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ADAPTIVE FUZZY OUTPUT-FEEDBACK CONTROL FOR A SINGLE-LINK FLEXIBLE ROBOT MANIPULATOR DRIVEN DC MOTOR

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Abstract. *This paper deals with the problem of adaptive fuzzy output-feedback control for a single-link robotic manipulator coupled to a brushed direct current (DC) motor. The controlled system contains actuator saturation and unmeasured states. To address output constraint and input constraint, a barrier Lyapunov function and an auxiliary design system are employed, respectively. Fuzzy logic systems are used to approximate the unknown nonlinear functions. Stability proof of the overall closed-loop system is given via the Lyapunov method. The control method can guarantee that all the signals in the closed-loop system are bounded and the system output can follow the given bounded reference signal.*
Keywords: Single-link flexible-joint robot, Fuzzy adaptive control, Stability analysis, Backstepping technique, Output constraints, Input saturation

1. **Introduction.** In the past years, the robust control model of robotic manipulators with joint flexibilities has attracted a lot of attention, and some effective control approaches have been proposed for the control of robotic manipulators [1-4], such as the adaptive sliding mode technique [2], the passivity approach in [3], and the proportional-derivative control approach [4]. While all the abovementioned results are based on state feedback control methodology. As we know, adaptive output-feedback control has been studied by many authors [5,6]. In [6] the authors proposed an adaptive neural output feedback control scheme for uncertain nonlinear systems subject to unknown hysteresis, external disturbances and unmeasured states. In practical robotic systems, the velocity measurements obtained through the tachometers are easily perturbed by noises. Therefore, in order to make the economic or physical constraints reduced to a minimum, the design of controlling flexible-joint robots without velocity measurements is important. By designing observer we can overcome the problem of velocity unmeasured in flexible-joint robot motion.

In addition, saturation is one of very important non-smooth nonlinearities arisen in actuator, and it can reduce the performance of a control system and sometimes even leads to the unsteadiness of the control systems. In resent years, based on the backstepping control design technique, some methods have been developed to solve the problem of actuator saturation [7,8]. In [8] the state feedback adaptive control method for a class of uncertain nonlinear systems with input saturation and external disturbance was proposed. However, the above-mentioned approaches require that the states variables are available for measurement, which limits the applicability of these control schemes in practical engineering. To the best of our knowledge, there are no results on the single-link robotic manipulator of velocity unmeasured with actuator saturation, which motivates us for this study. In this paper, the design of an adaptive fuzzy controller for a class of robot manipulators with input constraints and unmeasured states is addressed.

This paper is organized as follows. In Section 2, the problem statement and preliminaries of a single-link robotic manipulator are introduced. In Section 3, fuzzy state observer is designed. In Section 4, an adaptive fuzzy output feedback control method is presented. In Section 5 the stability analysis is given and the simulation illustrates the effectiveness of the proposed results. Finally, Section 6 draws the conclusions.

2. **Problem Statement and Preliminaries.** A single-link robotic manipulator coupled to a brushed direct current motor with a nonrigid joint is considered. The schematic diagram is depicted in Figure 1. When the joint is modeled as a linear torsional spring [9], from the Euler Lagrange equation, the equations of motion for such an electromechanical system can be derived as

$$\begin{aligned} &J_1\ddot{q}_1 + F_1\dot{q}_1 + K\left(q_1 - \frac{q_2}{N}\right) + mgd\cos q_1 = 0 \\ &J_2\ddot{q}_2 + F_2\dot{q}_2 - \frac{K}{N}\left(q_1 - \frac{q_2}{N}\right) = K_t i \\ &LDi + Ri + K_b\dot{q}_2 = u \end{aligned} \tag{1}$$

where q_1 and q_2 are the angular positions of the link and the motor shaft, i is the armature current, and u is the armature voltage. The inertias J_1, J_2, the viscous friction constants F_1, F_2, the spring constant K, the torque constant K_t, the back-emf constant K_b, the armature resistance R and inductance L, the link mass M, the position of the link's center of gravity d, the gear ratio N and the acceleration of gravity g can all be unknown.

