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A PRACTICAL DECISION-MAKING MODEL FOR DYNAMIC BILATERAL BARGAINING IN COOPERATIVE SPECTRUM SHARING

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Abstract. *In this paper, the problem of how to make decisions without any prior private information of both the primary user (PU) and the secondary user (SU) (both called decision-makers) during the dynamic bilateral bargaining process in cooperative spectrum sharing networks has been studied. A practical decision-making model (PDMM) has been proposed. In order to explain PDMM more clearly, firstly, an one-stage multi-slot bilateral bargaining model (OMBBM) has been given, where only the PU has a chance to propose a bid to the SU while the SU has no opportunity to propose a bid to the PU in each slot. Then, we have proposed PDMM, also called a multi-stage multi-slot bilateral bargaining model (MMBBM), to describe decision-makers' bidding strategies in the general case, where both decision-makers have a right to propose their own bids to each other for several times in each slot. An empirical belief updating rule has been used to update the accepted probabilities of the bids provided by both decision-makers at each stage based on the historical bidding data. The effects of PDMM can be shown by the simulation results.*
Keywords: Cooperative spectrum sharing networks, Bilateral bargaining, Incomplete information, Belief updating

1. **Introduction.** Cooperative spectrum sharing (CSS) is a very promising way to improve the spectrum efficiency [1], where cooperations between primary users (PUs) and secondary users (SUs) are necessary. However, due to the costs (e.g., battery energy expenditure) of relaying packets for PUs, SUs have no incentive to cooperate with PUs if SUs cannot obtain any compensation from the cooperations [2]. Therefore, a proper resource allocation mechanism is needed to satisfy the requirements of PUs and SUs in the cooperations. Adopting market mechanisms to study this resource allocation problem is a natural idea. As a result, there are some market mechanisms which have been introduced in several previous works about resource allocation problems in CSS [1, 2, 3, 4]. From the view of the number of decision-makers in CSS, the system models which consist of one PU and multiple SUs have been studied in [3, 4]. In [3], two auction mechanisms (the signal-noise ratio (SNR) auction and the power auction) have been proposed, the existences of Nash equilibrium for these two models have been proved. In [4], the interactions among one PU and multiple SUs have been modeled as a labor market using contract theory. The optimal contracts have been designed for both weakly and strongly incomplete information scenarios. However, in [1, 2], the authors consider that the general CSS with multiple PUs and multiple SUs could be decomposed into multiple pairs of one PU and one SU. This assumption is reasonable because each SU will only cooperate with PUs nearby for providing good relay services to these PUs [2]. Hence, under this assumption, non-cooperative bargaining theory is a very suitable mathematical tool to study the interaction between one PU and one SU in the cooperation, which is also a key point in this

paper. In addition, in [1, 2], a dynamic bargaining with incomplete information model between the PU and the SU has been proposed, where incomplete information means that the PU does not know the SU's energy cost but knows the probability distribution of the SU's energy cost. However, in practice, the PU and the SU may have no prior private information of its opponent's costs (including costs' probability distributions) and real profits. This practical case has not been considered in [1, 2]. Furthermore, this problem has not yet been mentioned in other previous works which are based on non-cooperative bargaining theory. In [5], the authors have analyzed the situation where the buyer's private information is uncertain to the seller, whereas the seller's private information is common knowledge. However, in some situations, the seller's private information cannot be known by the buyer. In [6], two decision-making models based on optimal stopping theory have been proposed on the side of the buyer, which means the seller's bargaining strategies have not been considered in these models. Therefore, we will propose a more practical decision-making model to analyze the interaction between the PU and the SU in the dynamic bilateral bargaining process.

In this paper, we assume decision-makers are rational and risk-neutral. This is a common assumption in decision-making models [2, 6], which means decision-makers will pursue their own maximal benefits and make decisions based on the expected profits. Then, PDMM for the dynamic bilateral bargaining process is proposed, where any prior private information of the decision-makers is not required. The remainder of this paper is organized as follows. The system model is described in Section 2. In Section 3, OMBBM firstly has been given and then PDMM has been proposed. In Section 4, an existing belief updating scheme has been used in our proposed models. The simulation results are shown in Section 5. Finally, Section 6 concludes this paper.

