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INTERVAL-VALUED FUZZY INFERENCE BASED ON FUZZY HIERARCHICAL VARIABLE WEIGHTS

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Abstract. *In interval-valued fuzzy inference, according to actual demand of fuzzy decision making, we present a new approach to calculate fuzzy hierarchical variable-weighted matching degrees between input facts and antecedent components of rules. Furthermore, an interval-valued fuzzy inference based on fuzzy hierarchical variable-weighted matching degrees is proposed, which not only realizes the incentive strategy to some antecedent components in fuzzy inference, but satisfies balance requirements of decision makers. An example is given to illustrate the effectiveness and feasibility of the algorithm.*
Keywords: Fuzzy inference, Interval-valued fuzzy sets, Incentive variable weight, Hierarchical variable-weighted matching degree

1. **Introduction.** When we use interval-valued fuzzy sets to express state values of input facts and antecedent components in fuzzy inference, the fuzziness and uncertainty in practice can be reflected very well. So the research interest of many scholars in interval-valued fuzzy inference theory grows as the days passed. In 1980s, based on CRI (Compositional Rule of Inference [1]) and Boole paradigm theory, Turksen and Yao proposed an interval-valued approximate reasoning mode and then put forward the interval-valued fuzzy inference theory [2]. Furthermore, Turksen and Zhong gave an interval-valued fuzzy inference based on similarity [3]. This method is simple and easy to understand, and has been widely used. In addition, some other scholars have also studied interval-valued fuzzy inference from various viewpoints [4-12]. In particular, Li and Xu proposed a fuzzy matching function different from the former in [13], and gave a fuzzy reasoning approach based on interval-valued fuzzy production rules. They used a fuzzy matching function to calculate matching degrees between input facts and antecedent components. Then the fuzzy rules with maximum matching degree are selected and the consequents of the rules are regarded as ultimate decision quantities. However, in the reasoning process of the method, the different influence degrees of antecedent components were not considered when the matching degrees were calculated; in other words, the weights of antecedent components should be assigned, and moreover the balance degrees of matching should also be taken into account. So in [14], we gave an interval-valued fuzzy reasoning approach based on fuzzy weighted-balanced matching degrees to revise the above defects. However, another problem has still not been solved. In a practical decision problem, when state values of all antecedent components are considered, some antecedent components need to be motivated; in other words, their weights should be enlarged when their state values increase. On the other hand, when we consider these antecedent components separately, their state values need to be balanced, that is, their variable weights should be reduced when their state values increase. Therefore, based on hierarchical variable weights, we present an approach to calculate fuzzy hierarchical variable-weighted matching degrees between inputs and antecedent components. This approach not only realizes the incentive strategy

to some antecedent components in fuzzy inference, but meets balance requirements of decision makers. The proposed algorithm makes the matching between input facts and antecedents of rules conform greatly to actual requirements, and furthermore a new idea of interval-valued fuzzy inference is given in this paper.

The paper is organized as follows. In Section 2, we propose such a new algorithm for fuzzy hierarchical variable-weighted matching degrees between inputs and antecedent components. In Section 3, we set out the fuzzy reasoning procedure based on hierarchical variable-weighted matching degrees, and furthermore give an example to analyze the practicability of the above method in Section 4, followed by conclusion in Section 5.

2. Fuzzy Hierarchical Variable-Weighted Matching Degrees.

2.1. Weighted interval-valued fuzzy production rules. In [14], we presented weighted interval-valued fuzzy production rules in order to simulate human reasoning and decision thinking better. The general form is as follows:

$$IF\ \left(P_1, t_1, v_1^0\right)\ and\ \left(P_2, t_2, v_2^0\right)\ and\ \cdots\ and\ \left(P_n, t_n, v_n^0\right)\ THEN\ Q\ \ CF\ w^0,$$

where P_1, $P_2, \cdots, P_n$ express n predications, and every predication includes a fuzzy quantifier and a fuzzy predicate. In view of the two aspects included in every predication, corresponding membership degree values $t_1, t_2, \cdots, t_n$ are assigned to them in order to show the determinacy degrees of these predications, and $t_1, t_2, \cdots, t_n \in I[0,1]$. The membership degree of a predication is calculated by the product of such two membership degree values of a fuzzy quantifier and a fuzzy predicate. $v_1^0, v_2^0, \ldots, v_n^0$ represent the influence degrees of these predictions respectively in reasoning process, in which $v_i^0 \in [0,1]$, $i = 1, 2, \cdots, n$, Q denotes a rule consequent, and w^0 is the value of determinacy factor CF that shows the degree of confidence of a rule and $w^0 \in I[0,1]$.

