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LS-SVM BASED CONSTRAINED INVERSE CONTROL FOR A CLASS OF NONLINEAR SYSTEMS

Nan Ji[1,2], Dezhi Xu[1,2] and Fei Liu[1,2]

[1]Key Laboratory of Advanced Process Control for Light Industry (Ministry of Education)
[2]Institute of Automation
Jiangnan University
No. 1800, Lihu Avenue, Wuxi 214122, P. R. China
lutxdz@126.com

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Abstract. *In this paper, constrained inverse control based on least squares support vector machines (LS-SVM) is proposed for the general unknown nonlinear systems. The proposed control method includes three parts which are LS-SVM, inverse controller and dynamic anti-windup compensator. In this control design, LS-SVM is used to obtain the Jacobian information of the unknown nonlinear system and the inverse controller is designed to control the dynamic system. At the same time, in order to solve the control input saturation of the dynamic nonlinear system, a dynamic anti-windup compensator is proposed for accommodating the reference. Due to the proposed scheme, the output tracking ability of the closed-loop control system can be guaranteed in spite of constrained unknown nonlinear process. Finally, the simulation results for a solid oxide fuel cell (SOFC) are provided to demonstrate the effectiveness of the proposed constrained control approach.*

Keywords: LS-SVM, Nonlinear system, Dynamic anti-windup, Constrained inverse control

1. **Introduction.** In industry environment, many systems are dynamic nonlinear processes in fact. The classical control methods based on linear systems are unstable and insufficient for nonlinear systems [1,2]. So alternate nonlinear control algorithms need to be found for good performance index and satisfactory stability. Neural network (NN) control based on adaptive or predictive techniques is a commonly used method in the last three decades [3,4]. However, there are some shortages of NN control, such as slow convergence speed and the overfitting phenomenon [5]. In recent years, SVM and LS-SVM based on statistical learning theory (SLT) are the new machine learning methods for nonlinear systems [6]. SVM and LS-SVM are all based on minimizing the structural risk principle. Compared with SVM, LS-SVM has the fast training speed, the simple calculation and the less uncertain training results, so LS-SVM is more suitable for computation of on-line implementation in nonlinear systems [7]. Moreover, compared with the classical NNs, the generalization ability of LS-SVM is more likely to be accepted by people [8].

Though, the method of LS-SVM has been widely used in many nonlinear systems, few methods of LS-SVM consider the actuator saturation problem. If the actuator saturation factor is not considered, controllers may appear saturated and unstable. So researchers have paid close attention to dynamical systems subject to input saturation. In linear systems, a general method to solve this problem of saturation is to treat the system as the sector nonlinearity [9]. The linear differential inclusion approach is another effective way for solving this problem, and this strategy puts the saturated linear feedback inside the convex hull of the set in auxiliary linear feedbacks [10]. However, in nonlinear systems, there is no general effective method to deal with the saturation problem due to complex dynamic characteristics of the nonlinear systems. In [11], a new method of anti-windup compensator for nonlinear systems was proposed in the condition that people must find the equivalent functions to replace the original unknown nonlinear function. However, in practice, it is very difficult to realize.

In this paper, LS-SVM based constrained inverse control is proposed for the general unknown nonlinear systems. This method utilizes Jacobian information via LS-SVM to build saturation compensator. An application of the proposed controller design for SOFC demonstrates the effectiveness of this algorithm. The rest of this paper is organized as follows. The description of LS-SVM is given in Section 2. In Section 3, an LS-SVM based constrained inverse controller is designed. In Section 4, simulation results for SOFC are presented to show the effectiveness of the proposed method. Finally, some conclusions are made at the end of this paper.

2. **LS-SVM Description.** LS-SVM is an effective method which makes the input data into the high dimensional feature space to predict the time series data. Moreover, it can create a linear regression function for the system [12]. LS-SVM can reduce the complexity of the calculation and improve the training speed. Due to these advantages, LS-SVM is widely used in many fields.

Firstly, we define the training samples $(\boldsymbol{x}_\kappa, y_\kappa)_{\kappa=1}^N$, $\kappa = 1, 2, 3, \ldots, N$, where $\boldsymbol{x}_\kappa$ is the sample input vector, y_κ is the corresponding target, and N is the number of samples. Then, the support vector regressive function can be described as follows

$$f(\boldsymbol{x}) = W^T \varphi(\boldsymbol{x}) + \gamma \tag{1}$$

where $\varphi(\boldsymbol{x})$ is a nonlinear function which maps the input data into higher dimensional feature space. W and γ are weight vector and bias, respectively.

