

## CHANNEL ESTIMATION BASED ON COMPRESSED SENSING IN FBMC/OQAM TRANSMISSION SYSTEM

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**ABSTRACT.** *Because of the imaginary valued intrinsic interference from neighbor symbols, the conventional preamble based channel estimation (CE) methods, interference approximation method (IAM) and interference cancellation method (ICM), are inefficient for filter bank multicarrier with offset quadrature amplitude modulation (FBMC/OQAM) system. In this paper, we propose a new way to estimate the sparse IEEE 802.22 multi-path channel based on compressed sensing (CS) for FBMC/OQAM transmission system. Two CS recovery algorithms, orthogonal matching pursuit (OMP) algorithm and compressive sampling matching pursuit (CoSaMP) algorithm, based preamble CE methods are proposed in this paper. The bit error rate (BER) and mean squared error (MSE) performance using least square (LS), and two CS based algorithms are given. Simulations results show that CS based preamble methods can provide significantly better BER and MSE than conventional preamble based LS method. CoSaMP based preamble CE method can obtain slightly better BER and MSE performance than OMP based preamble CE method with lesser time complexity under IEEE 802.22 channel.*

**Keywords:** FBMC/OQAM, Channel estimation, Compressed sensing, OMP, CoSaMP

**1. Introduction.** The filter bank multicarrier with offset quadrature amplitude modulation (FBMC/OQAM) [1,2] gets wide attention and extensive research recently, due to its higher spectral efficiency and increased robustness to frequency offset and Doppler spread. Because of the non-orthogonality between the real and imaginary FBMC/OQAM modulated signals that complicates the channel estimation (CE) process, the conventional OFDM CE methods cannot be directly applied to FBMC/OQAM. Some preamble based CE schemes [3] have been proposed to improve the CE performance in FBMC/OQAM system. The interference approximation method (IAM) and the interference cancellation method (ICM) are the two conventional preamble based CE methods. IAM aims at approximating the intrinsic imaginary interference from neighboring pilots. ICM is designed to cancel or avoid the interference. However, these two preamble based CE methods cannot reach the satisfactory effect, so further research is still needed.

Compressed sensing (CS) theory was proposed by E. Candes et al. [4] in 2006. It has been proved that if signal in a orthogonal space can be sparsely represented the signal sampling can use lower sampling frequency. With some recovery algorithm the signal can be restored with high probability, which contrasts with the Nyquist sampling theorem.

The main contributions of this paper are listed as follows.

1. For exploiting the spare channel characteristics, we employ the compressed sensing (CS) approach for channel estimation in FBMC/OQAM transmission system.
2. We propose effectively channel estimation methods based on preamble and CS theory, called IAM-OMP, IAM-CoSaMP, ICM-OMP and ICM-CoSaMP.

In the following, at first the system model is described and in Section 3 the CS recovery algorithms based on preamble channel estimation are introduced. Simulation results are given and analyzed in Section 4. Finally, Section 5 concludes this work.

**2. System Model.** The transmitted signal in FBMC/OQAM system can be written as [5]

$$s(t) = \sum_{m=0}^{N-1} \sum_n d_{m,n} g_{m,n}(t) \quad (1)$$

where  $d_{m,n}$  are real valued OQAM symbols, and  $g_{m,n}(t)$  represents the synthesis basis which is obtained by the time-frequency translated version of the prototype function  $g(t)$  in the following way

$$g_{m,n}(t) = g(t - n\tau_0) e^{i2\pi m F_0 t} e^{j\phi_{m,n}} \quad (2)$$

with  $N$  an even number of sub-carriers,  $F_0 = 1/T_0 = 1/2\tau_0$  the sub-carrier spacing,  $\phi_{m,n}$  an additional phase term.  $T_0$  represents the OFDM symbol duration, and  $\tau_0$  denotes the time offset between the real and imaginary parts of OQAM symbols. The double subscript  $(\cdot)_{m,n}$  denotes the  $(m, n)$ -th frequency-time (FT) point,  $m$  is the sub-carrier index and  $n$  the OQAM symbol time index.

