ICIC Express Letters ICIC International ©2016 ISSN 1881-803X
Volume **10**, Number **8**, August **2016** pp. 1879–1885

THE COMBINED WEIGHTED ARITHMETIC AVERAGING ALGORITHM TO MULTIPLE ATTRIBUTE DECISION-MAKING WITHIN TRIANGULAR FUZZY NUMBERS

Qingjian Zhou, Man Liu, Qiang Liu and Jia Jiao*

Department of Science
Dalian Minzu University
No. 18, Liaohe West Road, Jinzhou New District, Dalian 116600, P. R. China
{ Zqj68; liuman_dl; liuqiang }@dlnu.edu.cn; *Corresponding author: jiaojia@dlnu.edu.cn

Received December 2015; accepted March 2016

Abstract. *In this paper, the Combined Weighted Arithmetic Averaging Algorithm is proposed to solve the multiple attribute decision-making within triangular fuzzy numbers. Firstly, all the attribute values in the form of triangular fuzzy numbers can be translated into real numbers with their mathematical expectations. Secondly, the multiple attribute decision-making problems with triangular fuzzy attribute values are transformed into the problems with their mathematical expectations. Thirdly, the Combined Weighted Arithmetic Averaging Algorithm is adopted to rank all alternatives as well as select the best. Finally, a numerical example is given. The approach proposed in the paper can very well solve multiple attribute decision-making within triangular fuzzy numbers, and it can be seen that the approach is feasible and valid according to the illustrative example.*
Keywords: Multiple attribute decision-making, Triangular fuzzy number, Combined Weighted Arithmetic Averaging Algorithm

1. **Introduction.** As is well known, multiple attribute decision-making is characterized by a decision maker, who is called to rank all alternatives as well as select the best from a given alternative set. So far, multiple attribute decision-making has been one important part of modern decision sciences.

In many situations, a decision maker often cannot give the information with precise attribute values because of the complexity of the problems or lack of knowledge. So many attribute values have to be given in the form of fuzzy numbers, and triangular fuzzy numbers are one important form of fuzzy numbers. Then the problems of multiple attribute decision-making with triangular fuzzy numbers have arisen, and so far a lot of research has been done on it, as in [1-7,11-17].

In this paper, we mainly research on multiple attribute decision-making within triangular fuzzy numbers. Yen et al. gave the definition of triangular fuzzy numbers in the work of [8]. Li introduced some type of fuzzy number in [9]. Facchinetti et al. respectively gave a method to rank fuzzy triangular numbers in [10]. Zhou et al. gave the information entropy based algorithm to multiple attribute decision making with attribute values in the form of triangular fuzzy numbers in [14]. Xu introduced the method of possibility degree to multiple attribute decision-making with triangular fuzzy numbers in [17]. However, most of above methods are not easy to operate.

So in this paper, we will develop one new concise and practical decision analysis method to multiple attribute decision-making within triangular fuzzy numbers. In the first place, all the attribute values within triangular fuzzy numbers can be translated into real numbers with its mathematical expectation, and then the Combined Weighted Arithmetic Averaging Algorithm is adopted to rank all alternatives as well as select the best. At last, one numerical example is used to illustrate the feasibility and the validity of the approach proposed in the paper.

In order to do so, this paper is set out as follows. In Section 2, the decision-making process of the Combined Weighted Arithmetic Averaging Algorithm to multiple attribute decision-making within triangular fuzzy number attribute values is given. In Section 3, a numerical example is used to illustrate the validity of our approach. Conclusions and final remarks follow in Section 4.

2. **Decision Making Analysis.**

2.1. **Triangualr fuzzy number.**

Definition 2.1. *$\tilde{M} = [l, m, u]$ is called a triangular fuzzy number, and its membership function is the following:*

$$\mu_{\tilde{M}}(x) = \begin{cases} \dfrac{x-l}{m-l}, & x \in [l, m]; \\ \dfrac{u-x}{u-m}, & x \in [m, u]; \\ 0, & others. \end{cases}$$

Here, $0 < l < m < u$ are real numbers. Specially, when $l = m = u$, $\tilde{M}$ is degraded into a real number, as in [8,9].

