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# A LINEAR CLOSED-FORM ALGORITHM FOR TOA-BASED SOURCE LOCALIZATION BY CONSIDERING UNKNOWN TRANSMIT TIME

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Abstract. *A linear closed-form algorithm is proposed for TOA-based source localization when the transmit time is assumed to be unavailable. The source positions are represented as the algebraic solutions which avoid the problem of local optimum of the numerical calculation. By converting the positioning optimization model to the problem of linear least square estimation, the initial solution to source position estimation is obtained by using the proposed linear closed-form algorithm. Considering the initial solution as input parameter, the position refinement technique is designed to improve the source position. The simulations show that the positioning accuracy of the designed algorithm can be close to the Cramer-Rao lower bound (CRLB) of source position estimation. The positioning error of the proposed algorithm is unrelated with the transmit time.*
**Keywords:** Wireless sensor networks, Localization, Linear least square, Time of arrival

1. **Introduction.** Location awareness has received a great deal of interest in many wireless systems such as cellular networks, wireless local area networks, and wireless sensor networks due to its capability to provide wide range of add-on applications [1]. Location-based services such as location-based advertisement, and location-based social networking have become more important in order to enhance the future lifestyle. The wireless system applications offered by location awareness will enable ubiquitous and context aware network services which necessitate the location of the wireless device to be accurately estimated. Obtaining accurate location information becomes an important task in these wireless system applications.

Configuring GPS to obtain the positions of sensors is not suitable for wireless localization due to the expensive hardware costs, large volume and high energy consumption. So these limitations have inspired a new method to estimate the position of the source target by using the anchors' known positions. In the process of position estimation, the relative distance should be measured or estimated between the nodes. The most important kinds utilize the received signal strength (RSS) [2], angle of arrival (AOA) [3], and signal propagation time, respectively. RSS algorithms use the received signal power for object positioning; their accuracies are limited by the fading of wireless signals. AOA algorithms require either directional antennas or receiver antenna arrays. Signal propagation time based algorithms estimate the object location using the time it takes the signal to travel from the transmitter to the target and from there to the receivers. They achieve very accurate estimation of object location if combined with high-precision timing measurement techniques, such as ultrawideband (UWB) signaling, which allows centimeter and even submillimeter accuracy. The algorithms based on signal propagation time can be further classified into Time of Arrival (TOA) [4] and Time Difference of Arrival (TDOA) [5]. TOA algorithms employ the information of the absolute signal travel time from the transmitter to the target and thence to the receivers.

A number of localization algorithms became available for the wireless localization. Maximum likelihood (ML) estimator [6] is an asymptotically unbiased estimator for TOA based wireless localization problem. However, it is a nonlinear least squares estimator for Gaussian measurement noise. The cost function of the ML estimator is severely nonlinear and noncovex, so numerical solution of ML estimator strongly depends on the initialization. If the initialization is not sufficiently close to the global minimum, the numerical solution may converge to a local minimum or a saddle point causing a large estimation error. Therefore, determining an appropriate initialization point is a crucial problem in optimizing the ML cost function. As a result, some approaches have been introduced to address the shortcomings of ML problem. The semidefinite programming (SDP) in [7, 8] by convex relaxation technique is a solution to the ML convergence problem. In the semidefinite relaxation technique, the nonlinear and nonconvex ML problem is transformed into a convex optimization problem. The advantage of SDP technique is that its cost function does not have local minima and thus convergence to the global minimum is guaranteed. The downside is that the SDP technique is sub-optimal and cannot achieve the best possible performance in all conditions. It has a complicated structure and high computational complexity. Based on many approximations the linear analytical solutions in [9] are proposed to obtain the algebraic solutions of the target positions.

