

## CASCADING FAILURE MODEL ON AN INTERDEPENDENT SERVICE-INFRASTRUCTURE NETWORK BASED ON LOGISTICS NETWORK

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*ABSTRACT.* A logistics network consists of an infrastructure network for transport facilities and a service or virtual network for transport or logistics tasks. In order to theoretically analyze the chain reaction of emergency events that occurred on the logistics network with the complicated propagation mechanism of multi-layer interdependent networks, a cascading failure model on the interdependent network is proposed. This model described the interaction between the nodes on the same or the different networks. And it was used to analyze the delay time of logistics business caused by the node's failure on an interdependent network. The simulation results show that the delay time caused by the failure in an interdependent network is longer than that in a single network; however, it takes less time to lead to collapse in an interdependent network than in a single network.

**Keywords:** Interdependent network, Cascading failure, Logistics network, Connection strength, Infrastructure network, Service network

1. **Introduction.** With the rapid development of social economy, some infrastructure systems such as complex network become more and more complicated and mutually dependent. Once the complex systems are disturbed by any external or internal perturbations, the failures can spread very rapidly within the system, even to other correlative systems, and then cause the whole systems to lose their functions and to collapse. A logistics system is made up of a logistics network that can be divided into two layers of networks, an infrastructure network for transport facilities and a service or virtual network for transport or logistics tasks. As the emergency events such as floods, earthquakes, and sudden serious accidents occur on the logistics network frequently it may generate the chain reaction of the disasters spreading on the logistics network. Therefore, emergency management on the logistics system becomes one of the key factors to ensure the smooth connectivity of corresponding supply chain. It is a big challenge for the decision makers to determine the feasible provisions for possible emergencies. Consequently, to understand the mechanism of the chain reaction on the logistics networks is quite significant. From the theoretical point of view, recently the chain reaction problem of complex logistics networks in emergencies has increasingly become one of the hot topics, where the cascading failure problem is the most typical problem [1,2]. This problem has been studied extensively in critical infrastructure networks such as electric networks [3,4].

In the research field of vulnerability of traffic network, the cascading failure models in transport networks have been focused by many researches [5]. However, there still exist some insufficiencies in the current researches on cascading failures on logistics networks

due to the different characteristics of both the entity operations and the structure of networks compared to the other networks. For example, the directivity in the operation of the transport networks has been lack of consideration, and the impact of the level of relativity between network elements (nodes and edges) of interdependent networks on the spread features of cascading failures has rarely been studied. The events that occurred in the actual world have shown that interdependencies can increase system vulnerability [6]. In an interdependent network, a failure on one network may not only cause the cascading failures on the current network, but also cause the cascading failure on the other networks [7,8], and then cause more serious effects [9,10], and even lead to a catastrophic accident [11,12]. This influence on the propagation of cascading failures is mainly reflected in the redistribution rules for the load on the failure node. The ratio of load redistribution to the neighbor nodes depends on the node importance and the connection strength of the edges synthetically [13,14]. Until now the studies of the cascading failure models on interdependent networks for the logistics network have rarely been carried out.

For this reason, this paper focuses on modeling the cascading failure on interdependent networks with a two-layer structure of a logistics tasks-infrastructure network. This research aims to establish an approach to analyze and control the cascading failure dynamics propagation in logistics network with the characteristics of interdependent networks, and give some suggestions that can be taken to reduce the impact of logistics network's weaknesses on their vulnerabilities, and help to better shape emergency logistics management.

The rest of this paper is organized as follows. Section 2 gives a description of an interdependent service and infrastructure network based on logistics system. In Section 3, a cascading model on interdependent 2 networks is briefly introduced. In Section 4, we reveal the existence weaknesses or vulnerabilities of logistics network by the simulations. Section 5 is devoted to conclusions and remarks.

## 2. Network Description.

**2.1. Network structure.** A logistics network consists of at least 2 layers of networks, the transport infrastructure network and the transport service network. The former includes road and rail facilities, and the latter includes organization commands, information transmissions and logistics demands. The integrated network with the infrastructure network and the service network is defined as the interdependent network whose structure is shown as Figure 1, where network  $A$  and network  $B$  are short for the service network and the infrastructure network respectively.