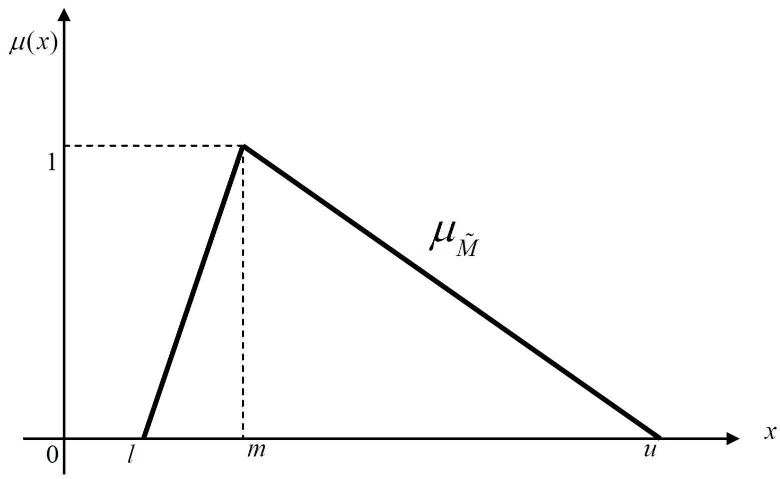

FIGURE 1. Membership function of triangular fuzzy number

Here, U is one fuzzy set of a given domain. For any $x \in U$, there is one corresponding $\mu(x) \in (0, 1)$, and then $\mu(x)$ is called as the membership function of x.

In Figure 1, l and u are respectively the lower and upper limits, and m is the most likelihood value. So the fuzzy number $\tilde{M}$ is expressed as $[l, m, u]$.

Triangular fuzzy number is an important form of fuzzy number and it can express uncertain information very well.

Property 2.1. *For two triangular fuzzy numbers $\tilde{a} = [a_l, a_m, a_u]$, $\tilde{b} = [b_l, b_m, b_u]$, and $\lambda \geq 0$,*

(1) $\tilde{a} = \tilde{b}$ if and only if $a_l = b_l$, $a_m = b_m$, $a_u = b_u$,

(2) $\tilde{a} + \tilde{b} = [a_l + b_l, a_m + b_m, a_u + b_u]$,

(3) $\lambda\tilde{a} = [\lambda a_l, \lambda a_m, \lambda a_u]$, and when $\lambda = 0$, $\lambda\tilde{a} = 0$.

2.2. **The mathematical expectation of triangular fuzzy number.**

Definition 2.2. $E\left(\tilde{M}\right) = \int_{-\infty}^{\infty} x \cdot \mu_{\tilde{M}}(x)dx = \int_{l}^{m} x \cdot \frac{x-l}{m-l}dx + \int_{m}^{u} x \cdot \frac{u-x}{u-m}dx$ *is called the mathematical expectation of triangular number* $\tilde{M}$*. As the mathematical expectation in probability, the mathematical expectation of triangular fuzzy number* $E\left(\tilde{M}\right)$ *expresses the averaging value of the triangular fuzzy number* $\tilde{M}$ *in probability meaning. So the mathematical expectation can well express the value of the corresponding triangular fuzzy number.*

So in this paper we will substitute the mathematical expectations of triangular fuzzy numbers for the triangular fuzzy numbers in the problems of multiple attribute with attribute values in the form of triangular fuzzy numbers. And then the problems to rank the alternatives with triangular fuzzy numbers can be transformed to rank the mathematical expectations.

2.3. **Combined Weighted Arithmetic Averaging Algorithm.** For some multiple attribute decision making problems, let $X = (x_1, x_2, \cdots, x_n)$ as one alternative set, $U = (u_1, u_2, \cdots, u_m)$ as one attribute set.

Definition 2.3. *Let* $CWAA : R^n \to R$*.*
If

$$CWAA_{\omega,w}(\alpha_1, \alpha_2, \cdots, \alpha_n) = \sum_{j=1}^{n} w_j \alpha_j,$$

where

$$w = (w_1, w_2, \cdots, w_n)^T$$

is the weight vector related to $CWAA$*, and*

$$w_j \in [0, 1], \quad \sum_{j=1}^{n} w_j = 1,$$

and b_j *is the jth number in the weighted data* $(n\omega_1\alpha_1, n\omega_2\alpha_2, \cdots, n\omega_n\alpha_n)$*, where*

$$\omega = (\omega_1, \omega_2, \cdots, \omega_n)$$

is the weight vector of $(\alpha_1, \alpha_2, \cdots, \alpha_n)$*, and*

$$\omega_j \in [0, 1], \quad \sum_{j=1}^{n} \omega_j = 1,$$

n *is the balance factor, and then* $CWAA$ *is called the Combined Weighted Arithmetic Averaging Algorithm.*

$CWAA$ considers not only the importance degree of each data, but also reflects the degree of the importance of the location of the data. So we can solve multiple attribute decision-making with triangular fuzzy numbers by $CWAA$.