In this paper an accurate linear closed-form algorithm is proposed to estimate the source position when considering the transmit time as unavailable. The accurate linear closed-form algorithm is designed by transforming the nonlinear equations to linear equations via subtraction of each equation from all others and obtains the initial solution of the source position. Then the refinement technique is proposed to improve the positioning accuracy. This paper contains a number of symbols. Following the convention, we represent the matrices as bold case letters. If we denote the matrices as $(*)$, $(*)^{-1}$ represents matrix inverse. If $(*)$ contains noise, $(*)^e$ would denote its estimated value while $\Delta(*)$ is the noise component. $\|*\|$ denotes $\ell_2$ norm. $\mathbf{A}_{[i,j]}$ denotes the element at the $i$th row and $j$th column of matrix $\mathbf{A}$. The rest of this paper is structured as follows. Section 2 presents the problem specification of TOA-based wireless localization. Section 3 derives the position CRLB for TOA-based soucce localization when the transmit time is assumed to be unknown. Section 4 describes linear estimation method of source position in detail. Section 5 analyzes the simulation results. The conclusion is represented in Section 6.

2. **Problem Specification.** Assume in a 2-dimensional (or 3-dimensional, for the sake of convenience, we assume in 2-dimensional, that the analysis method of 3-dimensional is the same with 2-dimensional) plane the known positions of anchors are $\mathbf{x}_i = [x_i \quad y_i]^T$, $i = 1, 2, \ldots, N$. These anchors with known positions are used to locate the position of an unknown source target node, which is denoted as $\mathbf{x} = [x \quad y]^T$. When the source node transmits the signal at the time $t_0$, the anchor $i$ receives the signal at time $t_i$. Considering the line of sight (LOS) propagation, we can obtain that

$$t_i = \frac{\|\mathbf{x}_i - \mathbf{x}\|}{c} + t_0 + \varepsilon_i \tag{1}$$

where $i = 1, 2, \ldots, N$, and $c$ is the speed of light. $\varepsilon_i$ is the noise which is a zero-mean white Gaussian process with known variance $\delta_i^2$. The zero-mean assumption is valid as long as the multipath effect. Although the parameters $\delta_i^2$ are usually unknown in practice, they can be determined for a particular signaling type in the TOA-based location system by channel measurement. Considering the transmit time $t_0$ of source node as unknown parameter, the source position estimation can be modeled as the well known maximum likelihood problem

$$\min_{\mathbf{x},t_0} \sum_{i=1}^{N} \frac{1}{\delta_i^2} \left( t_i - \frac{\|\mathbf{x}_i - \mathbf{x}\|}{c} - t_0 \right)^2 \tag{2}$$

The numerical method provides an accuracy solution to (2), but the solution of numerical method requires a reasonable initialization close to the true solution. If the initialization is not sufficiently close to the global minimum, the iterative algorithm may converge to a local minimum or a saddle point causing a large estimation error. As a result, semidefinite programming (SDP) techniques have been introduced to address the ML problem. The downside is that the SDP technique is sub-optimal and cannot achieve the best possible performance in all conditions. So based on many approximations, linear estimator having a closed-form solution is derived in this paper.

3. **Performance of CRLB.** The CRLB defines a lower bound on the variance of any unbiased estimator and is employed as a benchmark for evaluating the performance of estimators. The CRLB of the unknown parameters are the diagonal elements of the inverse of the Fisher information matrix. Since the transmit time of the source target is not available to the estimator, it should also be taken into account as an unknown parameter. Let us recall the vector of unknown parameters $\mathbf{\Phi} = [\mathbf{x}^T \quad t_0]^T$. Here when the transmit time $t_0$ is unknown, the FIM is denoted as $\mathbf{F}$, which is also rewritten as

$$\mathbf{F} = -\frac{\partial^2 \ln P(\mathbf{t}|\mathbf{\Phi})}{\partial \mathbf{\Phi}^T \partial \mathbf{\Phi}} \tag{3}$$

where

$$P(\mathbf{t}|\mathbf{\Phi}) = \prod_{i=1}^{M} \frac{1}{\sqrt{2\pi}\delta_i} \exp\left\{ -\frac{\left( t_i - \frac{\|\mathbf{x}_i - \mathbf{x}\|}{c} - t_0 \right)^2}{2\delta_i^2} \right\} \tag{4}$$