According to the model of LS-SVM, the performance index function of this constraint optimization problem is written as follows

$$J = \frac{1}{2}||W||^2 + \frac{C}{2}\sum_{\kappa=1}^{N} \zeta_\kappa \tag{2}$$

And the constraint condition is defined as

$$y_\kappa = W^T \varphi(\boldsymbol{x}_\kappa) + \gamma + \zeta_\kappa \tag{3}$$

where C is the regularization parameter and ζ_κ is the relaxation factor of nonsensitive loss function. Then, we define the Lagrange function which can be described as

$$L = J - \sum_{\kappa=1}^{N} \alpha_\kappa \left[W^T \varphi(\boldsymbol{x}_\kappa) + \gamma + \zeta_\kappa - y_\kappa \right] \tag{4}$$

where α_κ denotes the Lagrange multiplier of the function. Based on the Karush-Kuhn-Tucker (KKT) conditions, we can get some functions from Equation (4) as follows

$$\begin{aligned} & W = \sum_{\kappa=1}^{N} \alpha_\kappa \varphi(\boldsymbol{x}_\kappa) \\ & \sum_{\kappa=1}^{N} \alpha_\kappa = 0 \\ & \alpha_\kappa = C\zeta_\kappa \\ & y_\kappa = W^T \varphi(\boldsymbol{x}_\kappa) + \gamma + \zeta_\kappa \end{aligned} \tag{5}$$

According to Equation (5), we can obtain the equivalent linear system function which can be described as

$$\begin{bmatrix} 0 & \boldsymbol{I}^T \\ \boldsymbol{I} & \Omega + C^{-1}\boldsymbol{I} \end{bmatrix} \begin{bmatrix} \gamma \\ \boldsymbol{a} \end{bmatrix} = \begin{bmatrix} 0 \\ \boldsymbol{y} \end{bmatrix} \tag{6}$$

where $\boldsymbol{I} = [I_1, I_2, \ldots, I_N]^T$, $\boldsymbol{a} = [\alpha_1, \alpha_2, \ldots, \alpha_N]^T$, $\boldsymbol{y} = [y_1, y_2, \ldots, y_N]^T$ and $\Omega = Q(\boldsymbol{x}_i, \boldsymbol{x}_j) = \varphi(\boldsymbol{x}_i)^T \varphi(\boldsymbol{x}_j)$, $i, j = 1, 2, \ldots, N$. Here, we can select the Gaussian function $Q(\boldsymbol{x}_\kappa, \boldsymbol{x})$ which is defined as

$$Q(\boldsymbol{x}_\kappa, \boldsymbol{x}) = \exp\left(-\frac{\|\boldsymbol{x} - \boldsymbol{x}_\kappa\|^2}{2\tau_\kappa^2}\right), \ \kappa = 1, 2, \ldots, N \tag{7}$$

where τ_κ denotes the basis width parameter. So the nonlinear regression function can be written as follows

$$f(\boldsymbol{x}) = \sum_{\kappa=1}^{N} \alpha_\kappa Q(\boldsymbol{x}_\kappa, \boldsymbol{x}) + \gamma \tag{8}$$

3. **LS-SVM Based Constrained Inverse Controller Design.** In this section, we propose constrained inverse control based on LS-SVM. For the convenient computing, we make our research contents only to single-input-single-output (SISO) nonlinear systems. The extension of the proposed algorithm to multi-input-multi-output (MIMO) systems is very easy, so we will not discuss the method of MIMO systems here. The model of classical SISO nonlinear systems can be described as

$$y(k+1) = F\left[y(k), \ldots, y(k-n+1), u(k), \ldots, u(k-m+1), d(k)\right] \tag{9}$$

where k denotes the discrete time. $y(k)$, $u(k)$ and $d(k)$ are the controlled output, the control input and the disturbance, respectively. $F(\cdot)$ is the general function of the nonlinear systems, and n and m represent process orders.

In the process of actual control, the control input $u(k)$ is not infinitely great or infinitely little. Besides that, it also cannot change too fast in a small interval due to the inertia. So the control input $u(k)$ is subject to magnitude and rate constraints as follows

$$\begin{aligned} u_{\min}(k) \le u(k) \le u_{\max}(k) \\ \Delta u_{\min}(k) \le \Delta u(k) \le \Delta u_{\max}(k) \end{aligned} \tag{10}$$

where $\Delta u(k)$ denotes the increment of control input $u(k)$.

