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ADAPTIVE FUZZY CONTROL FOR MIMO NONLINEAR SYSTEM WITH UNMODELED DYNAMICS

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Abstract. *In this paper, an adaptive fuzzy control scheme is proposed for a class of multiple input and multiple output (MIMO) nonlinear systems with unmodeled dynamics. To solve the difficulties from the unmodeled dynamics, a dynamic signal is introduced. Fuzzy systems are used to approximate the packaged unknown nonlinearities, and an adaptive fuzzy control approach is developed via backstepping, which guarantees that all the signals in the closed-loop system are semi-globally uniformly ultimately bounded. Simulation results are used to show the effectiveness of the proposed control scheme.*

Keywords: Adaptive fuzzy control, MIMO nonlinear systems, Unmodeled dynamics

1. **Introduction.** During the past several decades, a large number of research results [1-6] have been obtained on adaptive backstepping control of multiple input and multiple output (MIMO) nonlinear systems which extensively existed in the practical engineering. For example, in [1], a robust stabilization control approach is proposed for a class of MIMO nonlinear systems via backstepping. Further, Chen et al. [2] consider the problem of adaptive backstepping tracking control of strict-feedback MIMO nonlinear systems with input constraints. Alternatively, by using universal approximators, such as fuzzy systems and radial basis functions neural networks, together with adaptive backstepping technique, many approximation-based adaptive backstepping control schemes [3, 4] are developed for nonlinear systems with unknown functions. It is well known that unmodeled dynamics frequently exist in practical systems and are often a source of instability of the control systems. Therefore, in the controller design process, the effect of unmodeled dynamics to the control systems could not be ignored, and there exist many significant results which have been reported in [5, 6] and the references therein.

Motivated by the above observations, in this paper, an adaptive fuzzy control approach is proposed for a class of MIMO nonlinear systems with unmodeled dynamics. During the controller design, fuzzy systems are introduced to approximate the packaged unknown nonlinearities. Then, an adaptive fuzzy control scheme is derived via backstepping, which can guarantee the semi-global boundedness of resulting closed-loop systems. The main advantage of this research is that only one adaptive law is required to be updated online for each subsystem. Finally, a numerical example is provided to illustrate the effectiveness of the presented approach.

2. **Problem Formulation and Preliminaries.** In this paper, we consider a class of MIMO nonlinear systems. Its ith ($i = 1, 2, \ldots, N$) subsystem is

$$\begin{cases} \dot{z}_i = q_i(z_i, x_i), \\ \dot{x}_{ij} = g_{ij}\left(\underline{x}_{ij}\right) x_{ij+1} + f_{ij}\left(\underline{x}_{ij}\right) + \Delta_{ij}\left(\underline{x}_{ij}, z_i, t\right), \ i=1,2,\ldots,N; \ j=1,\ldots,n_i-1, \\ \dot{x}_{in_i} = g_{in_i}\left(\underline{x}_{in_i}\right) u_i + f_{in_i}\left(\underline{x}_{ij}\right) + \Delta_{in_i}\left(\underline{x}_{in_i}, z_i, t\right), \\ y_i = x_{i1}, \end{cases} \tag{1}$$

where $x_i = [x_{i1}, x_{i2}, \ldots, x_{in_i}]^T \in R^{n_i}$, $y_i \in R$ and u_i are the state vector, the scalar output and control input of the ith nonlinear subsystem, respectively; $\underline{x}_{ij} = [x_{i1}, x_{i2}, \ldots, x_{ij}]^T \in R^j$. $z_i \in R^{n_{i0}}$ in (1) denotes the unmeasured portion of the state. The z_i-dynamics in (1) is the unmodeled dynamics, $\Delta_{ij}(\cdot)$ is an uncertain dynamic disturbance, $f_{ij}(.)$ and $g_{ij}(.)$ are unknown smooth nonlinear functions with $f_{ij}(0) = 0$. It is supposed that $\Delta_{ij}(\cdot)$ and $q_i(\cdot)$ in (1) are uncertain Lipschitz continuous functions.

The objective of this paper is to design an adaptive fuzzy control scheme such that all the signals in the closed-loop system are semi-globally uniformly ultimately bounded.

