

GLOBAL EXPONENTIAL STABILITY OF SWITCHED LINEAR SINGULAR SYSTEMS WITH DWELL TIME SPECIFICATIONS

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ABSTRACT. *This paper will study the global exponential stability (GES) analysis for a class of switched linear singular systems under any switching signal with dwell time specifications. Unlike the classical dwell time method, the dwell time is an arbitrarily prespecified constant, which is not computed by Lyapunov functions of the subsystems. A novel method to construct multiple time-varying Lyapunov functions is proposed to analyze the GES for a class of switched singular systems under dwell time specifications. Finally, an example is given to illustrate the effectiveness of the proposed method.*

Keywords: Switched linear singular systems, Stability, Dwell time specifications

1. Introduction. In the literature, the switched systems have attracted much attention due to their significance both in theory and applications [1-5]. Many methodologies have been reported to study switched systems. Some sufficient conditions are provided to ensure the asymptotic stability of switched systems under arbitrary switchings [7, 8]. Multiple Lyapunov function method is an effective tool for studying the asymptotic stability of switched systems under a certain switching law [9-11].

On the other hand, singular systems have been intensively studied due to the important applications in, for example, circuit systems, robotics, and aircraft modeling [12]. It is shown that the behavior of state jumps degrades the system performance or even destabilizes the system. Recently, many efforts have been done to the study of switched singular systems. However, since stability, regularity, impulse elimination and state consistency of switched singular systems should be considered at the same time, the analysis of such system is more difficult. Therefore, the issue of the switched singular systems has not been well developed [13-17]. There are a few results on stability analysis under arbitrary switchings and some constrained switchings. The dwell-time-based is one of the switching signals with constraints and has received a considerable attention. For example, [18] utilized the average dwell-time approach to study the stability analysis of switched singular systems with time-varying delay. However, the average dwell time requires a minimum time period during which no switching occurs and is no less than a positive constant τ . It is shown that the minimum of admissible dwell time is computed by two mode-dependent parameters, i.e., the increase coefficient of the Lyapunov-like function at switching instants and the decay rate of the Lyapunov-like function during the running time of subsystems. Unlike the classical dwell time method, the dwell time is an arbitrarily prespecified constant, rather than obtained by calculating these two parameters. Although stability analysis with dwell time specifications has been obtained in [19], few results have been presented for the stability analysis of switched singular systems with dwell time specifications, which partly motivates our present work.

This paper will study the GES analysis under dwell time specifications for a class of switched singular systems. Multiple time-varying Lyapunov functions are constructed to

analyze the GES of switched singular systems under dwell time specifications. An example is provided to illustrate the effectiveness of the proposed method.

This paper is organized as follows. Some preliminaries are introduced in Section 2. The stability analysis and the main contribution for the switched linear singular system are presented in Section 3. Then, a numerical example is provided in Section 4. Conclusions are given in Section 5.

2. Problem Formulation. Consider a class of switched linear singular systems:

$$E_{\sigma(t)}\dot{x}(t) = A_{\sigma(t)}x(t), \quad x_0 = x(0), \tag{1}$$

where $x(t) \in R^n$ is the state and $x_0 \in R^n$ is the initial state; the function $\sigma(t) : R^+ \rightarrow M = \{1, 2, \dots, m\}$ is the switching signal which is assumed to be a piecewise constant or piecewise continuous (from the right) function depending on time or state or both; m is the number of models (called subsystems) of the switched system; E_i and $A_i, \forall i \in M$, are constant matrices with $rank(E_i) = r \leq n$. We assume that $\sigma(t) = \sigma(t_k) = i_k, i_k \in M, t \in [t_k, t_{k+1})$, where t_k is the switching instant, which means that the i_k th subsystem is activated when $t \in [t_k, t_{k+1})$. For simplicity, we use (E_i, A_i) to denote the i th subsystem. We call the set D_{τ_1} of all switching signal with dwell time specification $\tau_1 > 0$, that is the set of all $\sigma(t)$ for which the time interval between successive switching instants satisfies $t_k - t_{k-1} > \tau_1, i = 1, 2, \dots$.

Our goal is to analyze the GES for switched singular system (1) under $\sigma(t) \in D_{\tau_1}$.