A directed weighted graph  $G(V, E, W)$  is used to describe the hierarchical structure of the interdependent network. In this graph, it is supposed that the interdependent network

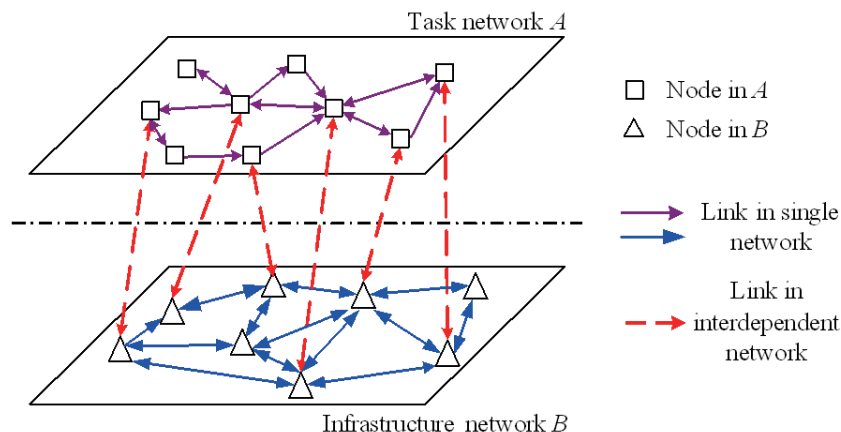


FIGURE 1. Structure of interdependent network  $G$

consists of 2 networks, namely network  $A$  for the logistics task or service network and network  $B$  for the transport infrastructure network, and the sizes of the two networks nodes are  $N_A$  and  $N_B$ , respectively. Let  $V_A = \{v_{Ai}|i = 1, 2, \dots, N_A\}$  and  $V_B = \{v_{Bi}|i = 1, 2, \dots, N_B\}$  represent the nodes sets of network  $A$  and network  $B$  respectively,  $E_A = \{e_{Aij}|i, j = 1, 2, \dots, N_A\}$  and  $E_B = \{e_{Bij}|i, j = 1, 2, \dots, N_B\}$  represent their edges sets,  $W_A = \{w_{Aij}|i, j = 1, 2, \dots, N_A\}$  and  $W_B = \{w_{Bij}|i, j = 1, 2, \dots, N_B\}$  represent the edge weight sets, where element  $w_{xij}$  represents the edge weight of node  $v_{xi}$  pointing to  $v_{xj}$  in network  $x$ ,  $x = A \vee B$ ,  $1 \leq i, j \leq N_x$ ,  $i \neq j$ .

To indicate the relationship between a node on network  $A$  and a node on network  $B$ , we define  $E_{AB} = \{e_{ABij}|i = 1, 2, \dots, N_A, j = 1, 2, \dots, N_B\}$  as the link relation set between nodes on networks  $A$  and those on  $B$ , where  $e_{ABij}$  means that there is a link relation between node  $v_{Ai}$  on network  $A$  and node  $v_{Bj}$  on network  $B$ ,  $v_{Ai} \in V_A$ ,  $v_{Bj} \in V_B$ ,  $e_{ABij} \in [0, 1]$ .  $e_{ABij} = 1$  or  $0$  means that there is or is no relation between  $v_{Ai}$  and  $v_{Bj}$ . Then  $E = E_A \cup E_B \cup E_{AB}$  indicates the edge set of the interdependent network,  $V = V_A \cup V_B$  is the nodes set of  $G$ , and  $W = W_A \cup W_B$  indicates the edge weight set of  $G$  accordingly.

**2.2. Basic attribute indexes.** Two basic attribute indexes used for this paper are defined, the node importance, and the edge connection strength. In this paper, the node importance is quantified by combining the weights of edges between the nodes. Suppose that the node importance of node  $v_{xi}$  is  $S_{xi}$ , the calculation method of  $S_{xi}$  is shown in Formula (1).

$$S_{xi} = \sum_{v_{xj} \in \Gamma_{xi}} w_{xij} \tag{1}$$

where  $\Gamma_{xi}$  is the adjacent node set of node  $v_{xi}$ ,  $x = A \vee B$ ,  $1 \leq i, j \leq N_x$ .