Assumption 2.1. [5] *For the dynamic disturbances Δ_{ij} in (1), there exist unknown non-negative smooth functions $\phi_{ij1}(\cdot)$ and $\phi_{ij2}(\cdot)$, such that*

$$|\Delta_{ij}| \le \phi_{ij1}\left(\left|\underline{x}_{ij}\right|\right) + \phi_{ij2}(|z_i|). \tag{2}$$

Assumption 2.2. [5] *The unmodeled dynamics in (1) is exponentially input-to-state practically stable (exp-ISpS); i.e., for the system $\dot{z}_i = q_i(z_i, x_i)$, there exists an exp-ISpS Lyapunov function $V_i(z_i)$ such that*

$$\alpha_{i1}(|z_i|) \le V_i(z_i) \le \alpha_{i2}(|z_i|), \tag{3}$$

$$\frac{\partial V_i(z_i)}{\partial z_i} q_i(z_i, x_i) \le -c_i V_i(z_i) + \mu_i(|x_{i1}|) + d_i, \tag{4}$$

where α_{i1}, α_{i2} and μ_i are of class K_∞-functions, c_i and d_i are known positive constants.

Assumption 2.3. [8] *For $1 \le j \le n_i$, the signs of $g_{ij}\left(\underline{x}_{ij}\right)$ are known, and there exist unknown positive constants b and b_M such that*

$$0 < b \le \left|g_{ij}\left(\underline{x}_{ij}\right)\right| \le b_M < \infty, \quad \forall \bar{x}_{ij} \in R^{n_i}. \tag{5}$$

Lemma 2.1. [7] *Let $f(x)$ be a continuous function defined on a compact set Ω. Then for any given constant $\varepsilon > 0$, there exists a fuzzy logic system $W^T S(x)$ such that $\sup_{x\in\Omega} \left|f(x) - W^T S(x)\right| \le \varepsilon$, where $W = [w_1, w_2, \ldots, w_N]^T$ is the ideal constant weight vector, $S(x) = [s_1(x), \ldots, s_N(x)]^T / \sum_{j=1}^N s_j(x)$ is the basis function vector, $N > 1$ is the number of the fuzzy rules and $s_j(x)$ are chosen as Gaussian functions, that is, $s_j(x) = \exp\left[\frac{-(x-\mu_i)^T(x-\mu_i)}{\eta_i^2}\right]$, $i = 1, 2, \ldots, N$ with $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{in}]^T$ being the center vector and η_i the width of the Gaussian function.*

In the next section, a backstepping-based adaptive control procedure will be proposed. Both the virtual control signals and adaption laws will be designed as

$$\alpha_{ij}(\bar{x}_{ij}) = -(\lambda_{ij} + 0.5)\bar{x}_{ij} - \frac{1}{2a_{ij}^2}\bar{x}_{ij}\hat{\theta}_i S_{ij}^T(X_{ij}) S_{ij}(X_{ij}), \tag{6}$$

$$\dot{\hat{\theta}}_i = \sum_{j=1}^{n_i} \frac{\gamma_i}{2a_{ij}^2} \bar{x}_{ij}^2 S_{ij}^T(X_{ij}) S_{ij}(X_{ij}) - \sigma_i \hat{\theta}_i, \tag{7}$$

where, for $1 \le i \le N$, $1 \le j \le n_i$, λ_{ij}, a_{ij}, γ_i and σ_i are positive design parameters, $X_{ij} = \left[\underline{x}_{ij}^T, \hat{\theta}_i, r_i\right]^T$ with $\underline{x}_{ij} = [x_{i1}, x_{i2}, \ldots, x_{ij}]^T$, and $\bar{x}_{ij}$ satisfies the following coordinate transformation:

$$\bar{x}_{ij} = x_{ij} - \alpha_{i(j-1)}, \tag{8}$$

where $\alpha_{i0} = 0$, $\hat{\theta}_i$ is the estimation of unknown constant θ_i which is defined as

$$\theta_i = \frac{1}{b} \parallel W_{ij} \parallel^2; \quad 1 \le i \le N, \ \ 1 \le j \le n_i, \tag{9}$$

where $\parallel W_{ij} \parallel$ denotes the norm of the ideal weight vector of fuzzy logic systems, which will be specified at the jth design step. Specifically, α_{in_i} denotes the control input u_i.

