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ANALYSIS AND DESIGN OF CONSENSUS BASED FORMATION CONTROL FOR LINEAR MULTI-AGENT SYSTEMS

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Abstract. *The paper studies the formation control problem of a general linear multi-agent system under the time-invariant and directed communication topology and with time-invariant relative position and angle constraints. A consensus based formation control protocol is proposed to make sure that the multi-agent system achieves a desired formation by local information exchange among agents. First of all, a state translation is used to transform the formation problem into a consensus problem. Then a state-linear-transformation continues to transform the consensus problem into a partial stability problem. By the partial stability theory, a sufficient and necessary criterion of formation and its consensus trajectory function are derived. Moreover, a design procedure of gain matrices based on the Lyapunov inequality and the Bilinear Matrix Inequality (BMI) is given. Finally, a numerical example is shown to illustrate the effectiveness of the results.*

Keywords: Multi-agent system, Formation, Consensus, Bilinear matrix inequality

1. **Introduction.** In recent years, the cooperative control problem of Multi-Agent Systems (MASs) has attracted increasing attention for its broad applications in many areas. One of its fundamental issues is the formation control which makes the multi-agent system form and maintain a desired geometry. Formation is a common phenomenon in nature, e.g., school of fish, and flock of birds. And there also exist many real applications in human society; e.g., vehicle formation [1, 2] in transportation domain can reduce vehicle distance and increase traffic density and road capacity, and wireless sensor formation [3] in exploration field can minimize the required number of nodes and maximize the sensing coverage.

Existing formation control approaches mainly include leader-following [4], behavior based [5], virtual structure [6, 7], etc. For the leader-following approach, there is at least one agent as a leader which moves along its predefined trajectory and other agents follow the leader according to the desired state offsets. The leader plays a key role and does not allow to be out of order. In the behavior based approach, the behavior of individual agent is decomposed into several basic behaviors and the result action of each agent is derived by a weighted sum of these basic behaviors. However, its mathematical formalization is difficult. In the virtual structure approach, the whole formation is treated as a single rigid body such that the relative positions between agents are fixed. The desired trajectories are not assigned to individual agent but to the formation as a whole.

For multi-agent systems, consensus [8] also is one of its important questions. It studies how a group of agents reaches a common state asymptotically with limited local information exchange. Actually, the consensus theory can also be used in formation control by selecting appropriate states. Compared with above formation control approaches, the consensus based approach is more reliable and robust when individual agent is likely to be out of order. Ren [9] applied the consensus based formation control approach to the second-order MAS, and pointed out that many existing leader-following, behavior based, and virtual structure approaches in the literature can be treated as the special cases of consensus based formation control approaches. Luo et al. [10] proposed two leader-following consensus protocols to deal with the formation control of second-order multi-agent network. Li et al. [11] studied the flight formation of multiple unmanned aerial vehicles based on consensus protocol by using improved artificial potential field method.

This paper aims to study the formation control problem for the general linear MAS under time-invariant communication topology and with time-invariant relative position and angle constraints. The contributions are addressed as follows. Firstly, a formation control protocol based on consensus theory is proposed. Secondly, a state translation is used to transform the formation control problem into a consensus problem. Then a linear transformation continues to transform the consensus problem into a partial stability problem. By the partial stability theory, a necessary and sufficient criterion and a trajectory function are obtained. Thirdly, a design procedure for gain matrices of the formation control protocol based on the Lyapunov inequality and the BMI is proposed.

The rest of paper is organized as follows. Section 2 states the formulation control problem of linear MAS and formulates it. Section 3 presents the analysis process for the necessary and sufficient criterion and trajectory function. In Section 4, a design procedure for gain matrices in the formation control protocol is proposed based on Lyapunov inequality and the BMI. Section 5 provides a numerical simulation to illustrate the main results. Final section concludes the paper.

2. **Problem Formulation and Statement.**

Suppose that a linear multi-agent system consists of N agents. The dynamics of agent is given as follows:

$$\dot{x}_i = Ax_i + Bu_i, \quad i = 1, \cdots, N, \tag{1}$$

where $x_i \in \mathbf{R}^n$ and $u_i \in \mathbf{R}^m$ are the state and control input of the ith agent respectively. $A \in \mathbf{R}^{n\times n}$ and $B \in \mathbf{R}^{n\times m}$ are parameter matrices.

A formation control protocol based on consensus theory is:

$$u_i = -K_1x_i + K_2h_i + W\sum_{j\in N_i} a_{ij}((x_j - h_j) - (x_i - h_i)), \tag{2}$$

where $K_1 \in \mathbf{R}^{m\times n}$, $K_2 \in \mathbf{R}^{m\times n}$ and $W \in \mathbf{R}^{m\times n}$ are gain matrices, $h_i \in \mathbf{R}^n$ are the offsets from the ith agent to the common reference point. The gain scalar $a_{ij} > 0$ is the communication weight when information is transferred form j to i.