2. **System Model.** Consider a CSS network with one PU pair including PU's transmitter (PT) and receiver (PR) and one SU pair including SU's transmitter (ST) and receiver (SR). An illustration of the system model has been shown in Figure 1. We further assume the PU operates in the slot transmission structure and let T denote the total slots. In Figure 1, h_p, h_s, h_{ps} and h_{sp} denote the channel gains of the links $PT - PR$, $ST - SR$, $PT - ST$ and $ST - PR$, respectively. All channel gains are fixed in each slot but may be different in different slots (block-fading). In CSS, the PU can provide some spectrum resources for the SU and the SU will help the PU to improve its data rates in return. A bargaining process is used to decide how much spectrum resource (e.g., channel used time) the PU will give to the SU in their cooperation, which has multiple bargaining stages in each slot.

As a result, there are three cases in each slot: (a) if the PU estimates that bargaining with the SU cannot improve its data rate before the start of the bargaining process, it will transmit its data by itself (also called no bargaining); (b) the PU bargains with the SU for the relay and an agreement is not reached until the last stage (also called bargaining but failure); (c) the PU bargains with the SU for the relay and an agreement is reached at some

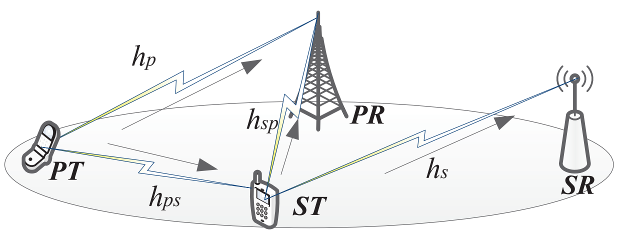


FIGURE 1. The cooperation between one PU pair and one SU pair

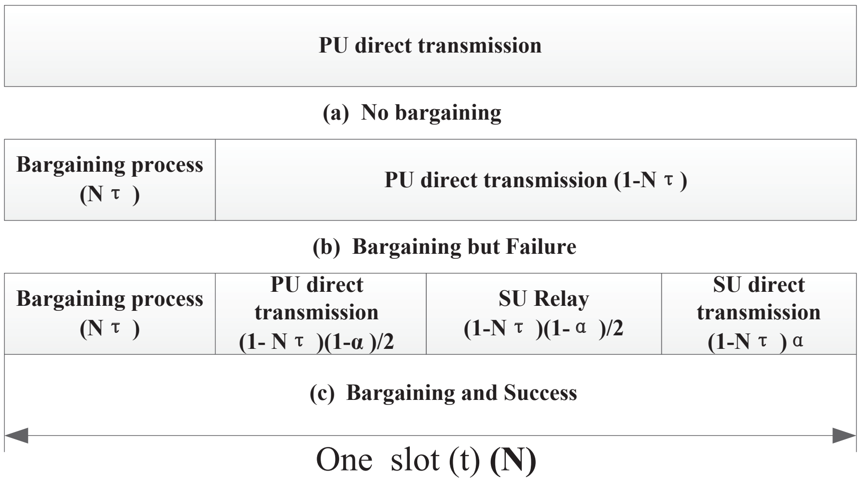


Figure 2. The three cases in each slot

stage (also called bargaining and success). An illustration of three cases is shown in Figure 2. In case (a) and case (b), the PU's transmission rate in slot t is $R_{dir}^t = \log_2(1 + P_t h_p^t)$; in case (c), under the cooperation with the SU, the PU's transmission rate in slot t is $R_r^t = \frac{1}{2}\log_2\left(1 + P_t h_p^t + \frac{P_t P_s h_{ps}^t h_{sp}^t}{P_t h_{ps}^t + P_s h_{sp}^t + 1}\right)$ [2]. The SU's transmission rate in slot t is $R_s^t = \log_2(1 + P_s h_s^t)$. Here, P_t and P_s denote the PU's transmitting power level and the SU's transmitting power level, respectively. The duration of each slot is normalized to be 1 and let N denote the maximal bargaining stages in each slot. τ ($\tau < 1$) denotes the overhead due to each bargaining stage.