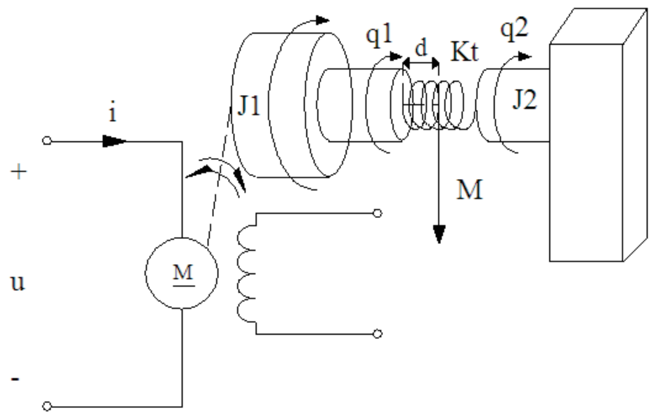

FIGURE 1. Single-link flexible-joint robot

Assume that only the link position q_1 is measured and $K_tK = J_1J_2NL$. We first try choice of the state variables $\xi_1 = q_1$, $\xi_2 = \dot{q}_1$, $\xi_3 = q_2$, $\xi_4 = \dot{q}_2$, $\xi_5 = i$. The dynamic Equation (1) becomes

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= -\frac{mgd}{J_1}\cos y - \frac{F_1}{J_1}\xi_2 - \frac{K}{J_1}\left(\xi_1 - \frac{\xi_3}{N}\right) \\ \dot{\xi}_3 &= \xi_4 \\ \dot{\xi}_4 &= \frac{K}{J_2N}\left(\xi_1 - \frac{\xi_3}{N}\right) - \frac{F_2}{J_2}\xi_4 - \frac{K_t}{J_2}\xi_5 \\ \dot{\xi}_5 &= -\frac{R}{L}\xi_5 - \frac{K_b}{L}\xi_4 - \frac{1}{L}u \\ y &= \xi_1 \end{aligned} \tag{2}$$

Denote $\xi_2 = Dy$ ($D = \frac{d}{dt}$ is the differentiation operator).

$$D^2 y = -\frac{mgd}{J_1}\cos y - \frac{F_1}{J_1}Dy - \frac{K}{J_1}\left(y - \frac{\zeta_3}{N}\right) \tag{3}$$

which implies that

$$\xi_3 = \frac{J_1 N}{K}\left(D^2 y + \frac{mgd}{J_1}\cos y + \frac{F_1}{J_1}Dy + \frac{K}{J_1}y\right) \tag{4}$$

$$\xi_4 = D\xi_3 = \frac{J_1 N}{K}\left(D^3 y + \frac{mgd}{J_1}D\cos y + \frac{F_1}{J_1}D^2 y + \frac{K}{J_1}Dy\right) \tag{5}$$

Differentiating (5) and substituting ξ_3 and ξ_4 from (4) and (5), one obtains

$$\begin{aligned}\xi_5 = \frac{J_1 J_2 N}{K_t K}\Bigg[&D^4 y + \left(\frac{F_1}{J_1} + \frac{F_2}{J_2}\right)D^3 y + \left(\frac{K}{J_1} + \frac{K}{J_2 N^2} + \frac{F_1 F_2}{J_1 J_2}\right)D^2 y + \frac{mgd}{J_1}D^2\cos y \\ &+ \left(\frac{F_1 K}{J_1 J_2 N^2} + \frac{F_2 K}{J_1 J_2}\right)Dy + \frac{mgdF_2}{J_1 J_2}D\cos y + \frac{mgdK}{J_1 J_2 N^2}\cos y\Bigg]\end{aligned} \tag{6}$$

Finally, differentiating (6) and substituting ξ_4 and ξ_5 from (4) and (5), we arrive at