2.4. **Steps of decision-making analysis.**

Step 1. Replace each triangular fuzzy number $\tilde{M} = [a_l, a_m, a_u]$ of the original decision matrix with its mathematical expectation $E\left(\tilde{M}\right)$, so the original matrix is transformed into the matrix composed by their mathematical expectations $A = (a_{ij})$, here $a_{ij} = E\left(\tilde{M}\right)$.

Step 2. Standardize the matrix $A = (a_{ij})$ into standardization matrix $R = (r_{ij})$.

In general there are two usual attribute types: benefit type and cost type. Let I_1, I_2 respectively represent the subscript sets of the benefit type and the cost benefit attributes.

In order to eliminate the influence on the decision result of different physical dimension, we standardize the original attribute value as:

$$r_{ij} = \frac{a_{ij}}{\max\limits_i(a_{ij})}, \quad i = [1, 2, \cdots, n], \quad j \in I_1, \tag{1}$$

$$r_{ij} = \frac{\min\limits_i(a_{ij})}{a_{ij}}, \quad i = [1, 2, \cdots, n], \quad j \in I_2. \tag{2}$$

So we can get the standardization matrix $R = (r_{ij})$.

Step 3. Obtain the overall value of each alternative by

$$z_i = CWAA_{\omega,w}(r_{ij}). \tag{3}$$

Step 4. Rank the alternatives and select the best by $z_i(\omega)$.

3. **Illustrative Example.** Let us consider the following example. One consumer is planning to buy one building from four buildings x_i, $i = 1, 2, 3, 4$ under four attributes:

u_1: the stand of location,
u_2: building area,
u_3: distance between work units and building,
u_4: natural environment.

And the evaluating results are shown in Table 1 with attribute values within interval numbers, where the stand of location, building area and natural environment are benefit attributes, and the distance between work units and building is cost attribute. Which building should be selected for the consumer?

Table 1. Decision making matrix table

	u_1	u_2	u_3	u_4
x_1	[7,8,10]	[5,6,8]	[7,8,9]	[5,7,8]
x_2	[5,8,10]	[4,5,7]	[4,6,7]	[6,8,10]
x_3	[6,7,9]	[6,7,8]	[7,8,9]	[6,8,9]
x_4	[6,8,9]	[6,7,8]	[6,8,10]	[5,7,9]

Step 1. Replace each attribute value of triangular fuzzy number of $\tilde{M} = [a_l, a_m, a_u]$ in the original decision matrix in Table 1 with its mathematical expectation and get the matrix composed by their mathematical expectations,

$$A = (a_{ij}) = \begin{pmatrix} 12.5 & 9.50 & 8 & 10 \\ 19.1667 & 8 & 8.5 & 16 \\ 11 & 7 & 8 & 11.5 \\ 8 & 7 & 16 & 14 \end{pmatrix}.$$

Step 2. Calculate the standardization matrix by Formulas (1) and (2)

$$R = (r_{ij}) = \begin{pmatrix} 0.6522 & 1 & 1 & 0.6250 \\ 1 & 0.8421 & 0.9412 & 1 \\ 0.5739 & 0.7368 & 1 & 0.7188 \\ 0.4174 & 0.7368 & 0.5 & 0.8750 \end{pmatrix}.$$

Step 3. According to the importance of four attributes, we set the weight vector of the data

$$\omega = (\omega_1, \omega_2, \omega_3, \omega_4) = (0.4, 0.3, 0.2, 0.1).$$

And to eliminate the influence of the injustice factors as far as possible, let the weighted vector related to $CWAA$

$$w = (w_1, w_2, w_3, w_4) = (0.1, 0.2, 0.3, 0.4),$$

and aggregate each alternative x_i by $CWAA$ algorithm, $i = 1, 2, 3, 4$.

