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DECOMPOSITION BASED ESTIMATION ALGORITHMS FOR MULTIVARIABLE OUTPUT ERROR MOVING AVERAGE SYSTEMS

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Abstract. *For a class of multivariable output error moving average (OEMA) systems, which include different types of parameter forms: a parameter vector and a parameter matrix, the conventional parameter estimation methods cannot be applied to these systems. In order to deal with this problem, a decomposition based estimation algorithm is presented. The basic idea is to employ the matrix transformation technique to decompose the multivariable OEMA system into several subsystems according to the output dimensions, to present a decomposition based least squares algorithm to estimate the parameters of each subsystem. The simulation results indicate that the proposed algorithm is effective.*

Keywords: System identification, Multivariable system, Decomposition, Least squares

1. **Introduction.** Multivariable modelling has been widely used in various systems [1, 2, 3, 4, 5], and arouses much attention in the system identification area. The identification approaches for multivariable systems mainly include the subspace methods, the least squares/stochastic gradient methods, the maximum likelihood methods, and the blind identification methods.

In a class of difference equation multivariable systems, there exist scalar polynomial coefficients and matrix polynomial coefficients, so the identification model of the systems naturally contains both a parameter vector and a parameter matrix, and the standard least squares method cannot be directly applied to the identification model. The hierarchical identification principle is an effective technique to solve the identification problems of this type of systems. The idea of the hierarchical identification principle is to interactively estimate the parameter vector and the parameter matrix of the multivariable systems [6, 7]. Based on the hierarchical identification principle, a series of works have been published, including the hierarchical least squares based iterative identification method for multivariable CARMA-like systems [8], for multivariable nonlinear systems [9], and the hierarchical gradient based iterative estimation algorithm for multivariable output error moving average systems [10].

In the previous study [11], the author explored the identification method of the single variable OEMA systems by using the data filtering-based and auxiliary model-based least squares algorithm. This paper investigates the parameterization of multivariable OEMA systems, to decompose a multivariable OEMA system into m subsystems (m is the output dimensions), each of which contains only a parameter vector, and to present a decomposition based least squares method to estimate the parameters of the system. The characteristics of the proposed method are that the least squares method can be directly applied to the m subsystems.

The rest of the paper is organized as follows. Section 2 demonstrates the identification problem of the multivariable OEMA system. Section 3 shows the system decomposition method and presents a decomposition based least squares (DLS) identification algorithm. Section 4 provides a numerical example for the proposed DLS algorithm. Finally, the concluding remarks are involved in Section 5.

2. **Problem Formulation.** Consider the following multivariable output error moving average (OEMA) model,

$$y(t) = \frac{Q(z)}{\alpha(z)}u(t) + D(z)v(t), \tag{1}$$

where $y(t) := [y_1(t),\ y_2(t), \cdots,\ y_m(t)]^T \in R^m$ is the system output vector, $u(t) \in R^r$ is the system input vector, $v(t) := [v_1(t),\ v_2(t), \cdots, v_m(t)]^T \in R^m$ is a white noise vector with the normal distribution $v_j(t) \sim N(0, \sigma^2)$, $j = 1, 2, \cdots, m$, $\alpha(z) \in R$ is the system characteristic polynomial in z^{-1} (z^{-1} is the unit backward shift operator: $z^{-1}y(t) = y(t-1)$), $Q(z) \in R^{m\times r}$ is a matrix polynomial in z^{-1}, and $D(z) \in R$ is a polynomial in z^{-1}, and they are defined as

$$\begin{aligned}
\alpha(z) &:= 1 + \alpha_1 z^{-1} + \alpha_2 z^{-2} + \cdots + \alpha_n z^{-n},\ \alpha_i \in R,\\
Q(z) &:= Q_1 z^{-1} + Q_2 z^{-2} + \cdots + Q_n z^{-n},\ Q_i \in R^{m\times r},\\
D(z) &:= 1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_n z^{-n},\ d_i \in R.
\end{aligned}$$

The diagram of this system is depicted in Figure 1. Note that there exist two types of coefficients: the matrix polynomial $Q(z)$, and the scalar polynomials $\alpha(z)$ and $D(z)$, respectively. The parameterization difficulty of the multivariable OEMA system is that we can not get a regression model, to which the standard estimation method can be directly applied, due to being unable to merge two different types of coefficients.