3. Bilateral Bargaining Model under Incomplete Information.

In OMBBM, when the PU provides a bid to the SU and the SU has only two choices (to accept the bid or to reject the bid) in each slot. As a result, no matter which strategy the SU chooses, the bargaining process in this slot will end and the bargaining process in the next slot begins. In PDMM (also called MMBBM), both the PU and the SU are allowed to propose the bids for several times in each slot, which is the general case of OMBBM.

3.1. One-stage multi-slot bilateral bargaining model.

Before the start of the bargaining process, the PU needs to make a decision about whether or not to bargain with the SU. Assume h_p, h_{ps} and h_{sp} in each slot can be obtained by some proper feedback mechanism [2] before the PU making a decision. In OMBBM ($N = 1$), consider if the PU sends $\alpha_{p,1}(t)$ as a bid to the SU, the SU will decide to accept $\alpha_{p,1}(t)$ or reject it. Here, $\alpha_{p,1}(t)$ ($0 \leq \alpha_{p,1}(t) \leq 1$) denotes the fraction of the remaining transmission time in each slot, where $\alpha_{p,1}(t) = 0$ means the PU will not give transmission time to the SU and $\alpha_{p,1}(t) = 1$ means the PU will give the SU total remaining transmission time; for example, in OMBBM, the total remaining transmission time is $(1 - \tau)$. If the SU rejects $\alpha_{p,1}(t)$, the PU will transmit its data by itself (case (b)), which means the PU's profit is $(1 - \tau)R_{dir}^t$. If the SU accepts $\alpha_{p,1}(t)$, the PU's profit is $(1 - \tau)\left(1 - \alpha_{p,1}(t)\right) R_r^t$ (case (c)). Therefore, when the PU gives $\alpha_{p,1}(t)$ to the SU, the PU's expected profit of bargaining with the SU in slot t is

$$U_p^t(\alpha_{p,1}(t)) = (1-\tau)R_{dir}^t Prob_{PR}^{t,1}(\alpha_{p,1}(t)) + (1-\tau)(1-\alpha_{p,1}(t))R_r^t Prob_{PA}^{t,1}(\alpha_{p,1}(t)) \quad (1)$$

where $Prob_{PA}^{t,1}(\alpha_{p,1}(t))$ and $Prob_{PR}^{t,1}(\alpha_{p,1}(t))$ denote the accepted and rejected probability of $\alpha_{p,1}(t)$ by the SU in slot t, respectively. Obviously, $Prob_{PA}^{t,1}(\alpha_{p,1}(t)) = 1 - Prob_{PR}^{t,1}(\alpha_{p,1}(t))$. If the PU finds $\max(1) > R_{dir}^t$, the PU will send $\alpha_{p,1}^*(t) = \arg\max(1)$ to the SU for the maximal profits, which means bargaining with the SU maybe improve the PU's profits compared to the case (a). Otherwise, the PU has no need to bargain with the SU in slot t

(case (a)) and goes to the next slot $t+1$. In Section 4, we will discuss the problems of how to calculate $Prob_{PA}^{t,1}(\alpha_{p,1}(t))$ only using the bargaining historical data and find $\alpha_{p,1}^*(t)$ of (1) in detail. After the SU receiving $\alpha_{p,1}^*(t)$, the SU can calculate its own profit and decides to accept $\alpha_{p,1}^*(t)$ or reject it. The SU's profit is $u_s(\alpha_{p,1}^*(t)) = (1-\tau)(\alpha_{p,1}^*(t)R_s^t - \frac{1+\alpha_{p,1}^*(t)}{2}P_sC)$, where C is the SU's energy cost per watt. $\frac{1+\alpha_{p,1}^*(t)}{2}P_sC$ denotes the SU's all energy cost including the energy costs of relaying the PU's data and transmitting the SU's own data [2]. Note that if the PU does not send any bid to the SU, the SU will obtain nothing ($u_s = 0$). Therefore, in OMBBM, if and only if $\alpha_{p,1}^*(t)$ makes $u_s(\alpha_{p,1}^*(t)) > 0$, then the SU will accept $\alpha_{p,1}^*(t)$. Otherwise, the SU will reject $\alpha_{p,1}^*(t)$. Here, we also assume the SU knows h_s by a proper feedback mechanism [2].