In the actual logistics network, many factors have impact on the edge connection strength, such as the capacity of roads, distance, transport mode. Thus,  $n$  attributes ( $n \geq 1$ ) are chosen to describe the edge connection strength. An assessment vector  $\Phi = \{\xi_1, \xi_2, \dots, \xi_n\}$  can be made up according to the selected  $n$  attributes, and then the edge connection strength  $r_{xij}$  between node  $v_{xi}$  and  $v_{xj}$  can be calculated with Formula (2).

$$r_{xij} = \frac{\sum_{\varepsilon=1}^n \xi_{xi\varepsilon} \xi_{xj\varepsilon} - n \bar{\Phi}_{xi} \bar{\Phi}_{xj}}{\sqrt{\sum_{\varepsilon=1}^n \xi_{xi\varepsilon}^2 - n \bar{\Phi}_{xi}^2} \sqrt{\sum_{\varepsilon=1}^n \xi_{xj\varepsilon}^2 - n \bar{\Phi}_{xj}^2}} \tag{2}$$

where  $\Phi_{xi} = \{\xi_{xi1}, \xi_{xi2}, \dots, \xi_{xin}\}$  and  $\Phi_{xj} = \{\xi_{xj1}, \xi_{xj2}, \dots, \xi_{xjn}\}$  represent the assessment vectors of node  $v_{xi}$  and node  $v_{xj}$  respectively,  $\xi_{xi\varepsilon}$  and  $\xi_{xj\varepsilon}$  are the attribute values,  $\bar{\Phi}_{xi}$  and  $\bar{\Phi}_{xj}$  are the average values of attributes in the vectors.  $r_{xij}$  can be calculated with Formula (2),  $r_{xij} \in [-1, 1]$ . It is predetermined that  $r_{xij} = 0$  when the denominator on the right of Formula (2) is equal to 0.

### 3. Cascading Failure Model.

**3.1. Load capacity.** Load capacity of nodes has important effect on the redistribution of the failure load, and its capacity is closely related to the degree of node importance. Prior studies usually used the “degree” as the index to identify the importance of nodes [2-4]. It might bring some deficiencies in practice because two nodes with the same degree were not equally important necessarily. Therefore, this paper utilizes the synthetical importance of nodes to quantify nodes’ load capacity. It assumed that load capacity  $C_{xi}$  of any node  $v_{xi}$  is calculated with Formula (3).

$$C_{xi} = S_{xi}^\alpha \left( \sum_{v_{xj} \in \Gamma_{xi}} S_{xj} \right)^{1-\alpha} \tag{3}$$

where parameter  $\alpha$  is used to control the influencing weight of nodes  $v_{xi}$  and its adjacent nodes,  $0 \leq \alpha \leq 1$ ,  $x = A \vee B$ .

**3.2. Propagation mechanism.** According to the previous studies [4,5], the propagation mechanism of cascading failure existed in a single network: when the allocated load amount (assuming it to be  $\Delta L_{xi \rightarrow xj}$ ) which adjacent nodes  $v_{xj}$  obtained from the failure node  $v_{xi}$  plus its own real-time load amount (assuming it to be  $L_{xj}(t)$ ) is beyond the load capacity  $C_{xj}$  of node  $v_{xj}$ , that is, the condition in Inequality (4) is satisfied, node  $v_{xj}$  becomes failure and then triggers a new failure transmission,  $x = A \vee B$ ,  $1 \leq i, j \leq N_x$ .

$$L_{xj}(t) + \Delta L_{xi \rightarrow xj} > C_{xj} \tag{4}$$

The propagation mechanism in an interdependent network is different from the single network obviously. Because it may due not only to their different network structures but also to their different propagation paths which the failed load would spread among interdependent networks, this paper brings forward a propagation mechanism of cascading failure in an interdependent network: in the case of interdependent network, its characteristics in attributes such as the node importance, and the difference in connection strength of the edge link should be considered. Therefore, its propagation mechanism of cascading failure is different from the case of a single network. The chain reaction process of cascading failure caused by the failed node on network  $B$  is shown in Figure 2.