Definition 2.1. [12] *The singular system $E\dot{x}(t) = Ax(t)$ or the pair (E, A) is said to be (i) regular if $\det(sE - A)$ is not identically zero; (ii) impulse-free if $\deg(\det(sE - A)) = rank(E)$; (iii) stable if all the roots of $\det(sE - A) = 0$ have negative real parts.*

Assumption 2.1. *All singular systems $(E_i, A_i), \forall i \in M$ are regular and impulse-free.*

Since $rank(E_i) = r \leq n$, we can find nonsingular matrices H_i and $N_i, i \in M$, such that (E_i, A_i) takes the following dynamics decomposition form [20]:

$$H_i E_i N_i = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} =: \bar{E}, \quad H_i A_i N_i = \begin{bmatrix} A_{11}(i) & A_{12}(i) \\ A_{21}(i) & A_{22}(i) \end{bmatrix} =: \bar{A}_i. \tag{2}$$

Noticing that the decomposition (2) can be obtained via a singular value decomposition on E_i and the decomposition is not unique. By Assumption 2.1, we know that $A_{22}(i) \in R^{(n-r) \times (n-r)}$ is nonsingular and the solution of system (1) is piecewise-smooth.

By the following state transformation in [17]:

$$\bar{x}(t) = \begin{bmatrix} \bar{x}_1(t) \\ \bar{x}_2(t) \end{bmatrix} = N_{i_k}^{-1} x(t), \quad t \in [t_k, t_{k+1}), \tag{3}$$

the switched singular system (1) is equivalent to the impulsive switched singular system:

$$\dot{\bar{x}}_1(t) = A_{11}(\sigma(t)) \bar{x}_1(t) + A_{12}(\sigma(t)) \bar{x}_2(t), \tag{4}$$

$$0 = A_{21}(\sigma(t)) \bar{x}_1(t) + A_{22}(\sigma(t)) \bar{x}_2(t), \tag{5}$$

$$\bar{x}(t_k^+) = \Gamma_{i_k i_{k-1}} \bar{x}(t_k^-) = \begin{bmatrix} I & 0 \\ -A_{22}^{-1}(i_k) A_{21}(i_k) & 0 \end{bmatrix} N_{i_k}^{-1} N_{i_{k-1}} \bar{x}(t_k^-), \tag{6}$$

for $i_k, i_{k-1} \in M, i_k \neq i_{k-1}$.

3. Stability Analysis. In this section, we will develop a piecewise time-varying Lyapunov function method to analyze the stability of the impulsive switched singular system (4)-(6).

Theorem 3.1. Consider system (1) satisfying Assumption 2.1. Suppose that there exist matrices $P_{i1} > 0$, $P_{i2} > 0$, and parameters $\mu_i > 1$, $\eta_i > 0$, $\tau_1 > 0$, such that

$$P_{i1} - P_{i2} > 0, \tag{7}$$

$$\begin{bmatrix} -\mu P_{i21} & \Pi_{i_1 i_2}^T P_{i12} \\ * & -\mu_{i_1} P_{i12} \end{bmatrix} \leq 0, \quad \forall i_1, i_2 \in M, \tag{8}$$

$$\mathfrak{s}_i P_{il} + P_{il} \hat{A}(i) + \hat{A}^T(i) P_{il} + 1/\tau_1 (P_{i1} - P_{i2}) + \eta_i P_{il} < 0, \quad l = 1, 2, \tag{9}$$

with $\Pi_{ij} = [I \ 0] N_i^{-1} N_j \begin{bmatrix} I \\ -A_{22}^{-1}(j) A_{21}(j) \end{bmatrix}$ and $\mu = \max \left\{ \frac{1}{\mu_i}, \forall i \in M \right\}$, $\hat{A}(i) = A_{11}(i) - A_{12}(i) A_{22}^{-1}(i) A_{21}(i)$, $\mathfrak{s}_i = 1/\tau_1 \ln(\mu_i)$. Then, system (1) is GES under $\sigma(t) \in D_{\tau_1}$.