The pulse  $g$  is designed so that the associated sub-carrier functions  $g_{m,n}$  are orthogonal in the real field, that is,

$$\Re \{ \langle g_{m,n} | g_{p,q} \rangle \} = \Re \left\{ \sum_t g_{m,n}(t) g_{p,q}^*(t) \right\} = \delta_{m,p} \delta_{n,q} \quad (3)$$

where  $\delta_{i,j}$  is the Kronecker delta,  $\delta_{m,p} = 1$  if  $m = p$  and  $\delta_{m,p} = 0$  if  $m \neq p$ . We can find that even in distortion-free channel and with perfect time and frequency synchronization, there still will be some purely imaginary inter-carrier interference at the output. We set interference weights

$$\langle g \rangle_{m,n}^{p,q} = -j \langle g_{m,n} | g_{p,q} \rangle \quad (4)$$

with  $\langle g_{m,n} | g_{p,q} \rangle$  a purely imaginary term for  $(m, n) \neq (p, q)$ .

The received signal is the convolution of the transmitted signal and the channel response plus noise. It is given by

$$r(t) = \sum_l^{L-1} s(t-l)h(t) + z(t) \quad (5)$$

where  $L$  is the number of sample-spaced channel taps,  $z(t)$  is the additive white Gaussian noise (AWGN) sample with zero mean and variance of  $\sigma^2$ , and  $h(t)$  is the multipath channel impulse response in time domain. The multipath channel impulse response can be described as

$$h(t) = \sum_{l=0}^{L-1} a_l(t) \delta(\tau - \tau_l) \quad (6)$$

where  $\tau_l$  is the delay of the  $l$ th path and  $a_l(t)$  is the complex amplitude of the  $l$ th path, which is assumed to be wide-sense-stationary (WSS) complex Gaussian process and independent between different paths.

**3. Preamble Channel Estimation Based on Compressed Sensing.** In this section, the preamble structures and CS theory for channel estimation are briefly described. We show two CS signal recovery algorithms, OMP [6] and CoSaMP [7].

In FBMC/OQAM, the preamble pilots exist in all sub-carriers, and the preamble sequence is superimposed on the data. Figures 1(a) and 1(b) show the IAM and ICM

preamble structures. Let us assume that a pilot symbol  $P_{m_0, n_0}$  is transmitted at a position  $(m_0, n_0)$  that is known by the receiver. The LS estimation is

$$\hat{H}_{m_0, n_0} = \frac{r_{m_0, n_0}}{P_{m_0, n_0}} = H_{m_0, n_0} + j \sum_{(m_0, n_0) \neq (m, n)} \frac{d_{m, n}}{P_{m_0, n_0}} \langle g \rangle_{m, n}^{m_0, n_0} \quad (7)$$

where  $j \sum_{(m_0, n_0) \neq (m, n)} \frac{d_{m, n}}{P_{m_0, n_0}} \langle g \rangle_{m, n}^{m_0, n_0}$  is imaginary interference.

CS theory states that a  $K$ -spares signal  $h$  can be stably recovered from linear measurement

$$y = \Phi h + z \quad (8)$$

where  $\Phi$  is a matrix with  $M$  rows and  $N$  columns,  $M \ll N$ , and  $z$  is noise. The premise is that the measurement matrix  $\Phi$  satisfies the restricted isometry property (RIP).

In this paper, we evaluate OMP and CoSaMP algorithms to estimate channel in FBMC/OQAM system. The two algorithms are shown as follows.

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**OMP algorithm**

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Input: residual  $\mathbf{r}$ , linear measurement  $\mathbf{y}$ , measurement matrix  $\mathbf{Y}$ , recovery matrix  $\mathbf{T}$ , sparse  $\mathbf{K}$

Output: channel estimator  $\mathbf{h}_{cs}$

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Initialize:  $\mathbf{T} = \phi$ ,  $\mathbf{Y} = \phi$ ,  $\mathbf{r} = \mathbf{y}$ , iterations  $\mathbf{i} = 1 : 1 : \mathbf{K}$