For x_1:

$$\begin{aligned}4\omega_1 r_{11} &= 4 \times 0.4 \times 0.6522 = 0.8899,\\ 4\omega_2 r_{12} &= 4 \times 0.3 \times 1 = 1.2,\\ 4\omega_3 r_{13} &= 4 \times 0.2 \times 1 = 0.8,\\ 4\omega_4 r_{14} &= 4 \times 0.1 \times 0.6250 = 0.25,\end{aligned}$$

then

$$b_1 = 1.2,\ b_2 = 0.8899,\ b_3 = 0.8,\ b_4 = 0.25.$$

By (3) get its overall value $z_1(\omega)$,

$$\begin{aligned}z_1 &= CWAA_{\omega,w}(r_{11}, r_{12}, r_{13}, r_{14})\\ &= 0.1 \times 1.2 + 0.2 \times 0.8899 + 0.3 \times 0.8 + 0.4 \times 0.25 = 0.6380.\end{aligned}$$

For x_2:

$$\begin{aligned}4\omega_1 r_{21} &= 4 \times 0.4 \times 1 = 1.6,\\ 4\omega_2 r_{22} &= 4 \times 0.3 \times 0.8421 = 1.0105,\\ 4\omega_3 r_{23} &= 4 \times 0.2 \times 0.9412 = 0.7530,\\ 4\omega_4 r_{24} &= 4 \times 0.1 \times 1 = 0.4,\end{aligned}$$

then

$$b_1 = 1.6,\ b_2 = 1.0105,\ b_3 = 0.7530,\ b_4 = 0.4.$$

By (3) get its overall value $z_2(\omega)$,

$$\begin{aligned}z_2 &= CWAA_{\omega,w}(r_{21}, r_{22}, r_{23}, r_{24})\\ &= 0.1 \times 1.6 + 0.2 \times 1.0105 + 0.3 \times 0.7530 + 0.4 \times 0.4 = 0.7479.\end{aligned}$$

For x_3:

$$\begin{aligned}4\omega_1 r_{31} &= 4 \times 0.4 \times 0.5739 = 0.9182,\\ 4\omega_2 r_{32} &= 4 \times 0.3 \times 0.7368 = 0.8842,\\ 4\omega_3 r_{33} &= 4 \times 0.2 \times 1 = 0.8,\\ 4\omega_4 r_{34} &= 4 \times 0.1 \times 0.7188 = 0.2875,\end{aligned}$$

then

$$b_1 = 0.9182,\ b_2 = 0.8842,\ b_3 = 0.8,\ b_4 = 0.2875.$$

By (3) get its overall value $z_3(\omega)$,

$$\begin{aligned}z_3 &= CWAA_{\omega,w}(r_{31}, r_{32}, r_{33}, r_{34})\\ &= 0.1 \times 0.9182 + 0.2 \times 0.8842 + 0.3 \times 0.8 + 0.4 \times 0.2875 = 0.6237.\end{aligned}$$

For x_4:

$$\begin{aligned}4\omega_1 r_{41} &= 4 \times 0.4 \times 0.4174 = 0.6678,\\ 4\omega_2 r_{42} &= 4 \times 0.3 \times 0.7368 = 0.8842,\\ 4\omega_3 r_{43} &= 4 \times 0.2 \times 0.5 = 0.4,\\ 4\omega_4 r_{44} &= 4 \times 0.1 \times 0.8750 = 0.35,\end{aligned}$$

then

$$b_1 = 0.8842,\ b_2 = 0.6678,\ b_3 = 0.4,\ b_4 = 0.35.$$

By (3) get its overall value $z_4(\omega)$,

$$\begin{aligned}z_4 &= CWAA_{\omega,w}(r_{41}, r_{42}, r_{43}, r_{44})\\ &= 0.1 \times 0.8842 + 0.2 \times 0.6678 + 0.3 \times 0.4 + 0.4 \times 0.35 = 0.4820.\end{aligned}$$

Step 4. Utilize z_i $(i = 1, 2, 3, 4)$ to rank the alternatives:

$$x_2 \succ x_1 \succ x_3 \succ x_4.$$

So the 2nd building is the best.

4. **Conclusions.** In this paper we mainly proposed a multiple attribute decision making algorithm with triangular fuzzy number attribute values. We have researched the following three aspects of work:

(1) to define the mathematical expectation of the triangular fuzzy number,

(2) the multiple attribute decision-making problems with triangular fuzzy numbers can be transformed into the problems with the mathematical expectations,

(3) to adopt the Combined Weighted Arithmetic Averaging Algorithm to rank all alternatives as well as select the best.