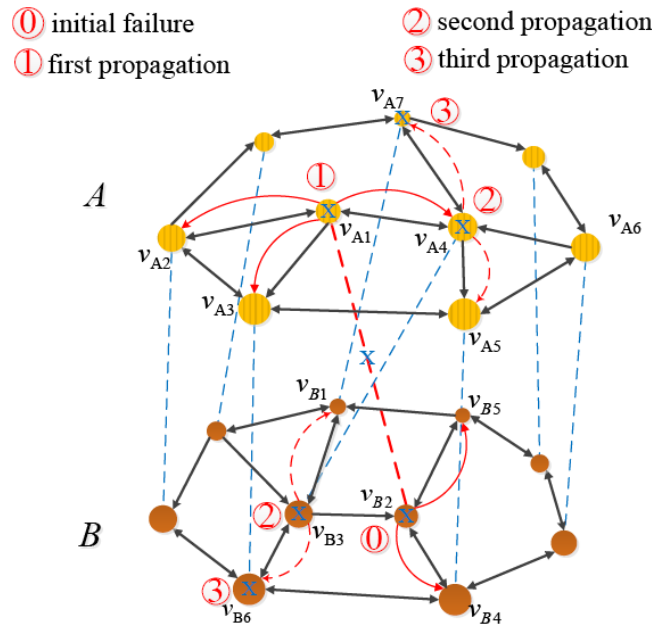


FIGURE 2. Cascading failure caused by the failed node on network  $B$

Firstly, assume that node  $v_{B2}$  on network  $B$  suddenly fails, the current load  $L_{B2}$  on node  $v_{B2}$  will be moved to its adjacency nodes. The load  $L_{B2}$  is reallocated to  $v_{Bu}$  by a certain proportion, according to connection strength  $r_{B2j}$  of edge links between nodes and their node importance  $S_{Bu}$  of adjacent nodes,  $u = 4, 5$ ,  $j = 1, 2, \dots, N_B$ . At the same time,  $r_{AB2i}$  spreads its failure to node  $v_{A1}$  on network  $A$  to some extent according to connection strength of the edge link, so may cause the failure of node  $v_{A1}$ . And  $v_{A1}$  is regarded as the first round of failure nodes (see ① in Figure 2),  $i = 1, 2, \dots, N_A$ .

Secondly, on the basis of failure of  $v_{A1}$ , the failure load will be reallocated and redistributed to nodes  $v_{Au}$  ( $u = 2, 3, 4$ ) on network  $A$  according to the connection strength of edge  $r_{A1i}$  and the node importance  $S_{Au}$ . Therefore, it may cause the failure of node  $v_{A4}$ . And after then, continuously it may cause the failure of node  $v_{B3}$  on network  $B$  according

to connection strength of edge  $r_{AB4j}$ ,  $i = 1, 2, \dots, N_A$ ,  $j = 1, 2, \dots, N_B$ . Here, nodes  $v_{A4}$  and  $v_{B3}$  are regarded as the second round of failure nodes (see ② in Figure 2).

Finally, the failed load on nodes  $v_{A4}$  and  $v_{B3}$  will be redistributed and transferred once more. Then the new failure nodes  $v_{A7}$  on network  $A$  and  $v_{B6}$  on  $B$  will be produced. These nodes are the third round of failure nodes (see ③ in Figure 2).

If the condition is satisfied, the failure process will be continued until the whole nodes in the networks ruin. To describe the dynamics process of redistribution and transmission of failure loads effectively, a failure propagation function  $F_{xi}(t)$  is defined according to Formula (4), as shown in Formula (5).

$$F_{xi}(t) = \begin{cases} 0, & L_{xi}(t) \leq C_{xi} \\ 1, & L_{xi}(t) > C_{xi} \end{cases} \quad (5)$$

where value 0 marks the state of normal operations of node  $v_{xi}$  at time  $t$ , and value 1 marks the failure state.