2.2. Hierarchical variable-weighted matching degrees between inputs and antecedent components. We gave a definition of distance between two interval-valued fuzzy sets on finite universes [14]. In this section, we use the distance to propose a new algorithm for hierarchical variable-weighted matching degrees between inputs and antecedent components of rules.

Definition 2.1. *Suppose $X = \{x_1, x_2, \cdots, x_m\}$ is a finite universe and $\bar{A}, \bar{B} \in \bar{\mathcal{F}}(X)$, i.e., $\bar{A} = [a_1, b_1] \times [a_2, b_2] \times \cdots \times [a_m, b_m] \in [0,1]^m$, $\bar{B} = [c_1, e_1] \times [c_2, e_2] \times \cdots \times [c_m, e_m] \in [0,1]^m$, then the distance between $\bar{A}$ and $\bar{B}$ is defined by*

$$d\left(\bar{A}, \bar{B}\right) = \frac{1}{m} \sum_{k=1}^{m} \left(\frac{|t_A^k - t_B^k|}{1 + h_A^k + h_B^k} \right),$$

where $t_A^k = \dfrac{a_k + b_k}{2}$, $t_B^k = \dfrac{c_k + e_k}{2}$, $h_A^k = b_k - a_k$, $h_B^k = e_k - c_k$, $k = 1, 2, \cdots, m$.

Particularly, the distance of two interval numbers is defined by

$$d([a_1, b_1], [a_2, b_2]) = \frac{|t_1 - t_2|}{1 + h_1 + h_2},$$

where $t_1 = \dfrac{a_1 + b_1}{2}$, $t_2 = \dfrac{a_2 + b_2}{2}$, $h_1 = b_1 - a_1$, $h_2 = b_2 - a_2$.

When we use the weighted interval-valued fuzzy production rules for fuzzy inference, it is necessary to consider balance requirements of the matching degrees between inputs and antecedent components and motivate certain matching degrees according to decision makers' preference.

For convenience, motivated input components are denoted by $\mathbb{R}_1 \triangleq (R_1, R_2, \cdots, R_r)$, and the other input components are denoted by $\mathbb{R}_2 \triangleq (R_{r+1}, R_{r+2}, \cdots, R_n)$. Accordingly, $\mathbb{P}_1 \triangleq (P_1, P_2, \cdots, P_r)$ and $\mathbb{P}_2 \triangleq (P_{r+1}, P_{r+2}, \cdots, P_n)$.

Firstly, the fuzzy weighted balance matching degree $m_i\left(\mathbb{R}_i, \mathbb{P}_i\right)$ is calculated to measure the matching degree between $\mathbb{R}_i$ and $\mathbb{P}_i$ $(i = 1, 2)$. The details are as follows:

$$m_1 \triangleq m_1\left(\mathbb{R}_1, \mathbb{P}_1\right) = \alpha\left(\mathbb{R}_1, \mathbb{P}_1\right) \cdot \sum_{j=1}^{r} v_j^0 \cdot \beta\left(R_j, P_j\right),$$

$$m_2 \triangleq m_2\left(\mathbb{R}_2, \mathbb{P}_2\right) = \alpha\left(\mathbb{R}_2, \mathbb{P}_2\right) \cdot \sum_{k=r+1}^{n} v_k^0 \cdot \beta\left(R_k, P_k\right),$$

where R_j and P_j are respectively the jth components of matching fact R and the antecedent P $(j = 1, 2, \cdots, r)$, R_k and P_k are respectively the kth components of matching fact R and the antecedent P $(k = r + 1, r + 2, \cdots, n)$, and R means

$$(R_1, s_1, v_1^0) \text{ } and \text{ } (R_2, s_2, v_2^0) \text{ } and \text{ } \cdots \text{ } and \text{ } (R_n, s_n, v_n^0),$$

where $R_1, R_2, \cdots, R_n$ express n predications and $s_1, s_2, \cdots, s_n$ are membership degree values of these predications.