3.2. **Multi-stage multi-slot bilateral bargaining model.** OMBBM is the foundation of understanding of MMBBM which is more complex than the former. Here, we assume the maximal bargaining stage in each slot is N $\left(N \leq \lfloor \frac{1}{\tau} \rfloor\right)$ (e.g., $1, 2, 3, \ldots, n, \ldots, N$) which is common knowledge to both the PU and the SU. The bargaining process in each slot will continue to the last stage unless an agreement is reached at some stage. An illustration of MMBBM is shown in Figure 3. Here, the PU must make a decision at the first stage in MMBBM. At first, we analyze this model from the PU's perspective at n ($n > 1$) stage. The PU will consider making a decision among three strategies when the PU

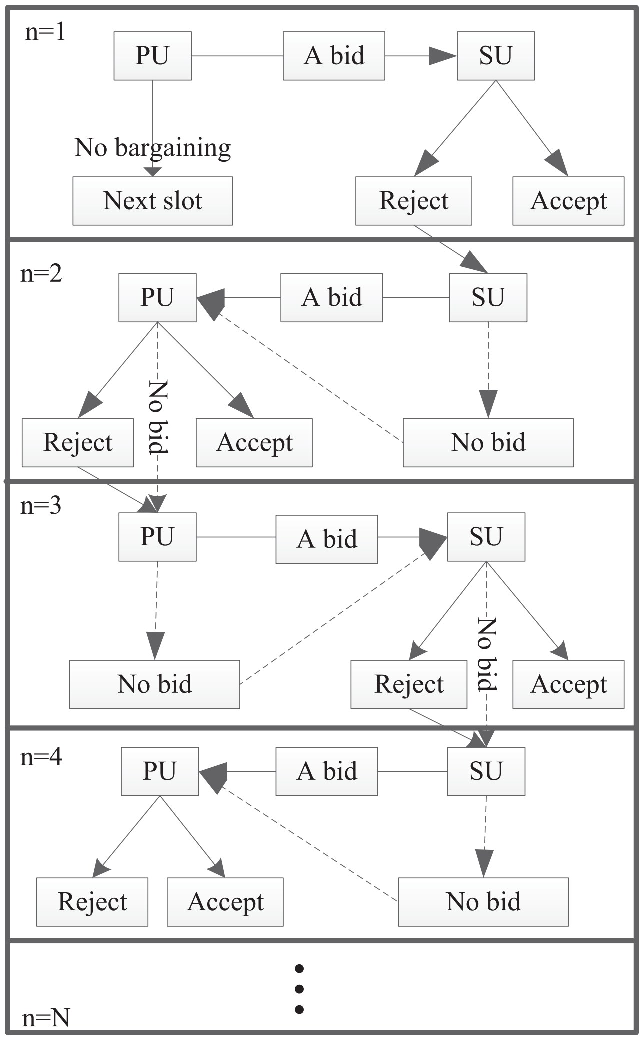


FIGURE 3. An illustration of MMBBM in each slot

receives the SU's bid $\alpha^*_{s,n-1}(t)$ at $n-1$ stage: (i) accepting $\alpha^*_{s,n-1}(t)$ at $n-1$ stage and the bargaining process in this slot ends; (ii) proposing a new bid to the SU at n stage after rejecting $\alpha^*_{s,n-1}(t)$ at $n-1$ stage; (iii) rejecting $\alpha^*_{s,n-1}(t)$ only and does not send a new bid to the SU at n stage. Here, $\alpha^*_{s,n-1}(t)$ also denotes the fraction of the remaining transmission time in each slot. If the PU accepts $\alpha^*_{s,n-1}(t)$ at $n-1$ stage, the PU's profit is $u_p\left(\alpha^*_{s,n-1}(t)\right) = (1-(n-1)\tau)\left(1-\alpha^*_{s,n-1}(t)\right)R^t_r$. Note that once the PU decides to bargain with the SU, the least profit is $(1-N\tau)R^t_{dir}$, which means no agreement has been reached after N stages and the PU will transmit its data by itself. Hence, the PU's profit is no less than $(1-N\tau)R^t_{dir}$. Assume if the PU's bid $\alpha_{p,n}(t)$ is accepted by the SU at stage n, the PU's profit is $(1-n\tau)(1-\alpha_{p,n}(t))R^t_r$. Therefore, if the PU gives a new bid $\alpha_{p,n}(t)$ to the SU, the PU's approximate expected profit at n stage in slot t is,

$$U^t_p(\alpha_{p,n}(t)) = \Psi R^t_{dir} Prob^{t,n}_{PR}(\alpha_{p,n}(t)) + (1-n\tau)(1-\alpha_{p,n}(t))R^t_r Prob^{t,n}_{PA}(\alpha_{p,n}(t)) \qquad (2)$$