The absolute value of inner product of  $\mathbf{Y}^T$  and  $\mathbf{r} : \mathbf{U} = \text{sum}(\text{abs}(\mathbf{Y}^T * \mathbf{r}))$  where  $*$  is inner product operation;

Record the position of  $\mathbf{U}$  and its value:  $[\mathbf{V}, \mathbf{P}] = \max(\mathbf{U})$ ;

Measurement matrix  $\mathbf{Y}$  expansion:  $\mathbf{Y} = [\mathbf{Y}, \mathbf{T}(:, \mathbf{P})]$ ;

Signal approximation with least squares:  $\mathbf{h}_{cs} = (\mathbf{Y}^T * \mathbf{Y})^{-1} * \mathbf{Y}^T * \mathbf{r}$ ;

Update residual:  $\mathbf{r} = \mathbf{y} - \mathbf{Y} * \mathbf{h}_{cs}$ ;

Record the position of projection:  $\mathbf{Y}_s(:, \mathbf{i}) = \mathbf{Y}(:, \mathbf{P})$ ;

End iteration

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**CoSaMP algorithm**

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Input: residual  $\mathbf{r}$ , linear measurement  $\mathbf{y}$ , measurement matrix  $\mathbf{Y}$ , recovery matrix  $\mathbf{T}$ , sparse  $\mathbf{K}$

Output: channel estimator  $\mathbf{h}_{cs}$

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Initialize:  $\mathbf{T} = \phi$ ,  $\mathbf{Y} = \phi$ ,  $\mathbf{r} = \mathbf{y}$ , iterations  $\mathbf{i} = 1 : 1 : 2\mathbf{K}$

The absolute value of inner product of  $\mathbf{Y}^T$  and  $\mathbf{r} : \mathbf{U} = \text{sum}(\text{abs}(\mathbf{Y}^T * \mathbf{r}))$  where  $*$  is inner product operation;

Sort the value:  $[\mathbf{V}, \mathbf{P}] = \text{sort}(\mathbf{U}, 'descend')$ ;  $\mathbf{P} = \mathbf{P}(1 : 2\mathbf{K})$ ;

Record the position for current iteration:  $\mathbf{S}_P = \text{union}(\mathbf{P}, \mathbf{Y})$ ;

Current measurement matrix:  $\mathbf{Y} = \mathbf{Y}(:, \mathbf{S}_P)$ ;

Signal approximation with least squares:  $\mathbf{h}_{cs} = (\mathbf{Y}^T * \mathbf{Y})^{-1} * \mathbf{Y}^T * \mathbf{r}$ ;

Update residual:  $\mathbf{r} = \mathbf{y} - \mathbf{Y} * \mathbf{h}_{cs}$ ;

Record the position of projection:  $\mathbf{Y}_s(:, \mathbf{i}) = \mathbf{Y}(:, \mathbf{P})$ ;

End iteration

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**4. Simulation Results.** In this section, we present simulation results to verify our analysis. BER and MSE performances of the CS based preamble channel estimation methods versus the LS method are discussed. The sparse  $K$  is 6, and  $N$  is 2048. The MSE is plotted with respect to the signal to noise ratio (SNR). The simulation channel parameters based on IEEE 802.22 standard are listed in Table 1.

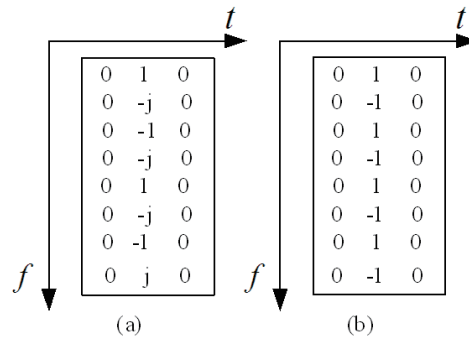


FIGURE 1. Preamble structures: (a) IAM, (b) ICM

TABLE 1. Simulation channel parameters

<b>Simulation channel</b>	IEEE 802.22. A channel with sampling frequency 6.86 MHz multi-paths delay (0 3 8 11 13 21) ( $\mu s$ ) relative power (0 -7 -15 -22 -24 -19) (dB)
<b>Channel coding</b>	Convolutional code ( $K = 7$ , with $g_1 = (133)_o$ , $g_2 = (171)_o$ and code rate=1/2)