Therefore, the elements of matrix $\mathbf{F}$ can be further represented as

$$\begin{cases} \mathbf{F}_{[1:2,1:2]} = \sum\limits_{i=1}^{N} \frac{1}{c^2\delta_i^2} \frac{(\mathbf{x}-\mathbf{x}_i)^T(\mathbf{x}-\mathbf{x}_i)}{\|\mathbf{x}-\mathbf{x}_i\|^2} \\ \mathbf{F}_{[1:2,3]} = \sum\limits_{i=1}^{N} \frac{1}{c\delta_i^2} \frac{(\mathbf{x}-\mathbf{x}_i)}{\|\mathbf{x}-\mathbf{x}_i\|} \\ \mathbf{F}_{[3,1:2]} = \mathbf{F}_{[1:2,3]}^T \\ \mathbf{F}_{[3,3]} = \sum\limits_{i=1}^{N} \frac{1}{\delta_i^2} \end{cases} \tag{5}$$

The CRLB of the unknown parameters are the diagonal elements of the inverse of the FIM. So the CRLB of the target position $\mathbf{x}$ is written as

$$\mathrm{CRLB}\left(\mathbf{x}_{[r]}\right) = \mathbf{F}_{[r,r]}^{-1} \tag{6}$$

where $r = 1, 2$. Given the FIM, the CRLB is obtained with

$$\mathrm{CRLB}(\mathbf{x}) = \mathbf{F}_{[1,1]}^{-1} + \mathbf{F}_{[2,2]}^{-1} \tag{7}$$

4. **Linear Estimator.** The proposed linear closed-form estimator is designed with two steps. The first step is to estimate the initial source location with the linear method. By eliminating the unknown parameter $t_0$, the position estimation problem is modeled as linear estimation to obtain the initial solution to source position. Secondly the initial source solution is further refined to improve the positioning accuracy. The source localization problem consists of estimating $\mathbf{x}$ from the observed time measurements $t_i$. In this section, the transmit time $t_0$ is assumed be unavailable. Weighting least square (WLS) solution is introduced to exploit the problem for TOA-based source localization.

4.1. **Initial source position.** To eliminate the unknown parameter $t_0$, (1) is rewritten as

$$\|\mathbf{x}_i - \mathbf{x}\| = c(t_i - t_0 - \varepsilon_i) \tag{8}$$

Squaring both sides of (8) and neglecting the high terms, (8) is rewritten as

$$-2x_i x - 2y_i y + x^2 + y^2 + 2c^2 t_i t_0 - c^2 t_0^2 = c^2 t_i^2 - x_i^2 - y_i^2 - 2c(t_i - t_0)\varepsilon_i \tag{9}$$

where $i = 1, 2, \ldots, N$. Subtracting the $N$th equation from the first $N-1$ equations, we obtain that

$$\begin{aligned} &2(x_N - x_i)x + 2(y_N - y_i)y + 2c^2(t_i - t_N)t_0 \\ &= c^2\left(t_i^2 - t_N^2\right) - x_i^2 - y_i^2 + x_N^2 + y_N^2 - 2c(t_i - t_0)\varepsilon_i + 2c(t_N - t_0)\varepsilon_N \end{aligned} \tag{10}$$

where $i = 1, 2, \ldots, N-1$. Let $\mathbf{z} = [x \quad y \quad t_0]$, (10) can be rewritten as matrix form

$$\mathbf{Az} = \mathbf{b} + \alpha \tag{11}$$

where the row vector of $\mathbf{A}$ is equal to $[2(x_N - x_i) \quad 2(y_N - y_i) \quad 2c^2(t_i - t_N)]$. The element value of $\mathbf{b}$ and $\alpha$ are equal to $[c^2\left(t_i^2 - t_N^2\right) - x_i^2 - y_i^2 + x_N^2 + y_N^2]$ and $[-2c(t_i - t_0)\varepsilon_i + 2c(t_N - t_0)\varepsilon_N]$, respectively. $\mathbf{A} \in \mathbb{R}^{N\times 3}$, $\mathbf{b} \in \mathbb{R}^N$ and $\alpha \in \mathbb{R}^N$. So the weighting least square (WLS) solution to (11) is