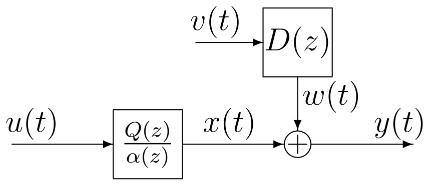

Figure 1. The multivariable OEMA systems

3. **The Decomposition Based Least Squares Algorithm.** In this section, we decompose the multivariable output error moving average (OEMA) system into m subsystems (m is the output dimensions), and directly apply a least squares algorithm to estimating the parameters of each subsystem. The description is as follows.

The internal variable $x(t)$ is the noise-free output vector,

$$x(t) = \frac{Q(z)}{\alpha(z)}u(t), \tag{2}$$

and we have

$$x(t) = [1 - \alpha(z)]x(t) + Q(z)u(t). \tag{3}$$

Then, Equation (1) can be written as

$$\begin{aligned}
y(t) &= x(t) + D(z)v(t), &(4)\\
&= [1 - \alpha(z)]x(t) + Q(z)u(t) + D(z)v(t). &(5)
\end{aligned}$$

Let

$$Q_i := \begin{bmatrix} Q_{i(1)} \\ Q_{i(2)} \\ \vdots \\ Q_{i(m)} \end{bmatrix}, \quad Q_i \in R^{m\times r},$$

$$Q_{(j)}(z) := Q_{1(j)}z^{-1} + Q_{2(j)}z^{-2} + \cdots + Q_{n(j)}z^{-n} \in R^{1\times r},$$

where $Q_{i(j)}$ $(j = 1, 2, \cdots, m) \in R^{1\times r}$ is the jth row elements of Q_i, and $Q_{(j)}(z)$ $(j = 1, 2, \cdots, m) \in R^{1\times r}$ is the jth row elements of $Q(z)$.

Then Equations (3) and (5) can be written as

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_m(t) \end{bmatrix} = [1-\alpha(z)] \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_m(t) \end{bmatrix} + \begin{bmatrix} Q_{(1)}(z) \\ Q_{(2)}(z) \\ \vdots \\ Q_{(m)}(z) \end{bmatrix} u(t),$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_m(t) \end{bmatrix} = [1-\alpha(z)] \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_m(t) \end{bmatrix} + \begin{bmatrix} Q_{(1)}(z) \\ Q_{(2)}(z) \\ \vdots \\ Q_{(m)}(z) \end{bmatrix} u(t) + D(z) \begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_m(t) \end{bmatrix}. \tag{6}$$

That is, system (6) can be decomposed into m subsystems

$$\begin{aligned} y_1(t) &= [1-\alpha(z)]x_1(t) + Q_{(1)}(z)u(t) + D(z)v_1(t), \\ y_2(t) &= [1-\alpha(z)]x_2(t) + Q_{(2)}(z)u(t) + D(z)v_2(t), \\ &\vdots \\ y_m(t) &= [1-\alpha(z)]x_m(t) + Q_{(m)}(z)u(t) + D(z)v_m(t). \end{aligned}$$

The jth subsystem can be represented as

$$\begin{aligned} y_j(t) &= [1-\alpha(z)]x_j(t) + Q_{(j)}u(t) + D(z)v_j(t) \\ &= [1-\alpha(z)]x_j(t) + [Q_{1(j)}z^{-1} + Q_{2(j)}z^{-2} + \cdots + Q_{n(j)}z^{-n}]u(t) + D(z)v_j(t), \\ &\quad j = 1, 2, \cdots, m. \end{aligned} \tag{7}$$