Lemma 2.2. [5] *If V_i is an exp-ISpS Lyapunov function for a control system, i.e., Equations (3) and (4) hold, then for any constant $\bar{c}_i$ in $(0, c_{i0})$, any initial condition $x_{i0} = x_{i0}(0)$, and any function $\bar{\mu}(x_{i1}) \geq \mu(|x_{i1}|)$ there exists finite time $T_{i0} = T_{i0}(\bar{c}_i, r_{i0}, z_{i0})$, nonnegative function $D_i(t)$ defined for all $t \geq 0$ and a signal described by*

$$\dot{r}_i = -\bar{c}_i r_i + \bar{\mu}_i(x_{i1}(t)) + d_i, \quad r_i(0) = r_{i0}, \tag{10}$$

such that $D_i(t) = 0$ for all $t \geq T_{i0}$,

$$V_i(z_i(t)) \leq r_i(t) + D_i(t). \tag{11}$$

For all $t \geq 0$, the solutions are defined. Without losing of generality, this paper takes $\bar{\mu}_i(.)$ as $\bar{\mu}_i(s) = s^2 \mu_{i0}(s^2)$, where $\bar{\mu}_i(.)$ is a nonegative smooth function. Therefore, the dynamical r_i defined by (10) becomes

$$\dot{r}_i = -\bar{c}_i r_i + x_{i1}^2 \mu_{i0}\left(\left|x_{i1}^2\right|\right) + d_{i0}, \quad r_i(0) = r_{i0}, \tag{12}$$

where μ_{i0} is a nonnegative smooth function.

3. **Adaptive Fuzzy Control Design.** For simplicity, the time variable t and the state vector $\underline{x}_{ij}$ are omitted from the corresponding functions and let $S_{ij}(X_{ij}) = S_{ij}$.

Step 1. Based on $\bar{x}_{i1} = x_{i1}$, let us first consider the subsystem

$$\begin{aligned} \dot{z}_i &= q_i(z_i, x_i), \\ \dot{x}_{i1} &= g_{i1} x_{i2} + f_{i1} + \Delta_{i1}(x_{i1}, z_i, t). \end{aligned} \tag{13}$$

To stabilize the subsystem (13), we consider a Lyapunov function as

$$V_{i1} = \frac{1}{2}\bar{x}_{i1}^2 + \frac{1}{\lambda_{i0}}\dot{r}_i + \frac{b}{2\gamma_i}\tilde{\theta}_i^2. \tag{14}$$

Then, the time derivative of V_{i1} is

$$\begin{aligned} \dot{V}_{i1} \leq\ & \bar{x}_{i1}(g_{i1}x_{i2} + f_{i1}) + |\bar{x}_{i1}|\phi_{i11}(|x_{i1}|) + |\bar{x}_{i1}|\phi_{i12}(|z_i|) - \frac{\bar{c}_i}{\lambda_{i0}} r_i \\ & + \frac{1}{\lambda_{i0}}\left(x_{i1}^2 \mu_{i0}\left(\left|x_{i1}^2\right|\right) + d_{i0}\right) - \frac{b}{\gamma_i}\tilde{\theta}_i \dot{\hat{\theta}}_i, \end{aligned} \tag{15}$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$. By Assumption 2.1 and $0 \leq |\eta| - \eta \tanh\left(\frac{\eta}{\epsilon}\right) \leq \delta\epsilon$, $\delta = 0.2785$, we obtain

$$|\bar{x}_{i1}|\phi_{i11}(|x_{i1}|) \leq \bar{x}_{i1}\hat{\phi}_{i11}(x_{i1}) + \epsilon'_{i11}, \tag{16}$$

where $\epsilon'_{i11} = 0.2785\epsilon_{i11}$ and $\hat{\phi}_{i11}(x_{i1}) = \phi_{i11}(|x_{i1}|)\tanh\left(\frac{\bar{x}_{i1}\phi_{i11}(|x_{i1}|)}{\epsilon_{i11}}\right)$.