Proof: The set $\{t_k\}$ generated by $\sigma(t) \in D_{\tau_1}$ denotes the switching sequences with $\tau_1 \leq t_k - t_{k-1}$, $k \in N$. Then, for each $\{t_k\}$ generated by $\sigma(t) \in D_{\tau_1}$ and $t \in [t_k, t_{k+1})$, let

$$\rho(t) = \frac{t - t_k}{t_{k+1} - t_k}, \quad \tilde{\rho}(t) = 1 - \rho(t), \quad \rho_1(t) = \frac{1}{t_{k+1} - t_k}, \quad \phi(t) = \mu_i^{\rho(t)-1}, \tag{10}$$

$$P_i(t) = \rho(t) P_{i1} + \tilde{\rho}(t) P_{i2}, \quad \forall i \in M, \tag{11}$$

where $\mu_i > 1$, $P_{i1} > 0$, $P_{i2} > 0$, $\forall i \in M$. It is obvious that $\rho(t_k^+) = 0$, $\rho(t_k^-) = 1$, $\mu_i^{-1} \leq \phi(t) \leq 1$, and $\dot{\phi}(t) \leq \phi(t) \frac{\ln(\mu_i)}{\tau_1}$.

For system (1), choose the following piecewise time-varying Lyapunov function:

$$V_i(t) = \phi(t) \bar{x}^T(t) \bar{E} \bar{P}_i(t) \bar{x}(t) = \phi(t) \bar{x}^T(t) \bar{E} \begin{bmatrix} P_i(t) & 0 \\ P_{i3} & P_{i4} \end{bmatrix} \bar{x}(t). \tag{12}$$

It follows from (12) that $V_i(t) = \phi(t) \bar{x}_1^T(t) P_i(t) \bar{x}_1(t)$. For $t \in [t_k, t_{k+1})$, from (7) and (8), taking derivative of $V_i(t)$ along the trajectory of system (1) yields

$$\begin{aligned} \dot{V}_i(t) &\leq \phi(t) \mathfrak{s}_i \bar{x}_1^T(t) P_i(t) \bar{x}_1(t) + \bar{x}_1^T(t) \frac{\phi(t)}{t_{k+1} - t_k} [P_{i1} - P_{i2}] \bar{x}_1(t) \\ &\quad + 2\phi(t) \bar{x}_1^T(t) P_i(t) [A_{11}(i) - A_{12}(i) A_{22}^{-1}(i) A_{21}(i)] \bar{x}_1(t) \\ &\leq \phi(t) \bar{x}_1^T(t) \left[\mathfrak{s}_i P_i(t) + P_i(t) \hat{A}(i) + \hat{A}^T(i) P_i(t) + \rho_1(t) (P_{i1} - P_{i2}) \right] \bar{x}_1(t) \\ &\leq \phi(t) \bar{x}_1^T(t) \left[\mathfrak{s}_i (\rho(t) P_{i1} + \tilde{\rho}(t) P_{i2}) + (\rho(t) P_{i1} + \tilde{\rho}(t) P_{i2}) \hat{A}(i) \right. \\ &\quad \left. + \hat{A}^T(i) (\rho(t) P_{i1} + \tilde{\rho}(t) P_{i2}) + 1/\tau_1 (P_{i1} - P_{i2}) \right] \bar{x}_1(t) \\ &= \phi(t) \bar{x}_1^T(t) \left[\rho(t) \mathfrak{s}_i P_{i1} + \rho(t) P_{i1} \hat{A}(i) + \mathfrak{s}_i \tilde{\rho}(t) P_{i2} + \tilde{\rho}(t) P_{i2} \hat{A}(i) \right. \\ &\quad \left. + \rho(t) \hat{A}^T(i) P_{i1} + \tilde{\rho}(t) \hat{A}^T(i) P_{i2} + \rho(t)/\tau_1 (P_{i1} - P_{i2}) + \tilde{\rho}(t)/\tau_1 (P_{i1} - P_{i2}) \right] \bar{x}_1(t) \\ &= \phi(t) \rho(t) \bar{x}_1^T(t) \left[\mathfrak{s}_i P_{i1} + P_{i1} \hat{A}(i) + \hat{A}^T(i) P_{i1} + 1/\tau_1 (P_{i1} - P_{i2}) \right] \bar{x}_1(t) \\ &\quad + \phi(t) \tilde{\rho}(t) \bar{x}_1^T(t) \left[\mathfrak{s}_i P_{i2} + P_{i2} \hat{A}(i) + \hat{A}^T(i) P_{i2} + 1/\tau_1 (P_{i1} - P_{i2}) \right] \bar{x}_1(t) \\ &\leq -\eta_i \phi(t) \bar{x}_1^T(t) (\rho(t) P_{i1} + \rho(t) P_{i2}) \bar{x}_1(t) \\ &= -\eta_i V_i(t), \end{aligned} \tag{13}$$