Define the parameter vectors and the information vectors as follows

$$\begin{aligned} \alpha &:= [\alpha_1, \alpha_2, \cdots, \alpha_n]^T \in R^n, \\ d &:= [d_1, d_2, \cdots, d_n]^T \in R^n, \\ Q_{(j)} &:= [Q_{1(j)}, Q_{2(j)}, \cdots, Q_{n(j)}] \in R^{1\times nr}, \\ \theta_j &:= [\alpha^T, Q_{(j)}]^T \in R^{n+nr}, \\ \Theta_j &:= [\alpha^T, Q_{(j)}, d^T]^T \in R^{2n+nr}, \\ \Theta &:= [\alpha^T, Q_{(1)}, Q_{(2)}, \cdots, Q_{(m)}, d^T]^T \in R^{2n+nrm}, \\ \phi_j(t) &:= \big[-x_j(t-1), -x_j(t-2), \cdots, -x_j(t-n), u^T(t-1), u^T(t-2), \\ &\qquad \cdots, u^T(t-n)\big]^T \in R^{n+nr}, \\ \chi_j(t) &:= [\phi_j^T(t), v_j(t-1), v_j(t-2), \cdots, v_j(t-n)]^T \in R^{2n+nr}. \end{aligned}$$

From (7), we get

$$x_j(t) = -\sum_{i=1}^{n} \alpha_i x_j(t-i) + \sum_{i=1}^{n} Q_{i(j)}u(t-i) = \phi_j^T(t)\theta_j, \tag{8}$$

$$y_j(t) = -\sum_{i=1}^{n} \alpha_i x_j(t-i) + \sum_{i=1}^{n} Q_{i(j)}u(t-i) + \sum_{i=1}^{n} d_i v_j(t-i) + v_j(t)$$

$$= \chi_j^T(t)\Theta_j + v_j(t). \tag{9}$$

For the unknown internal variable $x_j(t-i)$ in $\phi_j(t)$ and $\chi_j(t)$ and the unavailable noise variable $v_j(t-i)$ in $\chi_j(t)$, we replace them with their estimates $\hat{x}_j(t-i)$ and $\hat{v}_j(t-i)$. From (8) and (9), the estimates $\hat{x}_j(t-i)$ and $\hat{v}_j(t-i)$ can be computed by replacing $\phi_j(t)$ and $\chi_j(t)$ with their estimates $\hat{\phi}_j(t)$ and $\hat{\chi}_j(t)$, and replacing θ_j and Θ_j with their estimates $\hat{\theta}_j(t)$ and $\hat{\Theta}_j(t)$,

$$\hat{x}_j(t) = \hat{\phi}_j^T(t)\hat{\theta}_j(t), \tag{10}$$

$$\hat{v}_j(t) = y_j(t) - \hat{\chi}_j^T(t)\hat{\Theta}_j(t). \tag{11}$$

Define the following cost function

$$J(\Theta_j) := \sum_{i=1}^{t} \left[y_j(i) - \chi_j^T(i)\Theta_j\right]^2.$$

By replacing the unknown variables $x_j(t-i)$ and $v_j(t-i)$ in $\chi_j(t)$ with their estimates $\hat{x}_j(t-i)$ and $\hat{v}_j(t-i)$, according to the least squares principle [12], we can obtain the decomposition based least squares (DLS) algorithm for estimating $\hat{\Theta}_j(t)$:

$$\begin{aligned}
\hat{\Theta}_j(t) &= \hat{\Theta}_j(t-1)+L_j(t)\left[y_j(t) - \hat{\chi}_j^T(t)\hat{\Theta}_j(t-1)\right],\ j = 1, 2, \cdots, m, && (12)\\
L_j(t) &= P_j(t-1)\hat{\chi}_j(t)\left[1 + \hat{\chi}_j^T(t)P_j(t-1)\hat{\chi}_j(t)\right]^{-1}, && (13)\\
P_j(t) &= \left[I - L_j(t)\hat{\chi}_j^T(t)\right]P_j(t-1),\ P_j(0) = P_0 I, && (14)\\
\hat{\phi}_j(t) &= \left[-\hat{x}_j(t-1), -\hat{x}_j(t-2), \cdots, -\hat{x}_j(t-n), u^T(t-1), u^T(t-2),\right.\\
&\quad \left.\cdots, u^T(t-n)\right]^T, && (15)\\
\hat{\chi}_j(t) &= \left[\hat{\phi}_j^T(t), \hat{v}_j(t-1), \hat{v}_j(t-2), \cdots, \hat{v}_j(t-n)\right]^T, && (16)\\
\hat{x}_j(t) &= \hat{\phi}_j^T(t)\hat{\theta}_j(t), && (17)\\
\hat{v}_j(t) &= y_j(t) - \hat{\chi}_j^T(t)\hat{\Theta}_j(t), && (18)\\
\hat{\alpha}(t) &= \frac{1}{m}\sum_{j=1}^{m}\hat{\alpha}_{(j)}(t),\ \hat{\alpha}_{(j)}(t) = \hat{\Theta}_j(t)(1:n), && (19)\\
\hat{d}(t) &= \frac{1}{m}\sum_{j=1}^{m}\hat{d}_{(j)}(t),\ \hat{d}_{(j)}(t) = \hat{\Theta}_j(t)(n+nr+1:2n+nr), && (20)\\
\hat{\Theta}_j(t) &:= \left[\hat{\alpha}^T(t), \hat{Q}_{(j)}(t), \hat{d}^T(t)\right]^T, && (21)\\
\hat{\Theta}(t) &= \left[\hat{\alpha}^T(t), \hat{Q}_{(1)}(t), \hat{Q}_{(2)}(t), \cdots, \hat{Q}_{(m)}(t), \hat{d}^T(t)\right]^T. && (22)
\end{aligned}$$

The flowchart of computing the parameter estimates $\hat{\Theta}_j(t)$ by the DLS algorithm in (12)-(22) is shown in Figure 2.

4. **Example.** Consider a two-input two-output OEMA model,

$$y(t) = \frac{Q(z)}{\alpha(z)}u(t) + D(z)v(t),$$

where

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}, \quad u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}, \quad v(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix},$$

$$\alpha(z) = 1 - 0.35z^{-1}, \quad D(z) = 1 + 0.15z^{-1},$$

$$\begin{aligned} Q(z) &= \begin{bmatrix} 1.50 & 0.80 \\ 0.80 & 1.50 \end{bmatrix} z^{-1}, \\ \alpha &= -0.35,\ d = 0.15, \\ \Theta_1 &= [-0.35, 1.50, 0.80, 0.15]^T,\ \Theta_2 = [-0.35, 0.80, 1.50, 0.15]^T, \\ \Theta &= [-0.35, 1.50, 0.80, 0.80, 1.50, 0.15]^T. \end{aligned}$$

In simulation, the inputs $\{u_1(t)\}$ and $\{u_2(t)\}$ are taken as two persistent excitation signal sequences with zero mean and unit variances, $\{v_1(t)\}$ and $\{v_2(t)\}$ are taken as two white noise sequences with zero mean and variances σ_1^2 for $v_1(t)$ and σ_2^2 for $v_2(t)$. We take the data length $L = 3000$, apply the DLS algorithm to estimating the parameters of this example system, and the parameter estimates and the estimation errors are shown in Table 1, where the noise variances are $\sigma_1^2 = \sigma_2^2 = \sigma^2 = 0.20^2$ and $\sigma_1^2 = \sigma_2^2 = \sigma^2 = 0.40^2$. The estimation errors of the DLS algorithm are

$$\delta := \frac{\left\|\hat{\Theta}(t) - \Theta\right\|}{\|\Theta\|} \times 100\%.$$