By using the same derivations as [5], the following result holds:

$$|\bar{x}_{i1}|\phi_{i12}(|z_i|) \leq \bar{x}_{i1}\hat{\phi}_{i12}(x_{i1}, r_i) + \epsilon'_{i12} + \frac{1}{4}\bar{x}_{i1}^2 + d_{i1}(t), \tag{17}$$

where $\epsilon'_{i12} = 0.2785\epsilon_{i12}$, $d_{i1}(t) = \left(\phi_{i12} \circ \alpha_{i1}{}^{-1}(2D_i(t))\right)^2$ and

$$\hat{\phi}_{i12}(x_{i1}, r_i) = \bar{\phi}_{i12}(r_i)\tanh\left(\frac{\bar{x}_{i1}\bar{\phi}_{i12}(r_i)}{\epsilon_{i12}}\right)$$

with $\bar{\phi}_{i12}(r_i) = \phi_{i12} \circ \alpha_{i1}{}^{-1}(2r_i)$.

Substituting (16) and (17) into (15) results in

$$\begin{aligned} \dot{V}_{i1} \leq\ & \bar{x}_{i1}\left(g_{i1}x_{i2} + f_{i1} + \frac{1}{4}\bar{x}_{i1} + \hat{\phi}_{i11}(x_{i1}) + \hat{\phi}_{i12}(x_{i1}, r_i) + \frac{1}{\lambda_{i0}}\bar{x}_{i1}\mu_{i0}\left(\left|\bar{x}_{i1}^2\right|\right)\right) \\ & - \frac{\bar{c}_i}{\lambda_{i0}} r_i + \frac{d_{i0}}{\lambda_{i0}} + \sum_{k=1}^{2}\epsilon'_{i1k} + d_{i1}(t) - \frac{b}{\gamma_i}\tilde{\theta}_i\dot{\hat{\theta}}_i. \end{aligned} \tag{18}$$

Step j. $(2 \le j \le n_i)$ Similar procedures are taken for $i = 2, \ldots, n_i$ as those outlined in Step 1. The dynamics of $\bar{x}_{ij} = x_{ij} - \alpha_{i(j-1)}$ is given by

$$\dot{\bar{x}}_{ij} = g_{ij}x_{i(j+1)} + f_{ij} + \Delta_{ij} - \dot{\alpha}_{i(j-1)}, \tag{19}$$

where

$$\dot{\alpha}_{i(j-1)} = \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} \left(g_{ik}x_{i(k+1)} + f_{ik} + \Delta_{ik}\right) + \frac{\partial \alpha_{i(j-1)}}{\partial \hat{\theta}_i}\dot{\hat{\theta}}_i + \frac{\partial \alpha_{i(j-1)}}{\partial r_i}\dot{r}_i. \tag{20}$$

Consider a Lyapnov function candidate as $V_{ij} = \frac{1}{2}\bar{x}_{ij}^2$. Then, differentiating V_{ij} gives

$$\begin{aligned} \dot{V}_{ij} \le \bar{x}_{ij} \Bigg(& g_{ij}x_{i(j+1)} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} g_{ik}x_{i(k+1)} + f_{ij} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} f_{ik} \\ & - \frac{\partial \alpha_{i(j-1)}}{\partial r_i}\dot{r}_i \Bigg) + \left|\bar{x}_{ij}\bar{\Delta}_{ij}\right| - \bar{x}_{ij}\frac{\partial \alpha_{i(j-1)}}{\partial \hat{\theta}_i}\dot{\hat{\theta}}_i \end{aligned} \tag{21}$$

where $\bar{\Delta}_{ij} = \Delta_{ij} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}}\Delta_{ik}$. By using triangular inequality and Assumption 2.1, one has

$$\begin{aligned} \left|\bar{x}_{ij}\bar{\Delta}_{ij}\right| \le |\bar{x}_{ij}| & \left(\phi_{ij1}\left(\left|\underline{x}_{ij}\right|\right) + \sum_{k=1}^{j-1} \left|\frac{\partial \alpha_{i(j-1)}}{\partial x_k}\right| \phi_{ik1}\left(\left|\underline{x}_{ik}\right|\right) \right) \\ & + |\bar{x}_{ij}| \left(\phi_{ij2}(|z_i|) + \sum_{k=1}^{j-1} \left|\frac{\partial \alpha_{i(j-1)}}{\partial x_k}\right| \phi_{ik2}(|z_i|) \right). \end{aligned} \tag{22}$$