Secondly, we denote $\beta(R_j, P_j) = 1 - d(R_j, P_j)$, and calculate the balance degree $\alpha(\mathbb{R}_1, \mathbb{P}_1)$ of the matching degrees between input facts and antecedent components. The formula for calculation is as follows:

$$\alpha\left(\mathbb{R}_1, \mathbb{P}_1\right) = \frac{1}{1 + \sum\limits_{j=1}^{r} \left[\beta\left(R_j, P_j\right) - \frac{1}{r} \sum\limits_{i=1}^{r} \beta(R_i, P_i)\right]^2},$$

similarly,

$$\beta\left(R_k, P_k\right) = 1 - d\left(R_k, P_k\right),$$

$$\alpha\left(\mathbb{R}_2, \mathbb{P}_2\right) = \frac{1}{1 + \sum\limits_{k=r+1}^{n} \left[\beta(R_k, P_k) - \frac{1}{n-r} \sum\limits_{q=r+1}^{n} \beta(R_q, P_q)\right]^2}.$$

Thirdly, we select an incentive variable weight function in [15] in the following:

$$\gamma_i(m_1, m_2) = \begin{cases} e^{l(m_i - \bar{m})}, & m_1 > m_2, \\ 1, & \text{otherwise}, \end{cases}$$

where $\bar{m}$ is the arithmetic mean of m_1 and m_2, $l \geq 1$, and $i = 1, 2$. It is necessary to point out that the larger the value of l, the greater the incentive degree to m_1.

Finally, we can get the hierarchical variable-weighted matching degree M about every rule by calculating weighted average of γ_1 and γ_2.

3. **Fuzzy Inference Based on Hierarchical Variable-Weighted Matching Degrees.** The following steps should be carried out when the above algorithm is applied to a practical knowledge system:

(1) Some empirical knowledge is given by domain experts according to experience or heuristic knowledge;

(2) Empirical knowledge is fuzzified, and then a rule-based knowledge base is generated. In addition, the constant weight of every predication is assigned and a threshold λ for activating rules is determined according to actual problems;

(3) On the basis of above section, the hierarchical variable-weighted matching degrees of a given input fact and antecedents of rules are calculated. If some matching degrees are greater than or equal to the previously given threshold λ, then corresponding rules are activated and carried out, in which the determinacy degree of every rule consequent is determined by the product of matching degree and degree of reliability. If some matching degrees are smaller than λ, then the system does not carry out corresponding rules;

(4) If many rules are activated in the rule-based knowledge base, then we can get many determinacy degree values of rule consequents. Considering they are interval numbers, we

use the interval-valued sequencing approach in [16] to find the maximum interval value or quasi-maximum interval value, and consequently make a decision.

4. **Instance Analysis.** Suppose there is something wrong with a machine. Firstly, it is necessary to give some notations. All presumable phenomena are denoted by the set $P = \{P_1, P_2, \cdots, P_6\}$. The causes of the machine fault are shown as $Q = \{Q_1, Q_2, \cdots, Q_6\}$. In addition, we suppose that the values of balanced matching degrees of input facts and P_1, P_2 need to be motivated.

When the machine breaks down, the interval-valued data of determinacy degrees of the phenomena are given in the following:

$$R = \{(R_1, [0.80, 0.90], v_1^0), (R_2, [0.00, 0.00], v_2^0), (R_3, [0.60, 0.70], v_3^0), \\ (R_4, [0.00, 0.00], v_4^0), (R_5, [0.68, 0.76], v_5^0), (R_6, [0.00, 0.00], v_6^0)\}.$$

The constant weights of all predications are determined by domain experts, that is,

$$V = \{v_1^0, v_2^0, \cdots, v_6^0\} = \{0.26, 0.23, 0.04, 0.20, 0.15, 0.12\},$$

where $\sum_{i=1}^{6} v_i^0 = 1$.

Take $\lambda = 0.5$. There are six rules in the rule-based knowledge base as follows:

*Rule*1 *IF* $(P_1, [0.85, 0.95], v_1^0)$ *and* $(P_2, [0.25, 0.35], v_2^0)$ *and* $(P_3, [0.00, 0.00], v_3^0)$ *and* $(P_4, [0.00, 0.00], v_4^0)$ *and* $(P_5, [0.00, 0.00], v_5^0)$ *and* $(P_6, [0.00, 0.00], v_6^0)$ *THEN Q CF* $[0.90, 1.00]$;