And from the solution to the above example we can see the approach is precise and practical. In the future, we will continue to research on the other algorithms to attribute values in the form of triangular fuzzy numbers.

Acknowledgment. This paper is supported by the NSFC (11226157), Education Department Foundation of Liaoning Province of China (L2014549), Education Department Talents Project of Liaoning Province of China (LJQ2015029), Science and Technology Department Foundation of Liaoning Province of China (2015020021), and the Fundamental Research Funds for the Central Universities of China (DC201502050304, DC201502050409, DC201501043, DCPY2016058, DCPY2016061), and Doctor Support Foundation of DLNU (110090). Meanwhile, the authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

REFERENCES

[1] Z. S. Xu, *Uncertain Multiple Attribution Decision-Making: Methods and Applications*, Tsinghua University Press, Beijing, 2004.

[2] P. T. Harker and L. G. Vargas, The theory of ratioscale estimation: Saary's analytic hierarchy process, *Management Science*, vol.33, no.11, pp.1383-1403, 1987.

[3] J. Liu and S. F. Liu, Research on the ranking of multiple decision object for attribute value within interval numbers, *Chinese Journal of Management Sciences*, vol.18, no.3, pp.90-94, 2010.

[4] R. R. Yage and A. Kelman, Decision-making under various types of uncertainties, *Journal of Intelligent and Fuzzy System Applications in Engineering and Technology*, vol.3, no.4, pp.317-323, 1995.

[5] K. P. Lin and K. C. Hung, An efficient fuzzy weighted average algorithm for the military UAV selecting under group decision-making, *Knowledge-Based Systems*, vol.24, no.6, pp.877-889, 2011.

[6] D. F. Li, Multi-attribute decision-making models and methods using intuitionistic fuzzy sets, *Journal of Computer and System Sciences*, vol.70, no.1, pp.73-85, 2005.

[7] Z. S. Xu, A survey of preference relations, *Internatinal Journal of General Systems*, vol.36, no.2, pp.179-203, 2007.

[8] K. K. Yen, S. Ghoshray and F. Roig, A linear regression model using triangular fuzzy number coefficients, *Fuzzy Sets and Systems*, vol.106, no.2, pp.167-177, 1999.

[9] D. F. Li, *Fuzzy Multiobjective Many-Person Decision-Making and Games*, National Defence Industry Press, Beijing, 2003.

[10] G. Facchinetti, R. G. Ricci and S. Muzzioli, Note on ranking fuzzy triangular numbers, *International Journal of Intelligent Systems*, vol.13, pp.613-622, 1998.

[11] J. X. Ran, The research of the rank of triangular fuzzy numbers, *Journal of Minzu University of China (Natural Sciences Edition)*, vol.20, no.4, pp.37-42, 2011.

[12] F. J. Cabrerizo, S. Alonso and E. Herrera, A consensus model for group decision making problems with unbalanced fuzzy linguistic information, *International Journal of Information Technology and Decision Making*, vol.8, no.1, pp.109-131, 2009.

[13] Q. J. Zhou and J. Jiao, The ordered weighted averaging algorithm to multiple attribute decision making within triangular fuzzy numbers, *Applied Mathematical Sciences*, vol.8, no.56, pp.2763-2766, 2014.

[14] Q. J. Zhou, J. Jiao and D. L. Yang, The information entropy based algorithm to multiple attribute decision making with attribute values in the form of triangular fuzzy numbers, *ICIC Express Letters, Part B: Applications*, vol.5, no.5, pp.1279-1284, 2014.

[15] S. Laurent and A. J. Metz, Parenting styles and career decision-making among French and Korean adolescents, *Journal of Vocational Behavior*, vol.84, no.3, pp.345-355, 2014.

[16] Q. J. Zhou and J. Jiao, The weighted averaging algorithm to multiple attribute decision-making within triangular fuzzy numbers, *Proc. of 2013 International Conference on computational and Information Sciences*, pp.657-662, 2014.

[17] Z. S. Xu, A method for priorities of triangular fuzzy number complementary judgement matrices, *Fuzzy Systems and Mathematics*, vol.16, no.1, pp.47-50, 2002.