Subsequently, by following the similar estimation methods to (16) and (17) for terms at the right hand side of (22), we rewrite (21) as

$$\begin{aligned} \dot{V}_{ij} = \bar{x}_{ij} \Bigg(& g_{ij}x_{i(j+1)} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} g_{ik}x_{i(k+1)} + f_{ij} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} f_{ik} + \hat{\phi}_{ij1}\left(\underline{x}_{ij}, \hat{\theta}_i, r_i\right) \\ & + \hat{\phi}_{ij2}\left(\underline{x}_{ij}, \hat{\theta}_i, r_i\right) + \frac{\bar{x}_{ij}}{4}\left[1 + \sum_{k=1}^{j-1}\left(\frac{\partial \alpha_{i(j-1)}}{\partial x_k}\right)^2\right] - \frac{\partial \alpha_{i(j-1)}}{\partial r_i}\dot{r}_i \Bigg) + d_{ij}(t) \\ & + \sum_{k=1}^{2} \epsilon'_{ijk} - \bar{x}_{ij}\sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial \hat{\theta}_i}\dot{\hat{\theta}}_i. \end{aligned} \tag{23}$$

Now, choose the following Lyapunov function for the whole systems:

$$V = \sum_{i=1}^{N}\sum_{j=1}^{n_i} V_{ij} = \sum_{i=1}^{N}\sum_{j=1}^{n_i} \frac{1}{2}\bar{x}_{ij}^2 + \frac{b}{2\gamma_i}\tilde{\theta}_i^2. \tag{24}$$

By considering (18) and (23), we obtain

$$\begin{aligned} \dot{V} \le & \sum_{i=1}^{N}\sum_{j=1}^{n_i-1} \bar{x}_{ij}\left\{g_{ij}\alpha_{ij} + \hat{f}_{ij}(X_{ij})\right\} + \sum_{i=1}^{N} \bar{x}_{in_i}\left\{g_{in_i}u_i + \hat{f}_{in_i}(X_{in_i})\right\} - \sum_{i=1}^{N}\frac{\bar{c}_i}{\lambda_{i0}}r_i \\ & + \sum_{i=1}^{N}\frac{d_{i0}}{\lambda_{i0}} + \sum_{k=1}^{2}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\epsilon'_{ijk} + \sum_{i=1}^{N}\sum_{j=1}^{n_i} d_{ij}(t) - \sum_{i=1}^{N}\frac{b}{\gamma_i}\tilde{\theta}_i\dot{\hat{\theta}}_i, \end{aligned} \tag{25}$$

where the following result has been used:

$$-\sum_{i=1}^{N}\sum_{j=2}^{n_i}\bar{x}_{ij}\frac{\partial\alpha_{i(j-1)}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i \leq \sum_{i=1}^{N}\sum_{j=2}^{n_i}\bar{x}_{ij}\frac{\partial\alpha_{i(j-1)}}{\partial\hat{\theta}_i}\sigma_i\hat{\theta}_i - \sum_{i=1}^{N}\sum_{j=2}^{n_i}\bar{x}_{ij}\frac{\partial\alpha_{i(j-1)}}{\partial\hat{\theta}_i}\sum_{k=1}^{j-1}\frac{\gamma_i}{2a_{ik}^2}\bar{x}_{ik}^2S_{ik}^TS_{ik} + \sum_{i=1}^{N}\sum_{j=2}^{n_i}\frac{\gamma_i}{2a_{ij}^2}\bar{x}_{ij}^2S_{ij}^TS_{ij}\sum_{k=2}^{j}\left|\bar{x}_{ik}\frac{\partial\alpha_{i(k-1)}}{\partial\hat{\theta}_i}\right|, \tag{26}$$

and $\bar{f}_{ij}(X_{ij})$, $1 \leq i \leq N$, $1 \leq j \leq n_i$ are defined as