Define $\boldsymbol{x} = [Y(k), u(k), U(k-1), d(k)]$, where $Y(k) = [y(k), \ldots, y(k-n+1)]$ and $U(k-1) = [u(k-1), \ldots, u(k-m+1)]$ are the set of the past outputs and the set of the past control inputs, respectively. Using LS-SVM to identify the dynamic model of Equation (9), we can obtain the following LS-SVM identification as follows

$$y_m(k+1) = \hat{F}(\boldsymbol{x}) = \sum_{\kappa=1}^{N} \alpha_\kappa Q(\boldsymbol{x}_\kappa, \boldsymbol{x}) + \gamma \tag{11}$$

Define the new tracking error $e(k) = y^*(k) - y(k) - \xi(k)$. Since the dynamic constraints exist in the close-loop control system, an anti-windup compensator is designed to accommodate the reference trajectory $y^*(k)$ where $y(k)$ denotes output of the system. The compensation signal $\xi(k)$ is presented as the following

$$\xi(k) = \rho\xi(k-1) + \frac{\partial y_m(k+1)}{\partial u(k)}(u_c(k) - u(k)) \tag{12}$$

where $\rho < 1$. The algorithm of the inverse control law commonly known in the literature is as follows

$$u_c(k) = u(k-1) + \frac{\frac{\mu \partial y_m(k+1)}{\partial u(k)}}{\lambda + \left(\frac{\partial y_m(k+1)}{\partial u(k)}\right)^2} e(k) \tag{13}$$

where μ is a modification item. λ is a positive control effort weighting factor. Based on the above calculation, Jacobian information $\frac{\partial y_m(k+1)}{\partial u(k)}$ (sensitivity of plant output to controlled input) algorithm is as follows

$$\frac{\partial y_m(k+1)}{\partial u(k)} = \frac{\partial \hat{F}(\boldsymbol{x})}{\partial u(k)} = \sum_{\kappa=1}^{N} \alpha_\kappa Q(\boldsymbol{x}_\kappa, \boldsymbol{x}) \frac{\partial \boldsymbol{x}}{\partial u(k)} \cdot \frac{\boldsymbol{x}_\kappa - \boldsymbol{x}}{\tau_\kappa^2} \tag{14}$$

Based on the input constraints (10), the LS-SVM based constrained inverse controller is described as

$$u(t) = \mathrm{Sat}\left\{\left(u(k-1) + \mathrm{Sat}\left\{\left(u_c(k) - u(k-1)\right), T\Delta u_{\min}, T\Delta u_{\max}\right\}\right), u_{\min}, u_{\max}\right\} \tag{15}$$

where T is sampling time, and $\mathrm{Sat}(\cdot)$ function is defined as

$$\mathrm{Sat}(a,b,c) = \begin{cases} b & a \le b \\ a & b < a < c \\ c & a \ge c \end{cases}$$

4. **Simulation Results.** In this section, the proposed LS-SVM based constrained inverse control algorithm discussed above is applied to achieve safe fuel utilization and satisfy the operating constraints when the current load and voltage output are measurable online for SOFC. Its structure is shown in Figure 1.