$$\mathbf{z} = \left(\mathbf{A}^T \boldsymbol{\Sigma}_\alpha^{-1} \mathbf{A}\right)^{-1} \mathbf{A}^T \Sigma_\alpha^{-1} \mathbf{b} \tag{12}$$

where $\boldsymbol{\Sigma}_\alpha = E\left(\alpha^T \alpha\right)$. The element value of $\boldsymbol{\Sigma}_\alpha$ is represented as

$$\boldsymbol{\Sigma}_\alpha[i,j] = \begin{cases} 4c^2\left(t_N - t_0\right)^2 \delta_N^2 & i \neq j \\ 4c^2\left(t_N - t_0\right)^2 \delta_N^2 + 4c^2(t_i - t_0)^2 \delta_i^2 & i = j \end{cases} \tag{13}$$

where $\boldsymbol{\Sigma}_\alpha[i,j]$ represents the $i$th row and $j$th column element of covariance $\boldsymbol{\Sigma}_\alpha$. The estimation error in $\mathbf{z}$ is denoted as $\Delta\mathbf{z}$, which can be represented as

$$\Delta\mathbf{z} = \left(\mathbf{A}^T \boldsymbol{\Sigma}_\alpha^{-1} \mathbf{A}\right)^{-1} \mathbf{A}^T \boldsymbol{\Sigma}_\alpha^{-1} \alpha \tag{14}$$

where $\alpha$ is the noise vector, so $\Delta\mathbf{z}$ is unknown. When the noise terms in $\boldsymbol{\Sigma}_\alpha$ are negligible, it is straightforward to show that $\Delta\mathbf{z}$ has zero mean, because $\alpha$ is zero mean.

The evaluations of (12) need the weighting matrix $\boldsymbol{\Sigma}_\alpha^{-1}$ that relies on the transmit time $t_0$ which is not available. We preliminarily consider $\boldsymbol{\Sigma}_\alpha^{-1}$ as unit matrix $\mathbf{I}$. Then putting the initial estimation $t_0$ into (13) gives an approximated $\boldsymbol{\Sigma}_\alpha^{-1}$ and reusing it in (12) would produce a better solution of the source position. According to the definition of $\mathbf{z}$, we obtain the initial source position estimation $\mathbf{x}_i^e = \mathbf{z}(1:2)$ and the estimated transmit time $t_0^e = \mathbf{z}(3)$.

4.2. **Position refinement.** The initial solution of source position is obtained, but the substraction method enlarges the noises. The initial estimation $\mathbf{x}_i^e$ is not optimal, so the further refinement would improve the positioning accuracy. Let $\mathbf{x} = \mathbf{x}^e + \Delta\mathbf{x}$, which means that $x = x^e + \Delta x$ and $y = y^e + \Delta y$. Similarly $t_0 = t_0^e + \Delta t$. Substituting (9) with these equations, we can obtain that

$$\begin{aligned} &2\left(x^e - x_i\right)\Delta x + 2\left(y^e - y_i\right)\Delta y + 2c^2\left(t_i - t_0^e\right)\Delta t_0 \\ &= c^2\left(t_i - t_0^e\right)^2 - \left[\left(x_i - x^e\right)^2 + \left(y_i - y^e\right)^2\right] - 2c\left(t_i - t_0\right)\varepsilon_i \end{aligned} \tag{15}$$

where $i = 1, 2, \ldots, N$. Let $\Delta\mathbf{z} = \left[\Delta\mathbf{x}^T \quad \Delta t_0\right]^T$, then (15) is rewritten as matrix form