$$\hat{f}_{i1}(X_{i1}) = f_{i1} + \frac{1}{4}\bar{x}_{i1} + \hat{\phi}_{i11}(x_{i1}) + \hat{\phi}_{i12}(x_{i1}, r_i) + \frac{1}{\lambda_{i0}}\bar{x}_{i1}\mu_{i0}\left(\left|\bar{x}_{i1}^2\right|\right), \tag{27}$$

$$\begin{aligned}\hat{f}_{ij}(X_{ij}) = {} & g_{i(j-1)}\bar{x}_{i(j-1)} - \sum_{k=1}^{j-1}\frac{\partial\alpha_{i(j-1)}}{\partial x_{ik}}g_{ik}x_{i(k+1)} + f_{ij} - \sum_{k=1}^{j-1}\frac{\partial\alpha_{i(j-1)}}{\partial x_{ik}}f_{ik} + \hat{\phi}_{ij1}\left(\underline{x}_{ij}, \hat{\theta}_i, r_i\right) \\ & + \hat{\phi}_{ij2}\left(\underline{x}_{ij}, \hat{\theta}_i, r_i\right) + \frac{\bar{x}_{ij}}{4}\left[1 + \sum_{k=1}^{j-1}\left(\frac{\partial\alpha_{i(j-1)}}{\partial x_k}\right)^2\right] + \frac{\partial\alpha_{i(j-1)}}{\partial\hat{\theta}_i}\sigma_i\hat{\theta}_i - \frac{\partial\alpha_{i(j-1)}}{\partial r_i}\dot{r}_i \\ & -\frac{\partial\alpha_{i(j-1)}}{\partial\hat{\theta}_i}\sum_{k=1}^{j-1}\frac{\gamma_i}{2a_{ik}^2}\bar{x}_{ik}^2S_{ik}^TS_{ik} + \frac{\gamma_i}{2a_{ij}^2}\bar{x}_{ij}S_{ij}^TS_{ij}\sum_{k=2}^{j}\left|\bar{x}_{ik}\frac{\partial\alpha_{i(k-1)}}{\partial\hat{\theta}_i}\right|.\end{aligned} \tag{28}$$

Then, for any given $\varepsilon_{ij} \geq 0$ there exists a fuzzy logic system $W_{ij}^TS_{ij}(X_{ij})$ such that

$$\hat{f}_{ij}(X_{ij}) = W_{ij}^TS_{ij}(X_{ij}) + \delta_{ij}(X_{ij}), \tag{29}$$

where δ_{ij} refers to the approximation error and satisfies $|\delta_{ij}| < \varepsilon_{ij}$. Furthermore, by Young's inequality, one has

$$\bar{x}_{ij}\hat{f}_{ij}(X_{ij}) \leq \frac{b}{2a_{ij}^2}\bar{x}_{ij}^2\theta_iS_{ij}^TS_{ij} + \frac{1}{2}a_{ij}^2 + \frac{b}{2}\bar{x}_{ij}^2 + \frac{1}{2b}\varepsilon_{ij}^2, \tag{30}$$

where the unknown constant θ_i has been defined in (9).

Substituting (29) into (25) and using (30) produces

$$\begin{aligned}\dot{V} \leq {} & \sum_{i=1}^{N}\sum_{j=12}^{n_i}\bar{x}_{ij}\left(g_{ij}\alpha_{ij} + \frac{b}{2a_{ij}^2}\bar{x}_{ij}\theta_iS_{ij}^TS_{ij}\right) + \sum_{i=1}^{N}\sum_{j=1}^{n_i}\frac{b}{2}\bar{x}_{ij}^2 + \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left(\sum_{k=1}^{2}\epsilon'_{ijk}\right. \\ & \left. + \frac{1}{2}a_{ij}^2 + \frac{1}{2b}\varepsilon_{ij}^2 + d_{ij}(t)\right) + \sum_{i=1}^{N}\frac{d_{i0}}{\lambda_{i0}} - \sum_{i=1}^{N}\frac{1}{\lambda_{i0}}\bar{c}_ir_i - \sum_{i=1}^{N}\frac{b}{\gamma_i}\tilde{\theta}_i\dot{\hat{\theta}}_i.\end{aligned} \tag{31}$$