where $\hat{A}(i) = [A_{11}(i) - A_{12}(i) A_{22}^{-1}(i) A_{21}(i)]$.

From (5) and (6), one has that

$$\begin{aligned} \bar{x}_1(t_k^+) &= [I \ 0] \Gamma_{i_k i_{k-1}} \bar{x}(t_k^-) = [I \ 0] \Gamma_{i_k i_{k-1}} \begin{bmatrix} I \\ -A_{22}^{-1}(i_{k-1}) A_{21}(i_{k-1}) \end{bmatrix} \bar{x}_1(t_k^-) \\ &= \Pi_{i_k i_{k-1}} \bar{x}_1(t_k^-). \end{aligned} \tag{14}$$

According to Schur complement lemma, one has that (9) is equivalent to

$$-\mu P_{i_2 1} + \mu_{i_1}^{-1} \Pi_{i_1 i_2}^T P_{i_1 2} \Pi_{i_1 i_2} \leq 0. \tag{15}$$

Let $V(t) = \sum_{k=1}^m \theta_k(t) V_k(t, x)$ with $\theta_k(t) = \begin{cases} 1, & \text{if } \sigma(t) = k, \\ 0, & \text{otherwise.} \end{cases}$ Therefore,

$$\begin{aligned} V(t_k^+) &= \mu_{i_k}^{\rho(t_k^+)-1} \bar{x}_1^T(t_k^+) [\rho(t_k^+) P_{i_k 1} + \tilde{\rho}(t_k^+) P_{i_k 2}] \bar{x}_1(t_k^+) \\ &= \mu_{i_k}^{-1} \bar{x}_1^T(t_k^+) P_{i_k 2} \bar{x}_1(t_k^+) \\ &= \mu_{i_k}^{-1} \bar{x}_1^T(t_k^-) \Pi_{i_k i_{k-1}}^T P_{i_k 2} \Pi_{i_k i_{k-1}} \bar{x}_1(t_k^-) \\ &\leq \bar{x}_1^T(t_k^-) \mu P_{i_{k-1} 1} \bar{x}_1(t_k^-) \\ &= \mu \bar{x}_1^T(t_k^-) P_{i_{k-1} 1} \bar{x}_1(t_k^-) \\ &= \mu \bar{x}_1^T(t_k^-) [\rho(t_k^-) P_{i_{k-1} 1} + \tilde{\rho}(t_k^-) P_{i_{k-1} 2}] \bar{x}_1(t_k^-) \\ &= \mu \phi(t_k^-) \bar{x}_1^T(t_k^-) P_{i_{k-1}}(t_k^-) \bar{x}_1(t_k^-) \\ &= \mu V(t_k^-) \\ &\leq V(t_k^-). \end{aligned} \tag{16}$$

It is obvious that $V(t) \leq e^{-\eta_{\min} t} V(0)$ with $\eta_{\min} = \min \{\eta_1, \eta_2, \dots, \eta_m\}$, which implies that $\bar{x}_1(t)$ converges exponentially to zero. Since $\bar{x}_2(t) = -A_{22}^{-1}(\sigma(t)) A_{21}(\sigma(t)) \bar{x}_1(t)$ in (5), then $\bar{x}_2(t)$ also converges exponentially to zero. This indicates that system (1) is GES under any switching signal $\sigma(t) \in D_{\tau_1}$.

Remark 3.1. *In many actual applications, the minimum value of τ_1 is of interest. With fixed parameters $\mu_i > 1$ and $\eta_i > 0, \forall i \in M$, it can be obtained through following optimization procedure*

$$\begin{aligned} &\min \tau_1 \\ &\text{s.t. (7), (8) and (9).} \end{aligned}$$

4. Example. In this section, we present a numerical example to demonstrate the effectiveness of the proposed design method.