$$\mathbf{C}\Delta\mathbf{z} = \mathbf{d} + \beta \tag{16}$$

where the row vector of $\mathbf{C}$ is equal to $[2\left(x^e - x_i\right) \quad 2\left(y^e - y_i\right) \quad 2c^2\left(t_i - t_0^e\right)]$. The element of vector $\mathbf{d}$ is equal to $\left[c^2\left(t_i - t_0^e\right)^2 - \left[\left(x_i - x^e\right)^2 + \left(y_i - y^e\right)^2\right]\right]$. The element of vector $\beta$

is equal to $[-2c(t_i - t_0)\varepsilon_i]$. $\mathbf{C} \in \mathbb{R}^{N\times 3}$, $\mathbf{d} \in \mathbb{R}^N$ and $\beta \in \mathbb{R}^N$. So the WLS solution to (16) is

$$\Delta\mathbf{z} = \left(\mathbf{C}^T\mathbf{\Sigma}_\beta^{-1}\mathbf{C}\right)^{-1}\mathbf{C}^T\mathbf{\Sigma}_\beta^{-1}\mathbf{d} \tag{17}$$

Since $\varepsilon_i$ are independent for $i \neq j$, so the covariance matrix of $\beta$, $\mathbf{\Sigma}_\beta$ has elements

$$\mathbf{\Sigma}_\beta[i,j] = \begin{cases} 0 & i \neq j \\ 4c^2(t_i - t_0)^2\delta_i^2 & i = j \end{cases} \tag{18}$$

where $i,\ j = 1, 2, \ldots, N$. $\mathbf{\Sigma}_\beta$ relies on the estimated transmit time $t_0$ which can be approximated with $t_0^e$. Extracting from $\Delta\mathbf{z}$ we obtain the refined $\Delta\mathbf{x} = \Delta\mathbf{z}(1:2)$. The improved source position is denoted as $\mathbf{x}$, which can be represented as

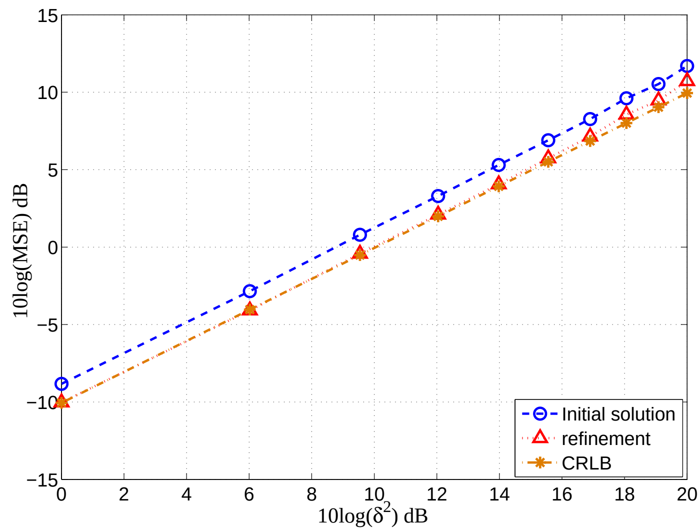
$$\mathbf{x} = \mathbf{x}^e + \Delta\mathbf{x} \tag{19}$$

In summary, we firstly estimate the initial source position by using the WLS method with (12) and obtain the initial source position estimation $\mathbf{x}_i^e$ and the transmit time $t_0^e$. Then the position refinement obtained with (17) is added to the initial source position for improving the positioning accuracy.

