From this result, an adjustment rule of parameter $\mathbf{a}(k)$ is obtained as

$$\mathbf{a}(k) = 2\beta_1^{-1} \mathbf{C}_{\Gamma_1\Gamma_1}^{-1} \mathbf{m}_{\Gamma_1} (m_y - \beta_1 \mathbf{b}^T \mathbf{m}_{\Delta} - \alpha^T \mathbf{m}_X) \quad (15)$$

Finally, for the parameter \mathbf{b} , in the same way, we differentiate an objective function with respect to parameter \mathbf{b} as

$$\frac{\partial J}{\partial \mathbf{b}} = \alpha^T \mathbf{m}_X \beta_1 \mathbf{m}_{\Delta} + \beta_1^2 \mathbf{a}^T \mathbf{m}_{\Gamma_1} \mathbf{m}_{\Delta} + \frac{1}{2} \beta_1^2 \mathbf{C}_{\Delta\Delta} \mathbf{b} - \beta_1 \mathbf{m}_{\Delta} m_y = 0 \quad (16)$$

We simply obtain an adjustment rule of the parameter $\mathbf{b}(k)$ from (16) as

$$\mathbf{b}(k) = 2\beta_1^{-1} \mathbf{C}_{\Delta\Delta}^{-1} \mathbf{m}_{\Delta} (m_y - \beta_1 \mathbf{a}^T \mathbf{m}_{\Gamma_1} - \alpha^T \mathbf{m}_X) \quad (17)$$

4. Adaptive D-IIR Filter. The parameter vectors β and \mathbf{b} rarely influence absolute stability of the D-IIR filter since they involve zeros of their transfer function and may be located anywhere in the z -plane [1]. However, the parameters give an impact on dynamic nature of the D-IIR filter such as the transient response, i.e., overshoot, rising time, settling time, and so on. We devise online parameter estimation for β and \mathbf{b} to realize adaptive D-IIR filter based AFC mechanism. Their elements are recursively adjusted by a parameter estimation rule for improving control performance due to unknown external disturbance in practice. By applying the steepest descent optimization, the adjustment rules for the parameter vectors β and \mathbf{b} are initially given respectively by

$$\beta(k+1) = \beta(k) - \eta \frac{\partial J}{\partial \beta} \quad (18)$$

$$\mathbf{b}(k+1) = \mathbf{b}(k) - \eta \frac{\partial J}{\partial \mathbf{b}} \quad (19)$$

where η is a learning rate. The partial differential equations in (18) and (19) are expanded by applying the chain rule as

$$\frac{\partial J}{\partial \beta} = \frac{\partial J}{\partial e} \frac{\partial e}{\partial x} \frac{\partial x}{\partial \beta} \quad (20)$$

$$\frac{\partial J}{\partial \mathbf{b}} = \frac{\partial J}{\partial e} \frac{\partial e}{\partial x} \frac{\partial x}{\partial \gamma} \frac{\partial \gamma}{\partial \mathbf{b}} \quad (21)$$

We solve the partial differential equations in (20) and (21) as $\partial e/\partial x = 1$, $\partial x/\partial \boldsymbol{\beta} = \boldsymbol{\Gamma}_2$, $\partial x/\partial \gamma = \beta_1$, $\partial \gamma/\partial \mathbf{b} = \boldsymbol{\Delta}$ and applying $J = \frac{1}{k} \sum_{i=1}^k e^2(i)$ calculates $\partial J/\partial e = e^2(k)/2k$. We apply these results to (20) and (21) and then have $\partial J/\partial \boldsymbol{\beta} = e^2(k)\boldsymbol{\Gamma}_2/2k$, $\partial J/\partial \mathbf{b} = e^2(k)\beta_1\boldsymbol{\Delta}/2k$. Finally by substituting the resulting equations to (18) and (19) we have the adjustment rules for the parameter vectors $\boldsymbol{\beta}$ and \mathbf{b} as

$$\boldsymbol{\beta}(k+1) = \boldsymbol{\beta}(k) - \tilde{\eta}(k)e^2(k)\boldsymbol{\Gamma}_2 \tag{22}$$

$$\mathbf{b}(k+1) = \mathbf{b}(k) - \tilde{\eta}(k)e^2(k)\beta_1\boldsymbol{\Delta} \tag{23}$$

where $\tilde{\eta}(k) = \eta/2k$ is a time-varying learning rate.