where $\Psi = 1-N\tau$. Note that if $\max(2) > \max\left(\Psi R^t_{dir}, u_p(\alpha^*_{s,n-1}(t))\right)$, the PU will reject $\alpha^*_{s,n-1}(t)$ and proposes a new bid $\alpha^*_{p,n}(t) = \arg\max(2)$ to the SU at stage n. In the other case, when $\max(2) \le \max(\Psi R^t_{dir}, u_p(\alpha^*_{s,n-1}(t)))$, if $\Psi R^t_{dir} > u_p(\alpha^*_{s,n-1}(t))$, the PU will reject SU's bid and does not propose a new bid to the SU (in Figure 3, using the *NO bid* to denote this case); otherwise, the PU will accept $\alpha^*_{s,n-1}(t)$ and the bargaining process ends at $n-1$ stage in this slot. For the PU, for $n=1$ stage in Figure 3, the PU needs to make a decision about whether or not to bargain with the SU like OMBBM, where the PU will bargain with the SU if $\max(2) > R^t_{dir}$; otherwise, no bargaining is in this slot and the next slot begins. Note that the PU also maybe propose a new bid to the SU, even if the SU does not propose a bid to the PU at $n-1$ stage (*NO bid* in Figure 3). Under this situation, $\max(2) > \Psi R^t_{dir}$ must be satisfied.

From the view of the SU, if the PU sends a bid $\alpha^*_{p,n}(t)$ to the SU at n stage, the SU will also estimate the expected profit if it proposes a new bid to the PU after rejecting $\alpha^*_{p,n}(t)$. However, if the SU's bid is rejected by the PU at $n+1$ stage, the SU's least profit is 0. This is because even if an agreement is not reached until N stage and the SU's profit is also 0. Hence, the SU's expected profit is

$$U^t_s(\alpha_{s,n+1}(t)) = u_s(\alpha_{s,n+1}(t))\, Prob^{t,n+1}_{SA}(\alpha_{s,n+1}(t)) \qquad (3)$$

where $Prob^{t,n+1}_{SA}(\alpha_{s,n+1}(t))$ denotes the probability of $\alpha_{s,n+1}(t)$ being accepted by the PU at $n+1$ stage in slot t and $u_s(\alpha_{s,n+1}(t)) = (1-(n+1)\tau)\left(\alpha_{s,n+1}(t)R^t_s - \frac{1+\alpha_{s,n+1}(t)}{2}P_sC\right)$. If $\max(3) > \max(u_s(\alpha^*_{p,n}(t)), 0)$, the SU will reject $\alpha^*_{p,n}(t)$ and proposes a new bid $\alpha^*_{s,n+1}(t) = \arg\max(3)$ to the PU at $n+1$ stage. In the other case, $\max(3) \le \max(u_s(\alpha^*_{p,n}(t)), 0)$, if $u_s(\alpha^*_{p,n}(t)) < 0$, the SU will reject $\alpha^*_{p,n}(t)$ and does not propose a new bid $\alpha^*_{s,n+1}(t)$ to the PU at $n+1$ stage (in Figure 3, using *NO bid* to denote this case); otherwise, the SU will accept $\alpha^*_{p,n}(t)$ and the bargaining process ends at n stage in slot t. Note that the SU also can propose a new bid to the PU, even if the PU does not propose a bid to the SU at n stage (*NO bid* in Figure 3), which requires $\max(3) > 0$.