$$\begin{aligned}D^5 y = u &- \left(\frac{R}{L} + \frac{F_1}{J_1} + \frac{F_2}{J_2}\right)D^4 y - \frac{mgd}{J_1}D^3\cos y \\ &- \left[\frac{R}{L}\left(\frac{F_1}{J_1} + \frac{F_2}{J_2}\right) + \frac{K_b}{L}\frac{K_t}{J_2} + \left(\frac{K}{J_1} + \frac{K}{J_2 N^2} + \frac{F_1 F_2}{J_1 J_2}\right)\right]D^3 y \\ &- \left[\frac{R}{L}\left(\frac{K}{J_1} + \frac{K}{J_2 N^2} + \frac{F_1 F_2}{J_1 J_2}\right) + \frac{F_1 K}{J_1 J_2 N^2} + \frac{F_2 K}{J_1 J_2} + \frac{K_b}{L}\frac{F_1 K_t}{J_1 J_2}\right]D^2 y \\ &- \left(\frac{R}{L} + \frac{F_2}{J_2}\right)\frac{mgd}{J_1}D^2\cos y - \left[\frac{R}{L}\left(\frac{F_1 K}{J_1 J_2 N^2} + \frac{F_2 K}{J_1 J_2}\right) + \frac{K_b}{L}\frac{F_1 K_t}{J_1 J_2}\right]Dy \\ &- \left(\frac{K}{N^2} + \frac{RF_2}{L} + \frac{K_b K_t}{L}\right)\frac{mgd}{J_1 J_2}D\cos y - \frac{R}{L}\frac{mgdK}{J_1 J_2 N^2}\cos y\end{aligned} \tag{7}$$

From (7), one has

$$\begin{aligned}\dot{x}_1 &= x_2 - \left(\frac{R}{L} + \frac{F_1}{J_1} + \frac{F_2}{J_2}\right)y \\ \dot{x}_2 &= x_3 - \left[\frac{R}{L}\left(\frac{F_1}{J_1} + \frac{F_2}{J_2}\right) + \frac{K_b}{L}\frac{K_t}{J_2} + \left(\frac{K}{J_1} + \frac{K}{J_2 N^2} + \frac{F_1 F_2}{J_1 J_2}\right)\right]y - \frac{mgd}{J_1}\cos y \\ \dot{x}_3 &= x_4 - \left[\frac{R}{L}\left(\frac{K}{J_1} + \frac{K}{J_2 N^2} + \frac{F_1 F_2}{J_1 J_2}\right) + \frac{F_1 K}{J_1 J_2 N^2} + \frac{F_2 K}{J_1 J_2} + \frac{K_b}{L}\frac{F_1 K_t}{J_1 J_2}\right]y \\ &\quad - \left(\frac{R}{L} + \frac{F_2}{J_2}\right)\frac{mgd}{J_1}\cos y \\ \dot{x}_4 &= x_5 - \left[\frac{R}{L}\left(\frac{F_1 K}{J_1 J_2 N^2} + \frac{F_2 K}{J_1 J_2}\right) + \frac{K_b}{L}\frac{K K_t}{J_1 J_2}\right]y - \left(\frac{K}{N^2} + \frac{RF_2}{L} + \frac{K_b K_t}{L}\right)\frac{mgd}{J_1 J_2}\cos y \\ \dot{x}_5 &= u(v) - \frac{R}{L}\frac{mgdK}{J_1 J_2 N^2}\cos y \\ y &= x_1\end{aligned} \tag{8}$$

The output y is required to remain in the set $|y| \leq k_{c1}$ (output constraint), $\forall t \geq 0$, where k_{c1} is a positive constant. v is the controller input to be designed, and $u(v(t))$ denotes the plant input subject to saturation type nonlinearly. Throughout this paper, it is assumed that the only output y is available for measurement.

$u(v(t))$ is described by

$$u(v(t)) = sat(v(t)) = \begin{cases} sign(v(t))u_M, & |v(t)| \geq u_M \\ v(t), & |v(t)| < u_M \end{cases} \tag{9}$$

where u_M is the bound of $u(t)$. Clearly, the relationship between the applied control $u(t)$ and the control input $v(t)$ has a sharp corner when $|v(t)| = u_M$. The saturation can be approximated by a smooth function defined as

$$g(v) = u_M \times \tanh\left(\frac{v}{u_M}\right) = u_M \frac{e^{v/u_M} - e^{-v/u_M}}{e^{v/u_M} + e^{-v/u_M}} \tag{10}$$

Then $sat(v(t))$ in (9) can be expressed as

$$sat(v) = g(v) + \rho(v) = u_M \times \tanh\left(\frac{v}{u_M}\right) + \rho(v) \tag{11}$$

where $\rho(v) = sat(v) - g(v)$ is a bounded function in time and its bound can be obtained as

$$|\rho(v)| = |sat(v) - g(v)| \leq u_M(1 - \tanh(1)) = D_1 \tag{12}$$

Note that in the section $0 \leq |v| \leq u_M$ the bound $\rho(v)$ increases from 0 to D_1 as $|v|$ changes from 0 to u_M, and outside this range the bound $\rho(v)$ decreases from D_1 to 0.