5. Numerical Simulation. We carry out simulation experiment to test the proposed D-IIR based AFC system. A mathematical model for a plant with a state vector $\boldsymbol{\zeta} = [\zeta_1 \ \zeta_2]^T$ is referred in [10], given by

$$\begin{cases} \begin{bmatrix} \zeta_1(k+1) \\ \zeta_2(k+1) \end{bmatrix} = \begin{bmatrix} \sigma_1(k) & 1 \\ -0.16 & -1 \end{bmatrix} \begin{bmatrix} \zeta_1(k) \\ \zeta_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ \sigma_2(k) \end{bmatrix} d(k) \\ y(k) = [\sigma_3(k) \quad 0] \begin{bmatrix} \zeta_1(k) \\ \zeta_2(k) \end{bmatrix} \end{cases} \tag{24}$$

where parameters σ_1 , σ_2 , and σ_3 are random variables whose numbers are uniformly distributed within $[0.5, 1]$ and an initial condition for a state vector $\boldsymbol{\zeta}$ is given as $\zeta(0) = [0 \ 0.1]^T$. A control objective in this simulation is to make the plant output y close to zero under the disturbance excitation. A random disturbance d in (24) is selected as a Gaussian mixture in which two zero-mean Gaussian signals with each different variance (0.5 and 0.2 respectively) are linearly mixed. Figure 3 shows time-histories of disturbance excitation in this simulation experiment. We build the proposed D-IIR based AFC framework against such disturbance nature. For parameter estimation of the D-IIR filter, initial conditions for its parameter vectors, \mathbf{a} , \mathbf{b} , $\boldsymbol{\alpha}$, and $\boldsymbol{\beta}$ are uniformly selected within $[-0.1, 0.1]$. As well, we let $n_1 = 26$, $m_1 = 18$, $n_2 = 13$, and $m_2 = 9$ in (1) and (2) from which the best control performance is obtained through the iterative learning routines. We test the constructed D-IIR filter based AFC system with the disturbance and also apply a conventional AFC approach adopted from [10] under the same simulation topology for a comparative study. Figure 4 shows time-histories of the system errors for the two AFC systems against the stochastic disturbance in Figure 3. From these results, we recognize that the D-IIR filter based AFC is obviously superior to the addressed AFC method in [10], in that maximum error level in the case of the proposed AFC is considerably reduced and its average error is dramatically smaller over a given control period. We calculate the norm of the system error defined by $e_{norm} = \sqrt{e^T e}$ for the two control methods, and have $e_{norm} = 6.3$ for the conventional AFC and $e_{norm} = 0.9$ for the proposed AFC. Apparently, this result proves that the D-IIR filter based AFC has over 85% improvement in the control point of view. These simulation results obviously conclude the proposed D-IIR filter effectively works as an AFC mechanism and the constructed AFC system is reliable to improve control performance against stochastic systems with unknown random disturbance.

6. Conclusions. This paper presents a novel D-IIR filter model linearly composed of two generic IIR filters in series. This D-IIR filter is applied to AFC framework against stochastic control systems with unknown random disturbance whose parameter is arbitrarily changed in practical applications. We derive the parameter estimation of it based on stochastic learning mechanism and additionally propose an adaptive AFC methodology in which a part of the D-IIR filter parameters is updated for reducing control error due to random disturbance in practice. We prove superiority of the proposed D-IIR filter based AFC system and its reliability in the control performance point of view through

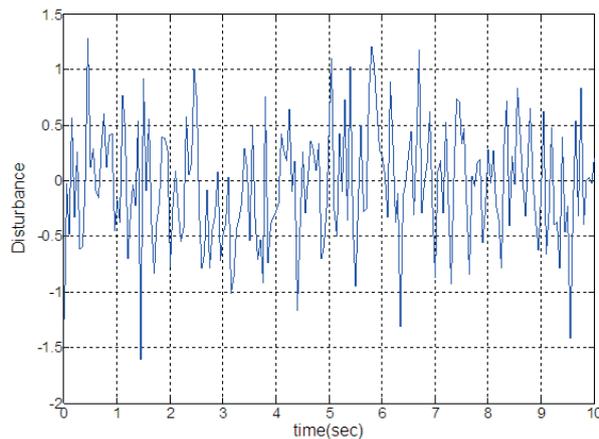


FIGURE 3. Random disturbance excitation

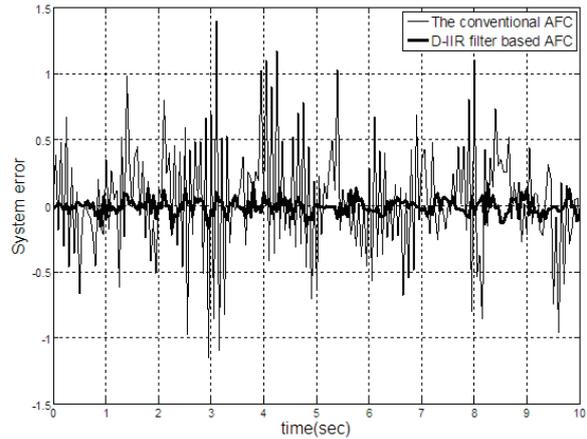


FIGURE 4. The two AFC system errors

simulation experiment. In particular, we observe that the average error is effectively mitigated and the maximum error level is also significantly reduced for a given control period in the case of the proposed AFC system. Future work will include a theoretical investigation about stability analysis of the D-IIR filter model and a real-time implementation for demonstrating its practical applicability in industry.

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