4. **Belief Updating Scheme.** In this section, we mainly discuss how to calculate the probabilities of the bids being accepted by the users online and the maximal value of (1), (2) and (3). As mentioned before, both the PU and the SU have no prior private information of each other. Therefore, we should build up updating beliefs of each other. Here, we adopt an empirical belief updating rule from the double auction model [7], which has been also used in [8] to set optimal reserve prices in spectrum double auctions. However, this rule has not yet been used in bargaining problems. From the respective of the PU, the SU always requires higher bids to pursuit higher profits. As a result, we have the following observations: if a bid $\rho > \alpha_{p,n}(t)$ is rejected, the bid $\alpha_{p,n}(t)$ will also be rejected; if a bid $\rho < \alpha_{p,n}(t)$ is accepted, the bid $\alpha_{p,n}(t)$ will also be accepted. Based

on these facts, the PU sends a bid $\alpha_{p,n}(t)$ to the SU and the probability of $\alpha_{p,n}(t)$ being accepted by the SU is,

$$Prob_{PA}^{t,n}(\alpha_{p,n}(t)) = \frac{\sum_{\rho \le \alpha_{p,n}(t)} \psi_A(\rho)}{\sum_{\rho \le \alpha_{p,n}(t)} \psi_A(\rho) + \sum_{\rho > \alpha_{p,n}(t)} \psi_R(\rho)} \tag{4}$$

where $\psi_A(\rho)$ denotes the accepted number of the bid ρ and $\psi_R(\rho)$ denotes the rejected number of the bid ρ in the PU's bid history until $n-1$ stage in the slot t. In general, in order to find $\alpha_{p,n}^*(t)$ for the PU at each stage in slot t, we can use the first derivative of (2). However, it is difficult to obtain the first order conditions of (2) because of the updating information. Note that if $\alpha_{p,n}(t) = 0$, (2) $= \Psi R_{dir}^t$; if $\alpha_{p,n}(t) = 1$, we also have (2) $= \Psi R_{dir}^t$. Therefore, we can find $\alpha_{p,n}^*(t)$ of (2) in the interval $(0,1)$ step by step if the PU wants to bargain with the SU and the search step length $k = 0.001$, which means $\alpha_{p,n}^*(t)$ is an approximate value in fact. Note that we need to initialize the search point which is closer to 0 in the process of finding $\alpha_{p,n}^*(t)$. In the simulation part, we initialize the first accepted bid $\alpha_{p,n=1}^*(t=1) = 0.02$ for different N and the curves of (2) have been shown.

From the respective of the SU, the PU always wants to use lower bids to pursue higher profits and then we also have following observations: if a bid $\kappa > \alpha_{s,n+1}(t)$ is accepted, the bid $\alpha_{s,n+1}(t)$ will also be accepted; if a bid $\kappa < \alpha_{s,n+1}(t)$ is rejected and the bid $\alpha_{s,n+1}(t)$ will also be rejected. Based on these facts, the SU sends a bid $\alpha_{s,n+1}(t)$ to the PU and the probability of $\alpha_{s,n+1}(t)$ being accepted by the PU is,

$$Prob_{SA}^{t,n+1}(\alpha_{s,n+1}(t)) = \frac{\sum_{\kappa \ge \alpha_{s,n+1}(t)} \mu_A(\kappa)}{\sum_{\kappa \ge \alpha_{s,n+1}(t)} \mu_A(\kappa) + \sum_{\kappa < \alpha_{s,n+1}(t)} \mu_R(\kappa)} \tag{5}$$