*Rule*2 *IF* $(P_1, [0.00, 0.00], v_1^0)$ *and* $(P_2, [0.00, 0.00], v_2^0)$ *and* $(P_3, [0.94, 0.98], v_3^0)$ *and* $(P_4, [0.00, 0.00], v_4^0)$ *and* $(P_5, [0.65, 0.75], v_5^0)$ *and* $(P_6, [0.00, 0.00], v_6^0)$ *THEN Q CF* $[0.96, 1.00]$;

*Rule*3 *IF* $(P_1, [0.00, 0.00], v_1^0)$ *and* $(P_2, [0.94, 0.98], v_2^0)$ *and* $(P_3, [0.00, 0.00], v_3^0)$ *and* $(P_4, [0.10, 0.20], v_4^0)$ *and* $(P_5, [0.00, 0.00], v_5^0)$ *and* $(P_6, [0.00, 0.00], v_6^0)$ *THEN Q CF* $[0.94, 0.98]$;

*Rule*4 *IF* $(P_1, [0.00, 0.00], v_1^0)$ *and* $(P_2, [0.00, 0.00], v_2^0)$ *and* $(P_3, [0.00, 0.00], v_3^0)$ *and* $(P_4, [1.00, 1.00], v_4^0)$ *and* $(P_5, [0.00, 0.00], v_5^0)$ *and* $(P_6, [0.26, 0.28], v_6^0)$ *THEN Q CF* $[0.98, 1.00]$;

*Rule*5 *IF* $(P_1, [0.00, 0.00], v_1^0)$ *and* $(P_2, [0.00, 0.00], v_2^0)$ *and* $(P_3, [0.00, 0.00], v_3^0)$ *and* $(P_4, [0.80, 0.90], v_4^0)$ *and* $(P_5, [0.35, 0.45], v_5^0)$ *and* $(P_6, [0.15, 0.25], v_6^0)$ *THEN Q CF* $[0.90, 0.96]$;

*Rule*6 *IF* $(P_1, [0.05, 0.15], v_1^0)$ *and* $(P_2, [0.00, 0.00], v_2^0)$ *and* $(P_3, [0.65, 0.75], v_3^0)$ *and* $(P_4, [0.00, 0.00], v_4^0)$ *and* $(P_5, [0.00, 0.00], v_5^0)$ *and* $(P_6, [0.90, 1.00], v_6^0)$ *THEN Q CF* $[0.96, 1.00]$.

The six fuzzy hierarchical variable-weighted matching degrees of R and the antecedent components are computed, and then the results are as follows:

$$M_1 = 0.67 > \lambda; \quad M_2 = 0.43 < \lambda; \quad M_3 = 0.22 < \lambda; \\ M_4 = 0.37 < \lambda; \quad M_5 = 0.41 < \lambda; \quad M_6 = 0.54 > \lambda.$$

Obviously, Rule 1 and Rule 6 should be activated and carried out. The determinacy degree values of two rule consequents are

$$Q_1 = 0.67 \times [0.90, 1.00] \approx [0.60, 0.67]; \\ Q_6 = 0.54 \times [0.96, 1.00] \approx [0.52, 0.54].$$

According to the order relations for interval numbers in [16], $Q_1 \gg Q_6$.

So we can conclude that the machine fault is most likely to be caused by Q_1, and the value of determinacy degree is $[0.60, 0.67]$. We compare this result with the one in [14]. By using the algorithm in [14], we obtain the final conclusion that the machine fault is most likely to be caused by Q_2. However, the balanced matching degree of R_1 and P_1 in Rule 2 is too small, and the matching degree should be motivated because of the importance of P_1 in fuzzy decision making. According to the above improved algorithm, the balanced matching degrees of input facts and P_1, P_2 are motivated and the constant weights of the first two predications are increased, which make the new reasoning result more credible.

5. **Conclusion.** In consideration of different importance of antecedent components of rules in reasoning process and incentive demand of some state values, based on weighted interval-valued fuzzy production rules and the definition of distance between two interval-valued fuzzy sets, we proposed a new approach to calculate fuzzy hierarchical variable-weighted matching degrees between input facts and antecedent components. We motivated some important antecedent components in fuzzy inference, and balanced the state values of these antecedent components so that the conclusion obtained by the given algorithm can conform greatly to actual requirements. An example illustrates the availability of the algorithm. In view of practicality of hierarchical interval-valued fuzzy inference, to discuss the selection problem of incentive variable weight functions and interval-valued sequencing approaches and their impact on the related calculation will be our next work in future.

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