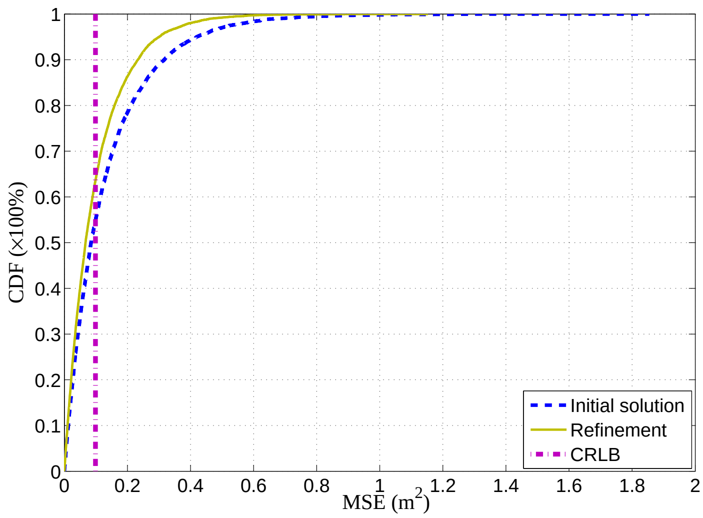
5. **Evaluation.** To test the performance of proposed algorithm, the simulations are conducted with MATLAB software. The anchors are deployed in a square area of 100 m × 100 m. In the region five anchors are set at the points $(20, 50)$, $(60, 95)$, $(65, 10)$, $(10, 80)$, $(10, 60)$. The source target is set at the point $(50, 50)$ in a prior. The noises between the source target and each anchor are subject to zero mean white Gaussian processes with zero mean and identical variance $\delta^2$. In order to evaluate the positioning performance in different conditions, mean square error (MSE) is used to evaluate the positioning accuracy. We verify the performance of the proposed method through Monte Carlo (MC) simulations and the number of samples used in the MC procedure is 10000. We firstly give the performance in terms of position MSE as the noises increase.

5.1. **Impacts of noises.** To test the impact noises, the transmit time is set at $t_0 = 5000$ ns in a prior. When the noise variance $\delta^2$ is varied from $1^2$ to $10^2$ (The noise in log scale is varied from 0 dB to 20 dB), Figure 1(a) plots the performance comparison of the initial solution and the position refinement under different noises. It is seen from Figure 1(a) that the position MSE is approximately linear with the noise in log scale. The MSE in log scale performance degrades with larger noises. The position MSE of the proposed refinement is less than that of the initial solution. For instance, when the noise variance $\delta^2$ is set to $1^2$ (noise variance in log scale is set to 0 dB), the position MSE in log scale with the refinement technique is about $-10.0$ dB, which is very close to the CRLB performance. However, the MSE in log scale of the initial solution is $-8.8$ dB. When all noise variance $\delta^2$ is set to $10^2$ (noise variance in log scale is set to 20 dB), the position MSE in log scale with the refinement technique is increased to 10.7 dB. However, the position MSE of the initial solution is 11.8 dB when the noise variance $\delta^2$ is set to $10^2$.

To further show the performance of the proposed algorithms with unknown transmit time, we plot the cumulative distribution function (CDF) in Figure 1(b) when the noise variance $\delta^2$ is set to $1^2$. It can be seen from Figure 1(b) that the proposed refinement algorithm performs well and that the estimation errors in over 65% of the simulated runs are less than the CRLB. However, the estimation errors of initial solution in only 48% of the simulated runs are less than the CRLB. 90% of the estimation error is less than 0.24 in the position refinement and 0.32 in the initial solution. Hence, the performance of the proposed refinement algorithm is quite good since the position refinement can reduce the localization error.