where $\mu_A(\kappa)$ denotes the accepted number of the bid κ and $\mu_R(\kappa)$ denotes the rejected number of the bid κ in the SU's bidding history until n stage in the slot t. Note that when $\alpha_{s,n+1}(t) = 0$, then (3) < 0; when $\alpha_{s,n+1}(t) = 1$, then (3) $= 0$. If the SU wants to propose $\alpha_{s,n+1}^*(t)$ to the PU, (3) > 0 must be satisfied at first. Therefore, $\alpha_{s,n+1}^*(t)$ of (3) can be also found in the interval $(0,1)$ step by step if the SU wants to bargain with the PU, which also means $\alpha_{s,n+1}^*(t)$ is an approximate value in fact. Here, we also set the search step length $k = 0.001$, initialize the first accepted bid $\alpha_{s,n+1=2}^*(t=1) = 0.98$ and show the curves of (3) in the simulation part.

5. **Simulation Results.** In order to show the performances of PDMM more clearly, we set $P_t = P_s = 1$, $T = 1000$, $\tau = 0.1$ and C is from 1 to 10. R_{dir}, R_r and R_s are uniformly distributed in $[1,5]$, $[15,20]$ and $[5,10]$, respectively. In fact, PDMM has no constraint on these parameters.

Firstly, we show the averaged profits over T of the PU and the SU at different Cs in Figure 4. We can find the bargaining process will bring both the PU and the SU into a win-win situation which has been also shown in [2], which means our proposed model is effective. With C increasing, the PU's and the SU's averaged profits are decreasing. Odd stages are good for the PU and even stages are beneficial to the SU. In Figure 5(a), we show the bids' changed process of both sides. Here, we set $C = 2$ and compare the bids of MMBBM ($N = 2$) with that of OMBBM ($N = 1$). In Figure 5(a), we can also find accepted bids in both models will be stable with t increasing. In MMBBM ($N = 2$), the PU will gradually increase its bids and the SU will also gradually decrease its bids for agreements. At last, the PU sends higher bids to the SU and the SU has no need to propose new bids to the PU at the second stage. When $N > 2$, the bids' changed processes are hard to be shown in figures. Therefore, the averaged agreed prices over the times of agreements (M) for different N have been shown in Figure 5(b). Here, we can find no agreement has been reached when $C \ge 10$.