Control objective: The control objective is to design an adaptive fuzzy output feedback controller based on actuator saturation to make the angular positions y follows a desired trajectory, as well as guarantee the boundness of all the signals of the closed-loop system.

3. Fuzzy State Observer Design.

Note that the state x_2, x_3, x_4, x_5 in system (1) are not available for feedback; therefore, a state observer should be established.

By employing the fuzzy logic systems to approximate the unknown functions $-\left(\frac{R}{L} + \frac{F_1}{J_1} + \frac{F_2}{J_2}\right)y$, $-\left[\frac{R}{L}\left(\frac{F_1}{J_1} + \frac{F_2}{J_2}\right) + \frac{K_b}{L}\frac{K_t}{J_2} + \left(\frac{K}{J_1} + \frac{K}{J_2N^2} + \frac{F_1F_2}{J_1J_2}\right)\right]y - \frac{mgd}{J_1}\cos y$, $-\left[\frac{R}{L}\left(\frac{K}{J_1} + \frac{K}{J_2N^2} + \frac{F_1F_2}{J_1J_2}\right) + \frac{F_1K}{J_1J_2N^2} + \frac{F_2K}{J_1J_2} + \frac{K_b}{L}\frac{F_1K_t}{J_1J_2}\right]y - \left(\frac{R}{L} + \frac{F_2}{J_2}\right)\frac{mgd}{J_1}\cos y$, $-\left[\frac{R}{L}\left(\frac{F_1K}{J_1J_2N^2} + \frac{F_2K}{J_1J_2}\right) + \frac{K_b}{L}\frac{KK_t}{J_1J_2}\right]y - \left(\frac{K}{N^2} + \frac{RF_2}{L} + \frac{K_bK_t}{L}\right)\frac{mgd}{J_1J_2}\cos y$, $-\frac{R}{L}\frac{mgdK}{J_1J_2N^2}\cos y$, respectively, system (8) can be expressed as

$$\begin{cases} \dot{x}_1 = x_2 + \theta_1^{*T}\varphi_1(y) + \varepsilon_1 \\ \dot{x}_2 = x_3 + \theta_2^{*T}\varphi_2(y) + \varepsilon_2 \\ \dot{x}_3 = x_4 + \theta_3^{*T}\varphi_3(y) + \varepsilon_3 \\ \dot{x}_4 = x_5 + \theta_4^{*T}\varphi_4(y) + \varepsilon_4 \\ \dot{x}_1 = u(v) + \theta_5^{*T}\varphi_5(y) + \varepsilon_5 \\ y = x_1 \end{cases} \tag{13}$$

where $|\varepsilon_i| \leq \varepsilon_i^*$, ε_i^* is an unknown constant.

In this paper, a fuzzy state observer is designed for (8) as follows

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + ky + \bar{F} + E_n g(v) \\ y = E_1^T \hat{x} \end{cases} \tag{14}$$

where $\hat{x} = [\hat{x}_1, \ldots, \hat{x}_5]^T$, $A = \begin{bmatrix} -k_1 & \\ \vdots & I_4 \\ -k_5 & \ldots \; 0 \end{bmatrix}$, $\bar{F} = [\theta_1^T\varphi_1(y), \ldots, \theta_5^T\varphi_5(y)]^T$, $k = [k_1, \ldots, k_5]^T$, $E_n^T = [0\ 0\ 0\ 0\ 1]$ and $E_1^T = [1\ 0\ 0\ 0\ 0]$.