(a) MSE performance with different noises

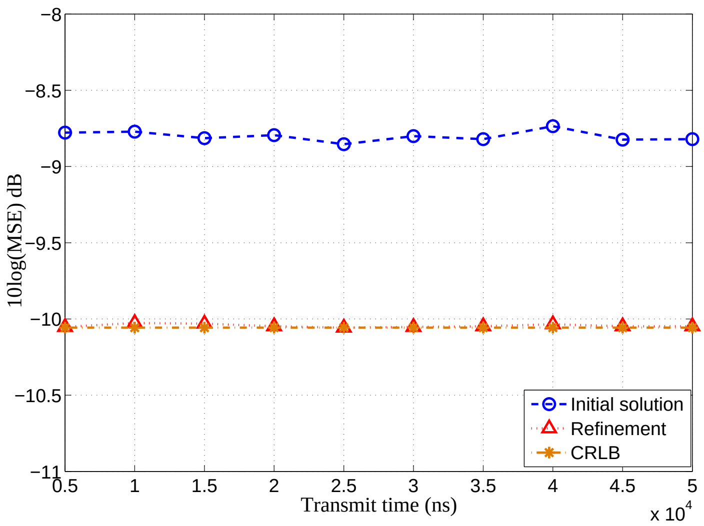



(b) CDF of MSE, $\delta^2 = 1^2$

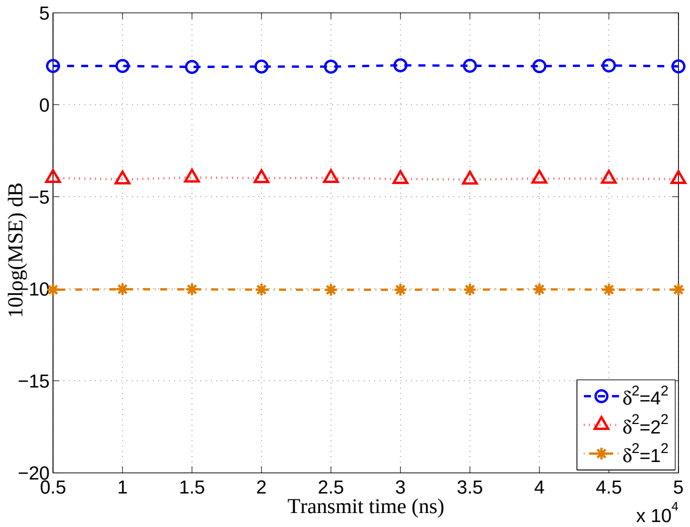
FIGURE 1. Impact of noises

5.2. **Transmit time.** When the transmit time is assumed to be unknown, the linear closed-form algorithm for the source position estimation is proposed. Apparently the estimation of source location is unrelated with the transmit time for the proposed algorithm. To test the MSE performance of different methods when the transmit time is unavailable, the noise variance $\delta^2$ is set to $1^2$, and Figure 2(a) plots the performance comparison of different methods as the transmit time increases. The order of different methods is the same as Figure 1(a). The refined position can attain the approximate position CRLB which provides optimal positioning accuracy. When the transmit time is set to 500 ns, the MSE in log-scale with the refinement method is $-10.1$ dB. When the transmit time is set to 5000 ns, the MSE in log-scale with the refinement method is also $-10.1$ dB. Due to the effective refinement the MSE in log-scale of the refinement is reduced by about 1.4 dB compared with that of the initial solution.

When the noise variance is set to $1^2$, $2^2$ and $4^2$, Figure 2(b) plots positioning MSE versus the transmit time using three different methods. It is seen from Figure 2(b) that MSE performance is unrelated with the transmit time. When the noises are set to $4^2$ and the transmit time is set to 500 ns, the position MSE of refinement method is 2.4 dB. When the noises are set to $4^2$ and the transmit time is set to 5000 ns, the position MSE




(a) MSE performance comparison when $\delta^2 = 1^2$




(b) MSE performance with different noises

FIGURE 2. Performance comparison with different transmit time

of the refinement method is also 2.4 dB. It is also seen from Figure 2(b) that the MSE performance degrades as the noise increases. When the noise variance is increased from $1^2$ to $4^2$, the MSE of refinement method is also increased from $-10.0$ dB to 2.4 dB.

6. **Conclusion.** In this paper, an accurate linear algebraic solution to the source position estimation is proposed by considering the transmit time as unavailable. By eliminating the nuisance parameters, the optimization problem is transformed into a linear least-squares estimation problem, so the initial solution to the source position estimation is obtained. Then the source position is further refined and its exact solution is obtained. The source position estimation of the algorithm is unrelated with the transmit time. The linear estimation method does not require iteration or initialization compared with the solution to the numerical calculation. Due to a large number of equality constraints and variables, the convex SDP algorithm runs slower than the linear algorithm. The proposed linear closed-form algorithm has the advantages of low computation complexity and high positioning accuracy compared with the convex SDP algorithm. The linear proposed closed-form algorithm can also be extended to multiple source nodes in the future work.

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