**3.3. Redistribution rule.** Suppose that an arbitrary node  $v_{xi}$  fails, and its adjacent node  $v_{xj}$  will obtain its failure load by the redistribution proportion  $P_{xij}$ ,  $P_{xij}$  can be determined with Formula (6):

$$P_{xij} = \left[ \beta \frac{S_{xj}}{\sum_{v_{x\eta} \in \{\Gamma_{xj} \setminus V_{fail}\}} S_{x\eta}} + (1 - \beta) \frac{C_{xj}}{\sum_{v_{x\eta} \in \{\Gamma_{xj} \setminus V_{fail}\}} C_{x\eta}} \right] \frac{r_{xij}}{\sum_{v_{x\tau} \in \Gamma_{xi}} r_{xi\tau}} \quad (6)$$

where  $V_{fail}$  is the node set of the failed nodes;  $\beta$  and  $1 - \beta$  represent the influence weights of node importance and load capacity of nodes affecting the redistribution proportion respectively.

**3.4. Failure measure.** An evaluation measure index is proposed that this indicator uses the time delay of the logistics business to measure the node failure destruction, shown as Formula (7):

$$D = \varphi \sum_{v_{xi} \in V_x} T_{xi} F_{xi} \quad (7)$$

where  $T_{xi}$  is a determined service time on a given logistics node, and  $\varphi$  represents the control parameter of business time delay.

**4. Simulation and Analysis.** Given an interdependent network it consists of network  $A$  with 300 nodes ( $N_A = 300$ ) and network  $B$  with 500 nodes ( $N_B = 500$ ). And the initial experimental parameters are produced. It is assumed that node  $v_{xi}$  possesses the load capacity  $C_{xi}$ , its initial load  $L_{xi}(0)$  is calculated with Formula (8):

$$L_{xi}(0) = \lambda_{xi} C_{xi} \quad (8)$$

where  $x = A \vee B$ ,  $\lambda_{xi}$  is the regulation parameter for initial node load, which meets  $0 \leq \lambda_{xi} \leq 1$ .

And the predetermined logistics business time  $T_{xi}$  is determined with Formula (9):

$$T_{xi} = \omega S_{xi} \quad (9)$$

where  $\omega$  is defined as the control parameter of the business time.

Then, other relevant experimental parameters are set as follows:  $\alpha = 0.5$ ,  $\beta = 0.5$ ,  $\omega = 1$ ,  $\varphi = 0.1$ , and  $r_{xi}$  and  $\lambda_{xi}$  are the random real numbers between  $[-1, 1]$  and  $[0, 1]$ , respectively. In addition, it is assumed that  $D_{max}^A$  and  $D_{max}^B$  are the maximum time delays of network  $A$  and  $B$  respectively, which occur after the network crash (i.e., all nodes have failed). Thus, according to Formulas (7) and (9),  $D_{max}^A = 49.6$ ,  $D_{max}^B = 273.4$ .