Next, by designing the virtual control α_{ij} in (6), we have

$$\bar{x}_{ij}g_{ij}\alpha_{ij} \leq -\lambda_{ij}b\bar{x}_{ij}^2 - \frac{b}{2}\bar{x}_{ij}^2 - \frac{b}{2a_{ij}^2}\bar{x}_{ij}^2\hat{\theta}_iS_{ij}^TS_{ij}. \tag{32}$$

Further, by combining (31) together with (32) and (9), we rewrite (31) as

$$\begin{aligned}\dot{V} \leq {} & -\sum_{i=1}^{N}\left(\sum_{j=1}^{n_i}\lambda_{ij}b\bar{x}_{ij}^2 + \frac{\sigma_ib}{2\gamma_i}\tilde{\theta}_i^2 + \frac{\bar{c}_i}{\lambda_{i0}}r_i\right) + \sum_{i=1}^{N}\frac{d_{i0}}{\lambda_{i0}} \\ & + \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left(\sum_{k=1}^{2}\epsilon'_{ijk} + \frac{1}{2}a_{ij}^2 + \frac{1}{2b}\varepsilon_{ij}^2 + d_{ij}(t) + \frac{\sigma_ib}{2\gamma_i}\theta_i^2\right) \\ \leq {} & -a_0V + b_0,\end{aligned} \tag{33}$$

where $a_0 = \min\{2\lambda_{ij}b, \bar{c}_i, \sigma_i, 1 \leq i \leq N, 1 \leq j \leq n_i\}$ and $b_0 = \frac{1}{2}\sum_{i=1}^{N}\delta_i^{*2} + \sum_{i=1}^{N}\frac{d_{i0}}{\lambda_{i0}} + \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left(\sum_{k=1}^{2}\epsilon'_{ijk} + \frac{1}{2}a_{ij}^2 + \frac{1}{2b}\varepsilon_{ij}^2 + d_{ij}(t) + \frac{\sigma_ib}{2\gamma_i}\theta_i^2\right)$, and the result $\tilde{\theta}_i\hat{\theta}_i \leq -\frac{1}{2}\tilde{\theta}_i^2 + \frac{1}{2}\theta_i^2$

has been used in the above inequality. Equation (33) means that all the signals in the closed-loop system are semi-globally uniformly ultimately bounded in mean square. The main result is summarized by the following theorem.

Theorem 3.1. *Under Assumptions 2.1-2.3, consider the closed-loop nonlinear system consisting of the system (1), controller (6), and adaptive law (7). Then, under the action of controller (6), for any initial conditions* $\left[\underline{x}_{ij}^T(0), \hat{\theta}_i(0)\right]^T \in \Omega_0$ *(where* Ω_0 *is an appropriately chosen compact set), all the signals in the closed-loop system are semi-globally uniformly ultimately bounded in the sense of mean square.*

4. **Simulation Example.** Consider the following second-order nonlinear systems

$$\begin{cases} \dot{z}_1 = -z_1 + 0.5x_{11}^2 + 0.5, \\ \dot{x}_{11} = x_{12} + x_{11}^2 \sin(x_{11}) + z_1 x_{11} \sin(x_{11}), \\ \dot{x}_{12} = u_1 + x_{11}x_{12} + z_1 x_{11} x_{12}, \\ y_1 = x_{11}, \end{cases} \quad \begin{cases} \dot{z}_2 = -z_2 + 0.5x_{21}^2 + 0.5, \\ \dot{x}_{21} = x_{22} + x_{21}^2 + z_2 x_{21} \sin(x_{21}), \\ \dot{x}_{22} = u_2 + x_{21} \sin(x_{22}) + z_2 x_{21} \sin(x_{22}), \\ y_2 = x_{21}. \end{cases}$$

Based on Theorem 3.1, choose the virtual control signal α_{ij} in (6) ($i = 1, 2,\ j = 1$), the actual controller u_1 and u_2 in (6) with $i = 1, 2,\ j = 2$ and the adaptive law in (7). The simulation is run with the initial conditions $[x_{11}(0), x_{12}(0), x_{21}(0), x_{22}(0)]^T = [0.4, -0.2, 0.3, -0.3]^T$, $\left[\hat{\theta}_1, \hat{\theta}_2\right]^T = [0, 0]^T$, the design parameters $k_{11} = k_{12} = k_{21} = k_{22} = 5$, $a_{11} = a_{12} = a_{21} = a_{22} = 2$, $\gamma_1 = \gamma_2 = 2$ and $\sigma_1 = \sigma_2 = 1$. The simulation results are shown in Figures 1 and 2. Apparently, Figures 1 and 2 show that all the signals in the closed-loop system are bounded.