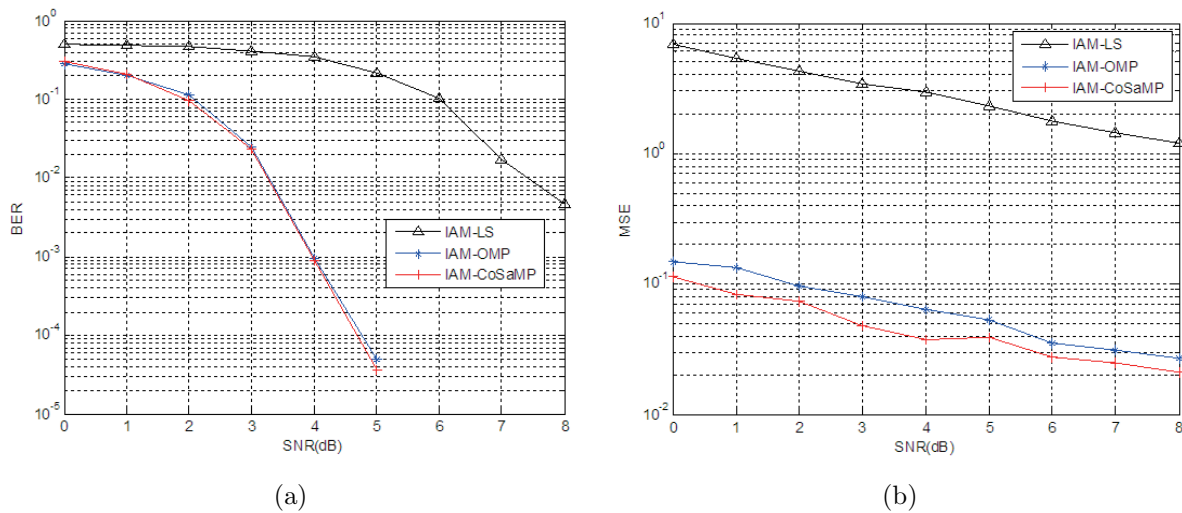


FIGURE 2. CE performances of LS, OMP and CoSaMP based IAM for (a) BER, (b) MSE

Figures 2(a) and 2(b) show the BER and MSE performance of IAM-LS, IAM-OMP and IAM-CoSaMP under IEEE 802.22 multi-paths channel model. It is obvious that, in both BER and MSE cases, CS based IAM approach can significantly outperform the LS method in the whole SNR range considered. In Figure 2(a), we can find that IAM-CoSaMP performs slightly better than IAM-OMP, but has a gain of about 3.1 dB compared with IAM-LS, when the BER of  $10^{-2}$  is considered. In Figure 2(b), IAM-CoSaMP performs the best MSE performance in the three schemes. It obtains about 1 dB SNR gain compared with IAM-OMP, when at the same MSE level. In addition, IAM-CoSaMP needs less simulation time than IAM-OMP; IAM-CoSaMP simulation time is 35.639836 s; IAM-OMP simulation time is 38.751858 s.

Figures 3(a) and 3(b) depict the BER and MSE performance of ICM-LS, ICM-OMP and ICM-CoSaMP under IEEE 802.22 multi-paths channel model. The trends of BER and MSE curves are the same as that in Figure 2. ICM-CoSaMP achieves the best MSE