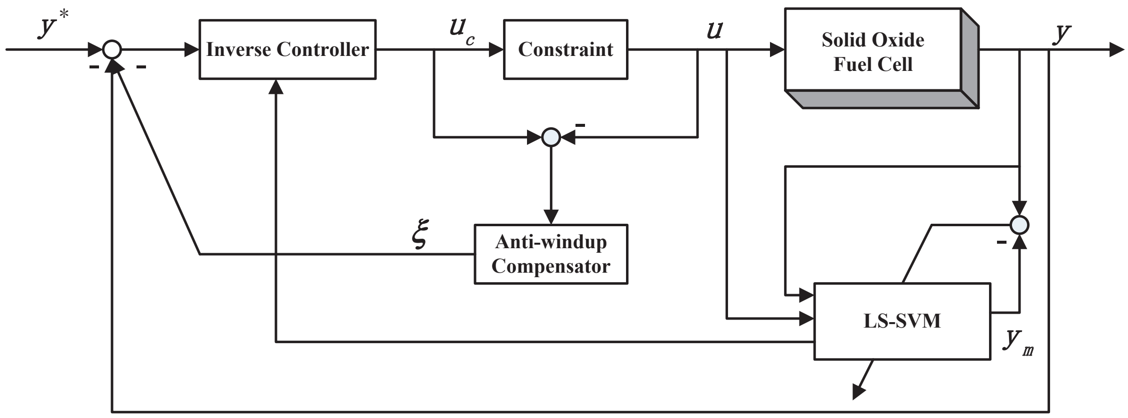

Figure 1. Structure of LS-SVM based constrained inverse controller

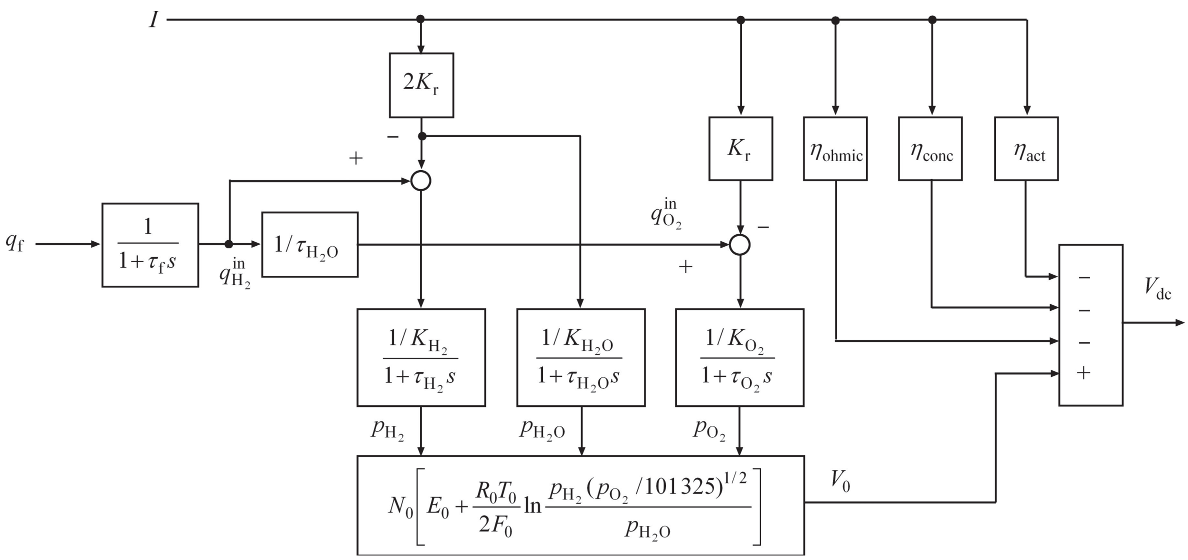

Figure 2. SOFC system dynamic model

In this paper, take the SOFC dynamical model widely accepted as the object of study [13,14], which is shown in Figure 2, where V_{dc} is the stack output voltage (V), q_f is the natural gas (e.g., H_2) flow rate (mol/s), and I expresses the measurable external current load (A); p_{H_2}, p_{O_2}, and $p_{\mathrm{H}_2\mathrm{O}}$ denote the partial pressures of hydrogen, oxygen, and water (Pa), respectively. $q_{\mathrm{H}_2}^{in}$ and $q_{\mathrm{O}_2}^{in}$ are the input flow rates of hydrogen and oxygen (mol/s), respectively. Applying Nernst's equation and taking into account ohmic, concentration, activation losses (i.e., η_{ohmic}, η_{conc}, and η_{act}), the stack output voltage V_{dc} is described as follows [15,16],

$$V_{dc} = V_0 - \eta_{ohmic} - \eta_{conc} - \eta_{act} \tag{16}$$

where

$$V_0 = N_0 \left[E_0 + \frac{R_0 T_0}{2F_0} \ln \frac{p_{\mathrm{H}_2}\sqrt{p_{\mathrm{O}_2}/101,325}}{p_{\mathrm{H}_2\mathrm{O}}} \right] \tag{17}$$

$$p_{\mathrm{H}_2} = \frac{1}{K_{\mathrm{H}_2}(1+\tau_{\mathrm{H}_2}s)} \left(\frac{1}{1+\tau_f s} q_f - 2K_r I \right) \tag{18}$$

$$p_{\mathrm{O}_2} = \frac{1}{K_{\mathrm{O}_2}(1+\tau_{\mathrm{O}_2}s)} \left(\frac{1/\tau_{\mathrm{H-O}}}{(1+\tau_f s)} q_f - K_r I \right) \tag{19}$$

$$p_{\mathrm{H}_2\mathrm{O}} = \frac{2}{K_{\mathrm{H}_2\mathrm{O}}(1+\tau_{\mathrm{H}_2\mathrm{O}}s)} K_r I \tag{20}$$

$$\eta_{ohmic} = Ir, \ \eta_{conc} = \partial + \beta \ln I, \ \eta_{act} = -\frac{R_0 T_0}{2F_0} \ln I \left(1 - \frac{I}{I_L}\right) \tag{21}$$