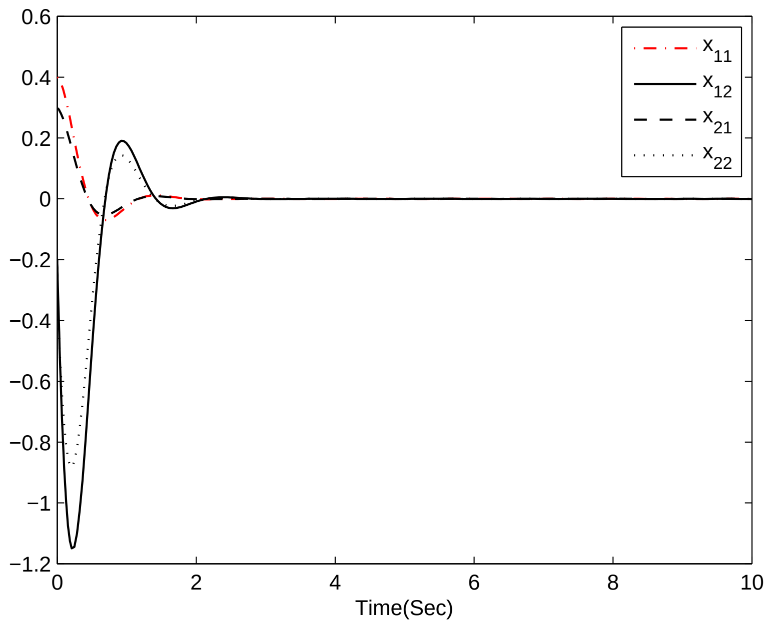

FIGURE 1. State variables x_{11}, x_{12}, x_{21} and x_{22}

5. **Conclusion.** In this paper, a fuzzy adaptive control approach has been proposed for a class of MIMO nonlinear systems with unmodeled dynamics and dynamics disturbances. The proposed controller guarantees that all the signals in the closed-loop system remain semi-globally uniformly ultimately bounded. Simulation results have been provided to illustrate the effectiveness of the proposed control scheme. Our future research will mainly focus on the output-feedback control for the original system (1) based on the result in this paper.

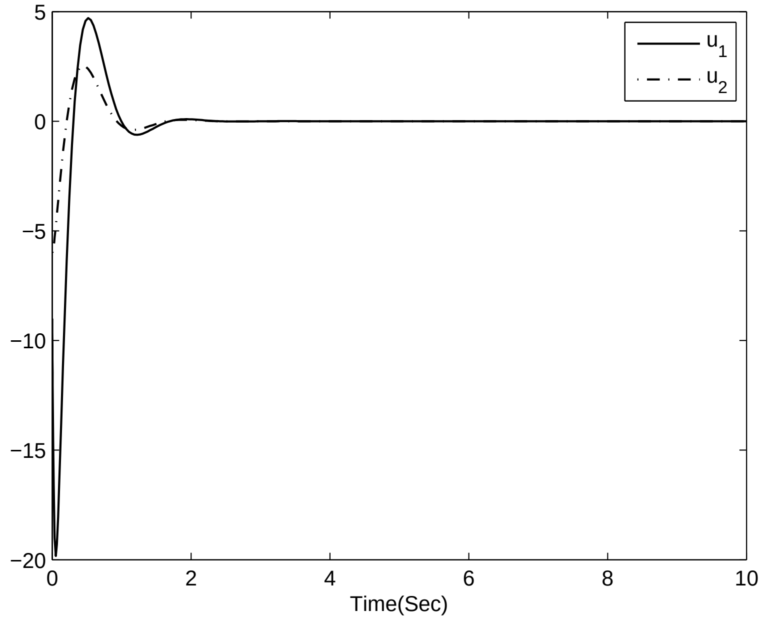

FIGURE 2. The actual control inputs u_1 and u_2

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