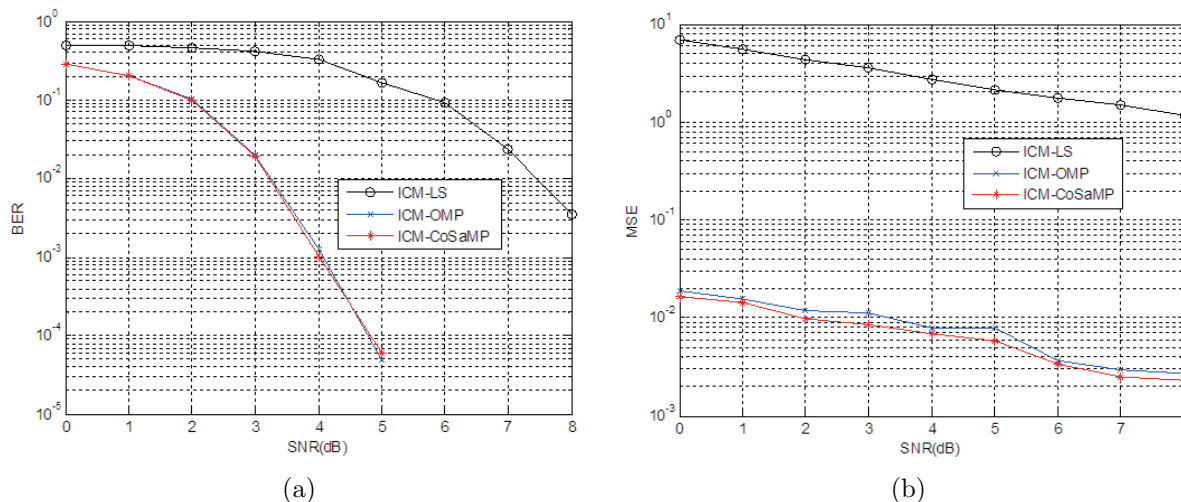


FIGURE 3. CE performances of LS, OMP and CoSaMP based ICM for (a) BER, (b) MSE

performance in the two preamble based CE scenarios. However, it takes about 37.927049 s to simulate. The simulation time is more than that in IAM-CoSaMP scheme.

As show in the simulation results, it can be verified that the CS based preamble approach can provide effectively channel estimation performance compared with conventional LS method in FBMC/OQAM system. CoSaMP based preamble approach needs less simulation time complexity than OMP based preamble approach.

**5. Conclusions.** In this paper, we have investigated the compressed sensing based preamble channel estimation for FBMC/OQAM transmission under IEEE 802.22 sparse multi-paths channel. BER and MSE performance are compared with LS, OMP and CoSaMP algorithms. Simulation results show that CS based preamble approach can achieve significantly better both BER and MSE performance than conventional LS based preamble method. It is verified that CS based preamble CE approach is an efficient method for channel estimation in FBMC/OQAM transmission system. In the future work, novel effective CS algorithm for channel estimation in FBMC/OQAM system needs to be studied.

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## REFERENCES

- [1] T. Maksymyuk and V. Pelishok, The LTE channel transmission rate increasing, *Proc. of 2012 International Conference on Modern Problems of Radio Engineering Telecommunications and Computer Science*, Lviv-Slavske, pp.251-252, 2012.
- [2] J. Nadal, C. A. Baghdadi and H. Lin, Hardware prototyping of FBMC/OQAM baseband for 5G mobile communication, *Proc. of the 25th IEEE International Symposium on Rapid System Prototyping*, New Delhi, India, pp.72-77, 2014.
- [3] D. Katselis and E. Kofidis, Preamble-based channel estimation for CP-OFDM and OFDM/QOAM systems: A comparative study, *IEEE Trans. Signal Processing*, vol.58, no.5, pp.2911-2917, 2010.
- [4] E. Candes, J. Romberg and T. Tao, Robust uncertainty principles: Exact signal recovery from highly incomplete frequency information, *IEEE Trans. Information Theory*, vol.52, no.4, pp.489-509, 2006.
- [5] C. Lele, J. P. Javaudin and R. Legouable, Channel estimation methods for preamble-based OFDM/OQAM modulations, *European Trans. Telecommunications*, vol.19, no.7, pp.741-750, 2008.

- [6] J. Tropp and A. Gilbert, Signal recovery from random measurements via orthogonal matching pursuit, *IEEE Trans. Information Theory*, vol.53, no.12, pp.4655-4666, 2007.
- [7] D. Needell and J. A. Tropp, CoSaMP: Iterative signal recovery from incomplete and inaccurate samples, *Communications of the ACM*, vol.53, no.12, pp.93-100, 2010.