The coefficient k_i is selected such that the polynomial $p(s) = s^5 + k_1s^4 + \cdots + k_5$ is a Hurwitz. Thus for a given $Q^T = Q > 0$, there exists a positive definite matrix $P^T = P > 0$ such that

$$A^TP + PA = -Q \tag{15}$$

Let $e = x - \hat{x} = [e_1, \ldots, e_5]^T$ be observer error, we have the observer error equation.

$$\dot{e} = Ae + \tilde{\Theta} + E_5\rho(v) + \varepsilon \tag{16}$$

where $\tilde{\Theta} = \left[\tilde{\theta}_1^T\varphi_1(y), \ldots, \tilde{\theta}_5^T\varphi_5(y)\right]^T$, $\varepsilon = [\varepsilon_1, \ldots, \varepsilon_5]^T$ and $\tilde{\theta}_i^* = \theta_i^* - \theta_i$, $i = 1, 2, \ldots, 5$.

4. **Adaptive Fuzzy Control Design.** In this section, in order to achieve the control objective, an adaptive fuzzy output-feedback controller and parameter adaptive laws are developed based on the backstepping design technique.

Step 1: Define the first error surface z_1 as

$$z_1 = y - y_r \tag{17}$$

Expressing x_2 in terms of its estimate $x_2 = \hat{x}_2 + e_2$, and substituting (13) into (17), we have

$$\begin{aligned} \dot{z}_1 &= \hat{x}_2 + e_2 + \theta_1^{*T}\varphi_1(y) + \varepsilon_1 - \dot{y}_r \\ &= z_2 + \alpha_1 + e_2 + \theta_1^T\varphi_1(y) - \tilde{\theta}_1^T\varphi_1(y) + \varepsilon_1 - \dot{y}_r \end{aligned} \tag{18}$$

Choose the first virtual control function α_1, and the parameter adaptive law θ_1 as

$$\alpha_1 = -\frac{5z_1}{4(k_{b1}^2 - z_1^2)} - c_1z_1 + \dot{y}_r - \theta_1^T\varphi_1(y) \tag{19}$$

$$\dot{\theta}_1 = \gamma_1\varphi_1(y)\frac{z_1}{k_{b1}^2 - z_1^2} - \sigma_1\theta_1 \tag{20}$$

where $c_1 > 0$ and $\sigma_1 > 0$ are design parameters.

Step i ($i = 2, 3, 4$): Define the ith error surface z_i as

$$z_i = \hat{x}_i - \alpha_{i-1} \tag{21}$$

Choose the ith virtual control function α_i and parameter adaptive law θ_i as follows.

$$\alpha_2 = -z_2 - c_2z_2 - \frac{1}{2}z_2 - H_2 - \frac{3}{2}\left(\frac{\partial\alpha_1}{\partial y}\right)^2 z_2 \tag{22}$$

$$\alpha_j = -z_{j-1} - c_jz_j - \frac{1}{2}z_j - H_j - \frac{3}{2}\left(\frac{\partial\alpha_{j-1}}{\partial y}\right)^2 z_j,\ j = 3, 4 \tag{23}$$

$$\dot{\theta}_i = \gamma_iz_i\varphi_i(y) - \sigma_i\theta_i,\ i = 2, 3, 4 \tag{24}$$

where $c_j > 0$ and $\sigma_i > 0$ are design parameters,

$$\begin{aligned} H_i = {} & \theta_i^T\varphi_i + k_ie_1 - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\dot{x}_k - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\theta_k}\dot{\theta}_k - \sum_{k=1}^{i}\frac{\partial\alpha_{i-1}}{\partial y_r^{(k-1)}}y_r^{(k)} \\ & -\frac{\partial\alpha_{i-1}}{\partial y}(\hat{x}_2 + \theta_1^T\varphi_1(y)), \quad i = 2, 3, 4. \end{aligned}$$

Step 5: In the last step, define the 5th error surface z_5 as

$$z_5 = \hat{x}_5 - \alpha_4 - \hbar \tag{25}$$

where $\hbar$ is an auxiliary design signal, which is obtained from the following dynamic system: $\dot{\hbar} = -\hbar + (g(v) - v)$.

Choose the actual control input v and parameter adaptive law θ_5 as

$$v = -c_5 z_5 - \frac{3}{2}\left(\frac{\partial \alpha_4}{\partial y}\right)^2 z_5 - \frac{1}{2} z_5 - z_4 - \hbar - H_5 \tag{26}$$

$$\dot{\theta}_5 = \gamma_5 z_5 \varphi_5(y) - \sigma_5 \theta_5 \tag{27}$$

where $c_5 > 0$ and $\sigma_5 > 0$ are design parameters.