The parameters of the SOFC model are summarized in Table 1 [16].

q_f, V_{dc} and I are redefined as the control input u, control objective y and disturbance d, respectively. The input-output relation of SOFC can be rewritten in the following unknown one-order nonlinear autoregressive with exogenous input (NARX) model:

$$y(k+1) = F(y(k), u(k), d(k)) \tag{22}$$

Table 1. Parameters in the SOFC system model

Parameter	Value	Unit	Representation
T_0	1273	K	Absolute temperature
F_0	96,485	C/mol	Faraday's constant
R_0	8.314	J/(mol K)	Universal gas constant
E_0	1.18	V	Ideal standard potential
N_0	384	–	Number of cells in series in the stack
K_r	0.996×10^{-3}	mol/(s A)	Constant, $K_r = N_0/4F_0$
K_{H_2}	8.32×10^{-6}	mol/(s Pa)	Valve molar constant for hydrogen
$K_{\mathrm{H}_2\mathrm{O}}$	2.77×10^{-6}	mol/(s Pa)	Valve molar constant for water
K_{O_2}	2.49×10^{-5}	mol/(s Pa)	Valve molar constant for oxygen
τ_{H_2}	26.1	s	Response time of hydrogen flow
$\tau_{\mathrm{H}_2\mathrm{O}}$	78.3	s	Response time of water flow
τ_{O_2}	2.91	s	Response time of oxygen flow
$\tau_{\mathrm{H-O}}$	1.145	–	Ratio of hydrogen to oxygen
r	0.126	Ω	Ohmic loss
τ_f	5	s	Time constant of the fuel processor
∂	0.05	–	Tafel constant
β	0.11	–	Tafel slope
I_L	800	A	Limiting current density

As an independent power source candidate, we must ensure that output voltage of the SOFC system is expected to be at a desired constant value. The external current load I directly affects the output voltage of the SOFC system. In normal working conditions, the current load I of the SOFC system is 300 A. The steady output of the voltage is 332.8 V. The safe fuel utilization ρ satisfies $[\rho_{\min}, \rho_{\max}] = [0.7, 0.9]$, and maintains operational constraints are considered as $[q_{f\,\min}, q_{f\,\max}] = [0, 1.2]$ mol/s and $[\dot{q}_{f\,\min}, \dot{q}_{f\,\max}] = [-0.7, 0.7]$ mol/s^2, respectively.

For the simulations, the modification item μ and the positive control effort weighting factor λ are selected as 1.7 and 2, respectively. Open-loop input-output data samples are obtained by exciting the open-loop SOFC system using designed sine signals $0.7823 + 0.3\sin(0.5t)\sin t$ and $300 + 50\sin(0.03t)\sin(0.04t)$ for the fuel and the current demand (i.e., q_f and I), respectively. For comparing the performance, the initial values of PID control parameters are $k_p(0) = 0.0312$, $k_i(0) = 0.000903$ and $k_d(0) = 0$.

In simulations, LS-SVM identification curves of SOFC are shown in Figure 3. From Figure 3, we observe that the test error is very small compared with the output value, so it demonstrates that LS-SVM can approximate the behavior of the SOFC system with good accuracy. Assuming that current load I changes as Figure 4 and a typical system response using the proposed algorithm is depicted in Figure 4, in which we can see that the tracking error is faster to a small neighborhood of zero than PID control, where PID controller parameters are $k_p = 0.0312$, $k_i = 0.000903$. Moreover, we find that the SOFC control system achieves desired utilization and satisfies the operating constraints when we adopt the proposed algorithm. At the same time, the typical PID control method cannot obtain the satisfactory utilization.