Consider the switched singular system (1) with

$$\begin{aligned} E_1 &= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -2 & 0 & -1 \\ 0 & -2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -3 & 1 & 3 \\ 1 & -3 & 2 \\ -1 & 1 & -3 \end{bmatrix}. \end{aligned}$$

Let $H_1 = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}, H_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}, N_1 = N_2 = I_3$. Then, by (2),

we obtain that $\bar{E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \bar{A}_1 = \begin{bmatrix} -2 & 0 & -1 \\ 2 & -2 & 0 \\ 0 & 1 & 2 \end{bmatrix}, \bar{A}_2 = \begin{bmatrix} -3 & 1 & 3 \\ 1 & -3 & 2 \\ 1 & 3 & -8 \end{bmatrix},$

$$\Pi_{12} = \Pi_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Let $\tau_1 = 0.05$, $\mu_1 = 1.02$, $\mu_2 = 1.02$, $\eta_1 = 0.01$, $\eta_2 = 0.01$. By Theorem 3.1, we obtain

$$P_{11} = \begin{bmatrix} 6.6208 & 0 \\ 0 & 6.6208 \end{bmatrix}, \quad P_{12} = \begin{bmatrix} 6.4937 & 0 \\ 0 & 6.4937 \end{bmatrix},$$

$$P_{21} = \begin{bmatrix} 5.4381 & 0 \\ 0 & 5.4381 \end{bmatrix}, \quad P_{22} = \begin{bmatrix} 5.3704 & 0 \\ 0 & 5.3704 \end{bmatrix}.$$

The simulation is carried out with the initial state $x(0) = [-2.3, -1.8, -3.6]^T$. Figure 1(a) shows the state trajectories of the switched system (1) under a randomly chosen switching signal of time, shown in Figure 1(b). By using Theorem 3.1, it can be clearly observed from Figure 1(a) that the exponential stability has been achieved. Thus, the simulation results well illustrate the effectiveness of the proposed method.

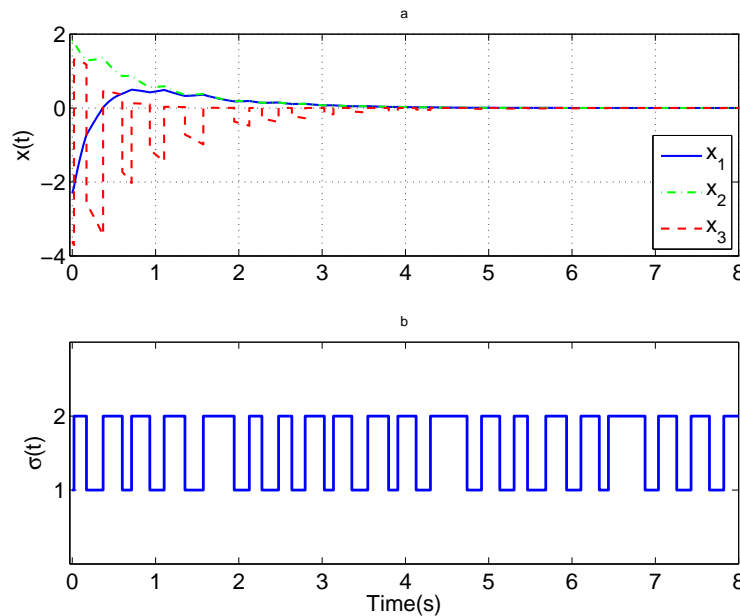


FIGURE 1. State trajectories of $x(t)$ and switching signal $\sigma(t)$

5. Conclusions. The issue of GES analysis for a class of switched linear singular systems with dwell time specifications has been studied in this paper. Unlike the classical dwell time method, the dwell time is an arbitrarily prespecified constant, which is not computed by Lyapunov functions of the subsystems. A novel method to construct multiple time-varying Lyapunov functions has been proposed to analyze the GES for a class of switched singular systems under dwell time specifications. Finally, an example is presented to demonstrate the effectiveness of the proposed method. How to design observers for switched singular systems will be an interesting topic for future research.

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