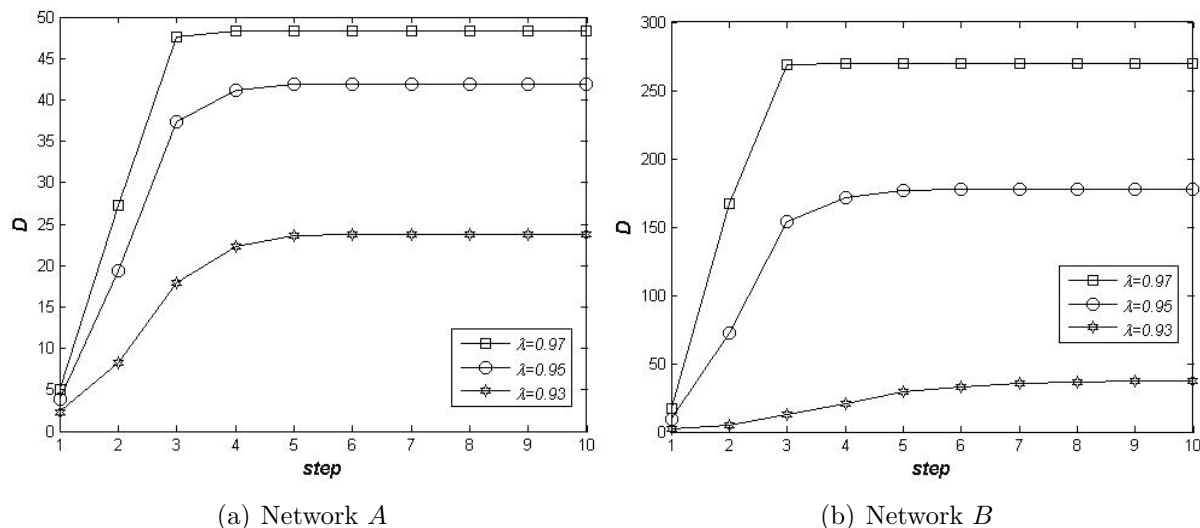


FIGURE 3. Cascading failure on a single network

**4.1. The case of single network.** The statistical results from the time delays caused by node failure on the single network are shown in Figure 3, where Figure 3(a) shows the data of network  $A$  and Figure 3(b) shows the data of network  $B$ . The abscissa is the number of spread steps and the ordinate is the delay time.

It is known from Figure 3(a) that the value  $D$  of network  $A$  increases as the value  $\lambda$  increases. Especially when  $\lambda = 0.97$  and the failure spreads to Step 3,  $D = D_{\max}^A$  which leads to the collapse of the entire network  $A$ ; and when  $\lambda \leq 0.95$ , no collapse happened on the network. The similar situation also occurred on network  $B$ . As shown in Figure 3(b), the value  $D$  increases as the increase of the value  $\lambda$ . When  $\lambda = 0.97$  and the failure spreads to Step 3,  $D = D_{\max}^B$  which leads to the collapse of the entire network  $B$ ; Similarly, when  $\lambda \leq 0.95$ , no collapse happened on network  $B$ .

**4.2. The case of interdependent networks.** The statistical results from the business time delays of the sub-network caused by the node failure of networks  $A$  and  $B$  under the interdependent networks are shown in Figure 4 and Figure 5 respectively. The abscissa is the number of spread steps and the ordinate is the delay time. For network  $A$ , it is known from Figure 4(a) that the value  $D$  on  $A$  increases as the increase of the value  $\lambda$ . When  $\lambda \geq 0.55$ , the collapse on network  $A$  occurred. However, when  $\lambda = 0.95$ , the network had collapsed in Step 2.

As shown in Figure 4(b), when  $\lambda \leq 0.65$ , the network did not go into collapse; and when  $\lambda \geq 0.75$ , the collapse occurred. Especially, when  $\lambda = 0.75$ , the network had crashed when the failure spread to Step 5. Meanwhile, when  $\lambda = 0.85$  and  $\lambda = 0.95$ , the network had crashed as the failure spread reaches to Step 4 and Step 3 respectively.

Similarly, for network  $B$ , shown in Figure 5(a) that when  $\lambda \geq 0.55$ , the network  $A$  collapsed. However, in contrast to the case of the single network (refer to Figure 3(a)), it is found that in the condition of the same load degree, that is, the same value  $\lambda$ , the effect of cascading failure on  $A$  caused by failure node spread in  $B$  in the case of the interdependent network is larger than that in the case of the single network.

In addition, in network  $B$ , the simulation results are shown in Figure 5(b). It is found that in the condition of the same value  $\lambda$ , the impact of failure node is much larger than that in the case of the single network (refer to Figure 3(b)), and also in the network  $A$  under the interdependent structure. Particularly, when  $\lambda = 0.95$ , network  $B$  had collapsed at Step 2.