5. **Stability Analysis.** In this section, the boundedness of all the signals in the closed-loop system will be verified. The main result is summarized in the following theorem.

Theorem 5.1. *For a single-link robotic manipulator system (8) with unmeasured states, the controller (26), state observer (14), together with the virtual control functions (19), (22) and (23), parameter adaptive laws (20), (24) and (27), guarantee that all signals of the closed-loop system are bounded, and the tracking errors converge to a small neighborhood of zero by suitably choosing the design parameters.*

Proof: Consider the Lyapunov function candidate

$$V = e^T P e + \frac{1}{2}\log\frac{k_{b1}^2}{k_{b1}^2 - z_1^2} + \frac{1}{2}\sum_{i=2}^{5} z_i^2 + \sum_{i=1}^{5}\frac{1}{2\gamma_i}\tilde{\theta}_i^T\tilde{\theta}_i \tag{28}$$

The time derivative of V is

$$\dot{V} = -e^T Q e + e^T P(\varepsilon + \Theta + E_n \rho(v)) + \frac{z_1 \dot{z}_1}{k_{b1}^2 - z_1^2} + \sum_{i=2}^{5} z_i \dot{z}_i - \sum_{i=1}^{n}\frac{1}{\gamma_i}\tilde{\theta}_i^T\dot{\theta}_i \tag{29}$$

By using Young's inequality, we have

$$2e^T P\varepsilon + 2e^T P\rho(v) \le 2\|e\|^2 + M_1 \tag{30}$$

$$2e^T P\tilde{\Theta} \le \|e\|^2 + \|P\|^2 \sum_{k=1}^{5}\tilde{\theta}_5^T\theta_5^T \tag{31}$$

where $M_1 = \|P\|^2 D_1^2 + \|P\|^2\|\varepsilon^*\|^2$. From (30) and (31), we have

$$\begin{aligned}
\dot{V} \le & -(\lambda_{\min}(Q) - 3)\|e\|^2 + \|P\|^2\sum_{i=1}^{5}\tilde{\theta}_i^T\theta_i^T + M_1 \\
& + \frac{z_1}{k_{b1}^2 - z_1^2}\left(z_2 + \alpha_1 + e_2 + \theta_1^T\varphi_1(y) - \tilde{\theta}_1^T\varphi_1(y) + \varepsilon_1 - \dot{y}_r\right) \\
& + \sum_{i=2}^{4} z_i\left(z_{i+1} + \alpha_i + \tilde{\theta}_i^T\varphi_i(y) - \tilde{\theta}_i^T\varphi_i(y) + H_i - \frac{\partial\alpha_{i-1}}{\partial y}\left(e_2 + \tilde{\theta}_1^T\varphi_1(y) + \varepsilon_1\right)\right) \\
& + z_5\left(g(v) + \tilde{\theta}_5^T\varphi_5(y) - \tilde{\theta}_5^T\varphi_5(y) + H_5 - \frac{\partial\alpha_{i-1}}{\partial y}\left(e_2 + \tilde{\theta}_1^T\varphi_1(y) + \varepsilon_1\right) - \dot{\hbar}\right) \\
& - \sum_{i=1}^{n}\frac{1}{\gamma_i}\tilde{\theta}_i^T\dot{\theta}_i
\end{aligned} \tag{32}$$

Substituting (23) and (26) into (32) yields, and using the Young's inequality, we have

$$\begin{aligned}
\dot{V} \le & -\left(\lambda_{\min}(Q) - \frac{11}{2}\right)\|e\|^2 + \|P\|^2\sum_{i=1}^{5}\tilde{\theta}_i^T\tilde{\theta}_i + M_1 \\
& - \frac{c_1 z_1^2}{k_{b1}^2 - z_1^2} - \sum_{i=1}^{5}\frac{\sigma_i}{2\gamma_i}\tilde{\theta}_i^T\tilde{\theta}_i + \sum_{i=1}^{5}\frac{\sigma_i}{2\gamma_i}\|\theta_i^*\|^2 + \frac{5}{2}{\varepsilon_1^*}^2 - \sum_{i=2}^{5} c_i z_i^2 + 2\tilde{\theta}_1^T\tilde{\theta}_1 + \frac{1}{2}\sum_{i=2}^{5}\tilde{\theta}_i^T\tilde{\theta}_i
\end{aligned} \tag{33}$$