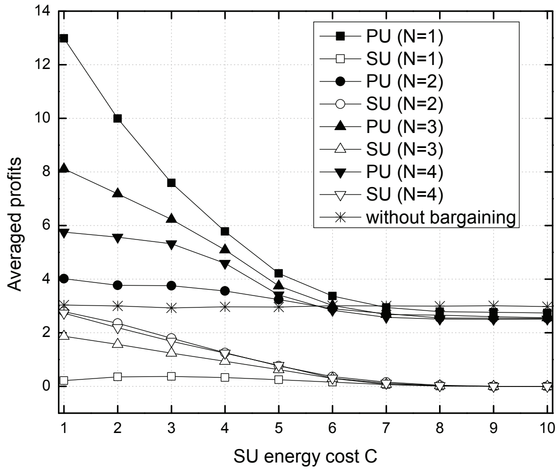
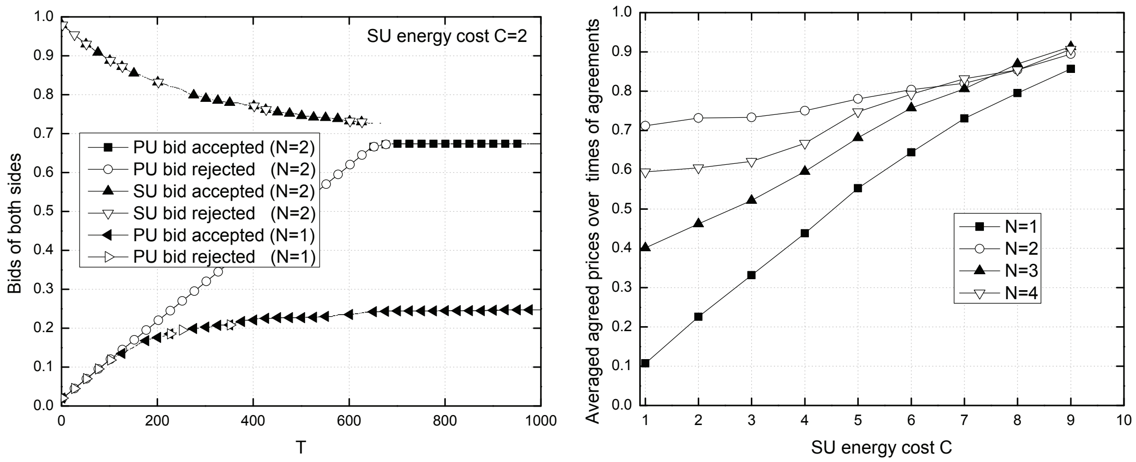
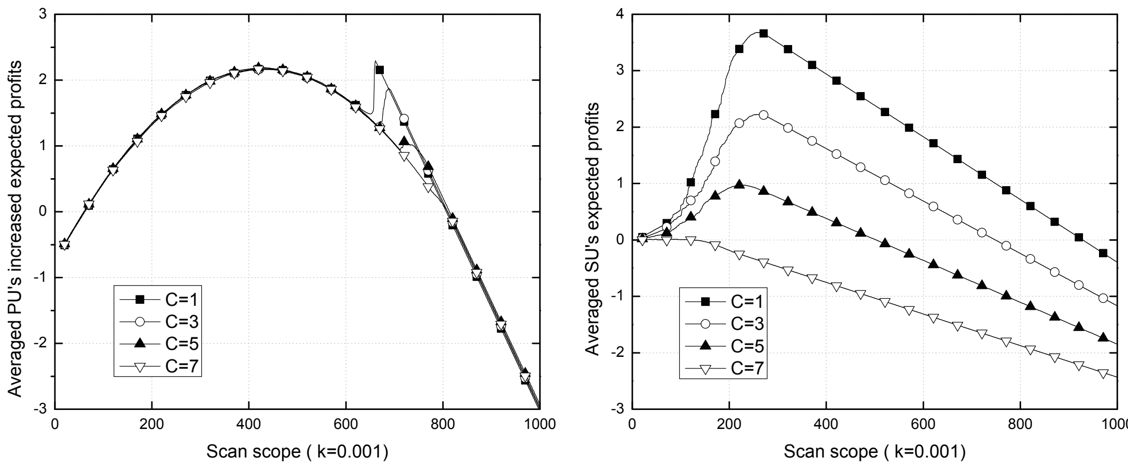


FIGURE 4. Decision-makers' averaged profits over T



(a) Changes of both sides' bids (b) Averaged agreed prices over M

FIGURE 5. Bargaining process and averaged prices in agreements for different Ns



(a) PU's averaged increased expected profits (b) SU's averaged expected profits

FIGURE 6. Averaged expected profits over T for different Cs ($N = 2$)

In order to understand the process of searching the optimal bids of (1), (2) and (3) in the interval $(0, 1)$ more clearly, taking $N = 2$ for example, in this model, both the PU and SU have no more than one opportunity to propose their own bids. Here, we show the averaged values over T of $((2) - R_{dir}^t)$ which denotes the increased expected profits in Figure 6(a) and (3) in Figure 6(b) for each bid in $(0, 1)$. Note that the PU's bids are increasing from lower bids to higher bids and the SU's bids are on the contrary.

6. **Conclusion.** In this paper, we have proposed an online bargaining decision-making model (also called PDMM), in which there is no need to assume the types of decision-makers and probability distributions of decision-makers' private information. In PDMM, decision-makers can reach agreement when the SU's energy costs are not too high. Therefore, PDMM is more practical than other bargaining decision-making models, which can be used in practical CSS networks. The situation in which one PU sequentially bargains with multiple SUs one by one will be investigated in our future works.

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