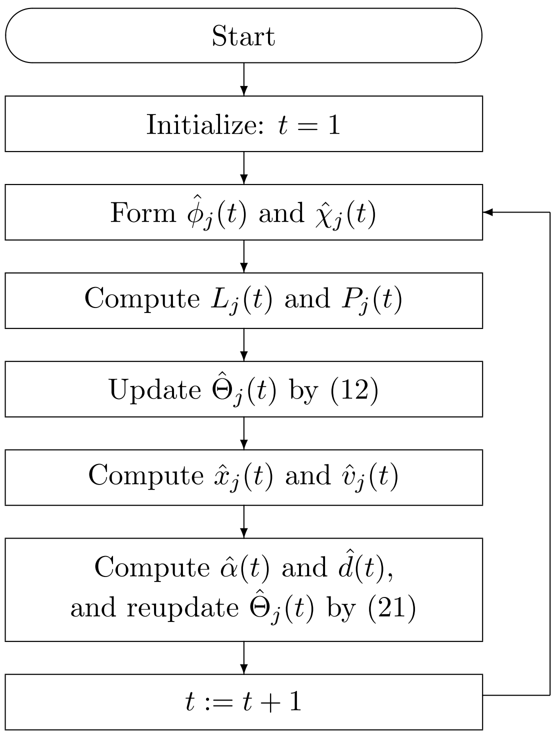

Figure 2. The flowchart of computing the DLS estimates $\hat{\Theta}_j(t)$

From Table 1, we can draw the following conclusions.

- The proposed DLS algorithm can give close to accurate parameter estimates under the low noise levels – see the estimation errors of the last column in Table 1.
- As shown in Table 1, with the same data lengths, the smaller the white noise variances are, the faster the convergence rate of the parameter estimates is.
- With the increasing of t, the estimation errors are generally small – see the estimation errors in Table 1.

5. **Conclusions.** In this paper, we propose a decomposition based least squares (DLS) algorithm to estimate the parameters of a multivariable OEMA model. Since there are two types of coefficients in the system, we employ the matrix transformation technique

Table 1. The estimates and errors under different σ^2

σ^2	t	α_1	$Q_1(1,1)$	$Q_1(1,2)$	$Q_1(2,1)$	$Q_1(2,2)$	d_1	δ (%)
0.20^2	100	−0.41744	1.42609	0.81640	0.64101	1.42601	0.12348	8.39289
	200	−0.37350	1.51552	0.76474	0.63826	1.45843	0.10413	7.35228
	500	−0.33178	1.51815	0.81614	0.76152	1.48505	0.09704	3.02752
	1000	−0.35011	1.50972	0.78433	0.74737	1.48043	0.08597	3.58000
	2000	−0.35027	1.48125	0.80705	0.79409	1.49556	0.09089	2.58182
	3000	−0.34557	1.49258	0.80522	0.78880	1.49852	0.08961	2.55772
0.40^2	100	−0.37949	1.27929	0.94667	0.99335	1.23140	0.22629	17.73893
	200	−0.36166	1.44165	0.88561	0.82794	1.34981	0.17322	7.65787
	500	−0.36501	1.46063	0.88036	0.79817	1.45825	0.12492	4.23159
	1000	−0.35410	1.46781	0.85412	0.75401	1.47239	0.10135	3.94605
	2000	−0.34156	1.51779	0.80308	0.77499	1.49155	0.08334	3.05717
	3000	−0.34388	1.51007	0.80609	0.78421	1.47637	0.08444	2.98559
True values		−0.35000	1.50000	0.80000	0.80000	1.50000	0.15000	

to decompose the system into several subsystems according to the output dimensions, and get regression models, to which the standard least squares algorithm can be applied directly. The estimates of some parameters for the DLS algorithm are obtained by taking an average of their estimates from all subsystems, so the parameter estimation accuracy of the DLS algorithm is high.

In the future research, we will further focus on the decomposition based identification methods for multivariable nonlinear systems, especially, for multivariable block-oriented nonlinear systems.

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