Then (33) becomes

$$\dot{V} \le -cV + D \tag{34}$$

where $c = \min\{(\lambda_{\min}(Q) - \frac{11}{2})/\lambda_{\min}(P), c_1, 2c_l, 2\gamma_1(\frac{\sigma_1}{2\gamma_1} - 2 - \|P\|), 2\gamma_l(\frac{\sigma_l}{2\gamma_l} - \frac{1}{2})\}$, $l = 2, \ldots, 5$.

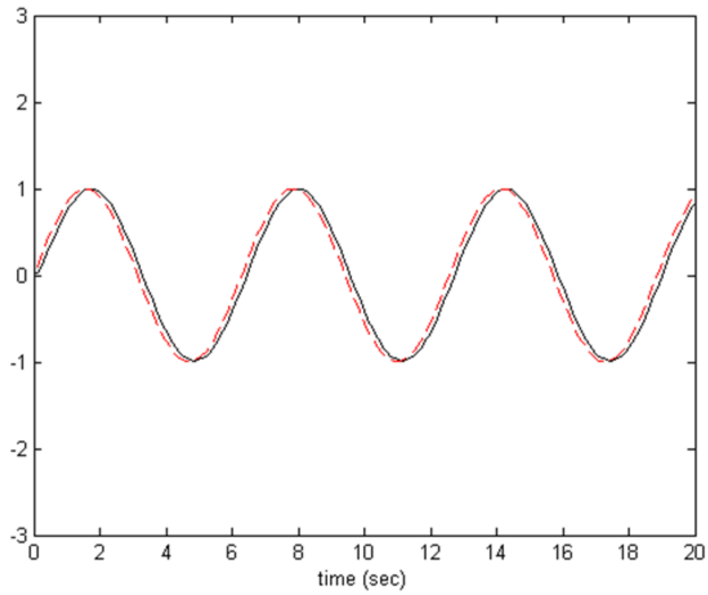

Figure 2. y (solid line) and y_r (dotted line)

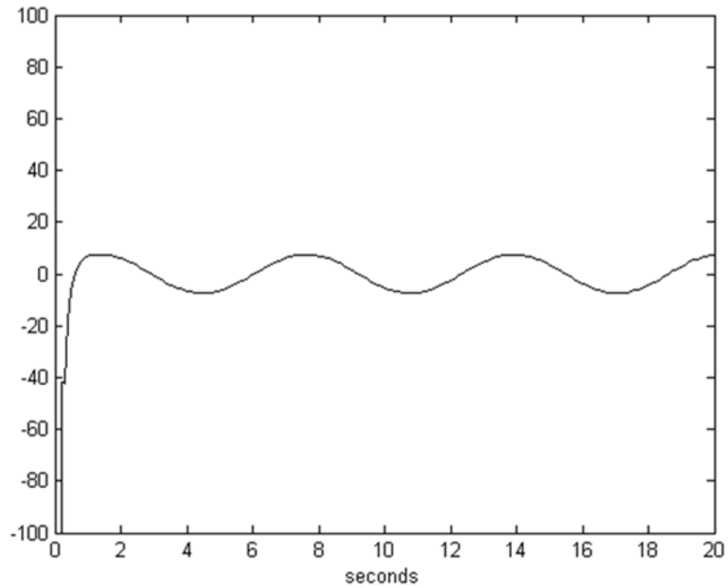

Figure 3. Input $u(v)$

The simulation results are shown in Figure 2 and Figure 3, where Figure 2 expresses the trajectories of the output and tracking signal; Figure 3 shows the trajectory of $u(v)$.

6. **Conclusions.** In this paper, the adaptive fuzzy controller design has been investigated for a class of robot manipulators with input constraints and unmeasured states. By utilizing the backstepping technique and the designed state observer, a new adaptive fuzzy control scheme has been developed. It has been proved all signals of the closed-loop system are bounded. The proposed control algorithm can not only solve the problems of input saturation and output constraints, but also can solve the problem for a single-link robotic manipulator with immeasurable states.

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