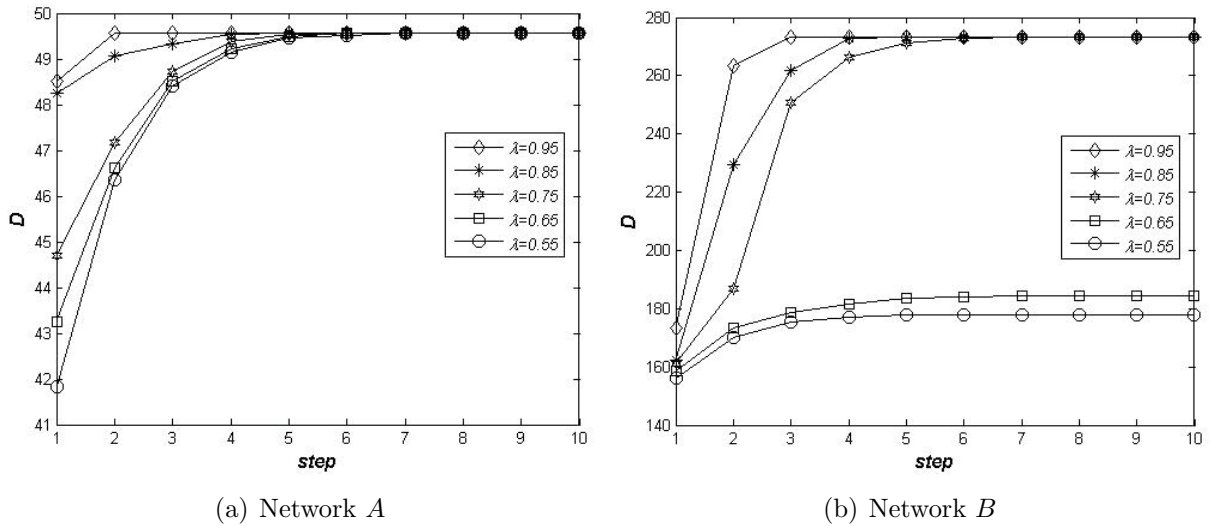


FIGURE 4. Cascading failure on interdependent network caused by  $A$

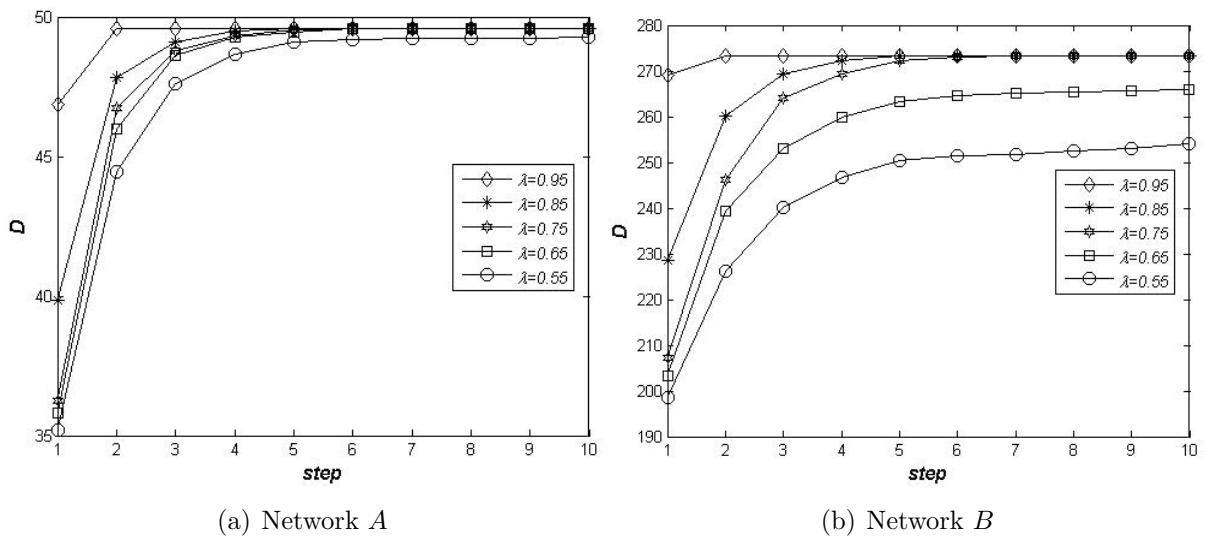


FIGURE 5. Cascading failure on interdependent network caused by  $B$

**5. Conclusions.** The significance of this proposed cascading failure model is that this model can theoretically describe the mechanism of the complicated dynamic transition process of the cascading failure on the logistics network with the features of the interdependent network effectively. And by simulation analysis the cascading failure process on the logistics network can be illustrated and evaluated. This model is helpful to provide the reasonable theoretical explanation of the characteristics of damage transitions for the decision maker of emergency provision for logistics systems.

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