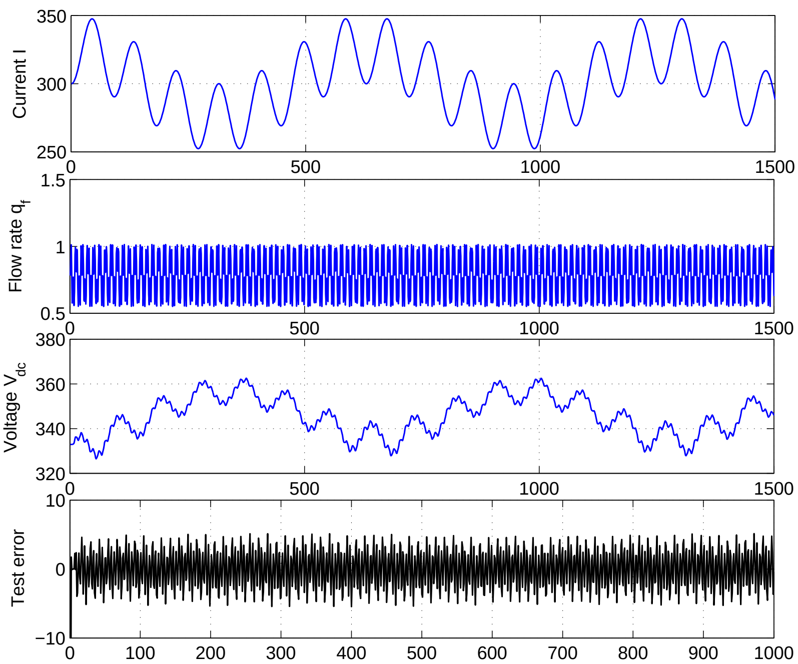

FIGURE 3. LS-SVM identification curves of SOFC

5. **Conclusions.** In this paper, the LS-SVM based constrained inverse control is proposed for a class of unknown nonlinear systems, which provides an effective alternative to the nonlinear processes. A dynamic constraints unit with anti-windup scheme is adopted to control input within an effective and safe range as long as possible. It is worth mentioning that this proposed control approach of SOFC is particularly useful while the explicit analytical model of SOFC is difficult to obtain. The simulation results have validated

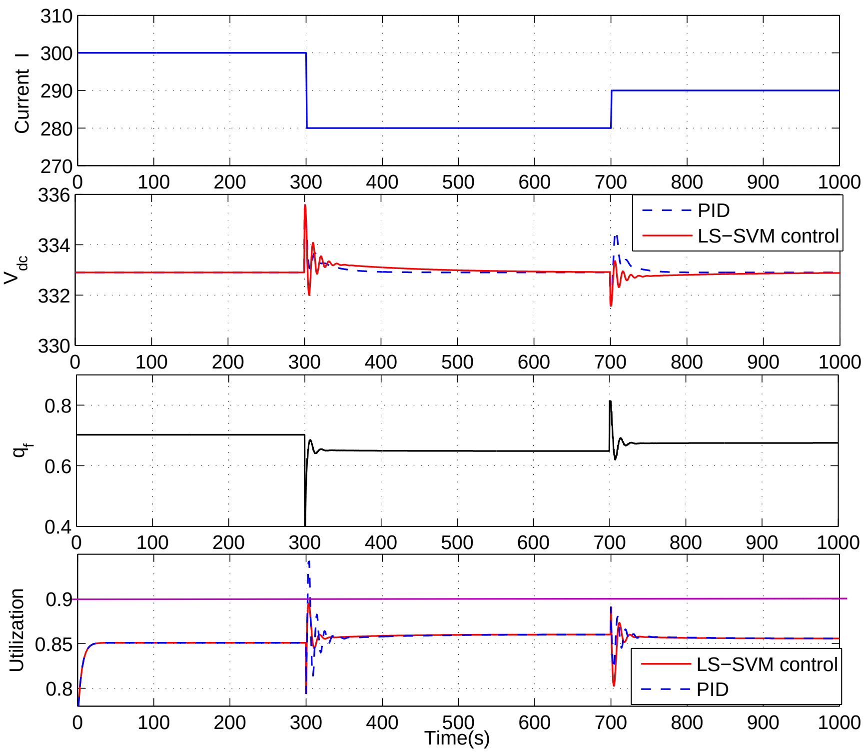

FIGURE 4. SOFC response curves

the proposed LS-SVM based constrained nonlinear control algorithm. In future, research topic is to introduce the instrumental variable method to further counteract the effect of noise. Furthermore, we will introduce adaptive technology into the proposed algorithm of this paper.

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