A multi-agent system with information exchange is often described as a weighted directed graph $G = (V, E, A)$ which is composed of a vertex set $V = \{1, 2, \cdots, N\}$, an edge set $E \subset V \times V$, and the weighted adjacency matrix $A = [a_{ij}]_{N\times N}$. Every agent is looked as a vertex. If there exists information transformation from the jth agent to the ith agent, it corresponds to there is an edge from the jth vertex to the ith vertex and the corresponding communication weight $a_{ij} > 0$. $d_i = \sum_j a_{ij}$ is the indegree of the ith agent. The Laplacian matrix of a directed graph is defined as $L = D - A$ and represents the communication topology, where $D = \mathrm{diag}(d_i)$ is the diagonal matrix of agent indegrees. Due to the limited communication range, each agent only can communicate with neighbor agents and a set $N_i = \{j \in V : (j, i) \in E\}$ is used to denote the index set of the neighboring agents of the ith agent.

The formation control protocol (2) consists of three parts which play different roles. The first part is a state feedback which can be applied to regulating the dynamics of each agent independently if the state of agent is measurable. On the contrary, if the state

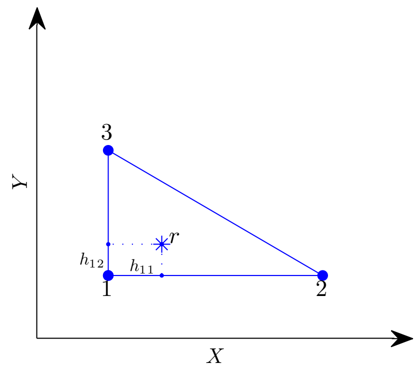

FIGURE 1. The triangle formation

is unmeasurable or the dynamics need not be regulated, the gain matrix K_1 is equal to zero matrix directly. The second part is a compensation term to guarantee achieving the desired formation. The third part is a cooperative term, which depends on the relative states between the agent itself and its neighbours. For system (1) and protocol (2), obviously, if $h_i = 0$, the formation control problem becomes a consensus problem and the formation control protocol is the same or similar as the consensus protocol in our previous works [12, 13].

The state of agent can be position, altitude, angle, velocity, climb rate and angular velocity, etc. [9]. Generally speaking, the formation mainly refers to the position formation in two or three dimension space. Figure 1 shows a triangle formation which includes three agents $\{1, 2, 3\}$ and a common reference point r in the two-dimensional plane. It indicates how to describe the desired formation by the formation vectors h_i, $i = 1, \cdots, N$. The common reference point r may be arbitrarily chosen, e.g., the formation center in Figure 1. The vector $h_1 = [h_{11}, h_{12}]^T$ is the position offset between the first agent and the reference point r. Stack the states of N agents into a vector form, i.e., $x = \left[x_1^T, x_2^T, \ldots, x_N^T\right]^T$, and define a new formation vector $h = \left[h_1^T, h_2^T, \ldots, h_N^T\right]^T$. Substituting protocol (2) into system (1), the system can be rewritten in a compact form:

$$\dot{x} = Mx + Q, \tag{3}$$

where

$$M = I_N \otimes (A - BK_1) - L \otimes (BW),$$
$$Q = (I_N \otimes (BK_2) + L \otimes (BW))h,$$

I_N is the identity matrix of order $N \times N$, and the symbol $\otimes$ is the Kronecker product. The entries of Laplacian matrix $L = [l_{ij}]_{N\times N}$ is:

$$l_{ij} = \begin{cases} \sum\limits_{k\in N_i} a_{ik}, & j = i \\ -a_{ij}, & j \neq i,\ j \in N_i \\ 0, & j \neq i,\ j \notin N_i. \end{cases} \tag{4}$$

Definition 2.1. *Under a given communication topology, the MAS is said to achieve formation specified by vector h via protocol (2) if for any initial state $x_i(0)$, it satisfies*

$$\lim_{t\to+\infty} \|(x_i(t) - h_i) - (x_j(t) - h_j)\| = 0, \quad \forall i, j \in \{1, 2, \cdots, N\}. \tag{5}$$

3. Formation Analysis.

First of all, according to the description of formation control protocol, a state translation $\hat{x}_i = x_i - h_i$ is introduced to transform the formation problem into a consensus problem. So system (3) is rewritten as follows:

$$\dot{\hat{x}} = M\hat{x} + (I_N \otimes (A - BK_1 + BK_2))h. \tag{6}$$

Then, a linear state transformation is adopted and the corresponding transformation matrix is:

$$P =: \begin{bmatrix} \tilde{P}_0 \\ \mathbf{1}_N^T \end{bmatrix} \otimes I_n, \tag{7}$$

where $\mathbf{1}_N$ is all 1 collum vector of order N, and matrix $\tilde{P}_0 \in \mathbf{R}^{(N-1)\times N}$ is:

$$\tilde{P}_0 = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -1 \end{bmatrix}.$$

The inverse matrix P is:

$$P^{-1} =: \begin{bmatrix} \hat{P}_0 & N^{-1}\mathbf{1}_N \end{bmatrix} \otimes I_n, \tag{8}$$

where

$$\hat{P}_0 = \frac{1}{N} \begin{bmatrix} N-1 & N-2 & \cdots & N-(N-1) \\ -1 & N-2 & \cdots & N-(N-1) \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -2 & \cdots & N-(N-1) \\ -1 & -2 & \cdots & -(N-1) \end{bmatrix}.$$

The constructed linear state transformation is:

$$\bar{x} = P\hat{x}. \tag{9}$$

Obviously, the error variables $\bar{x}_i = \hat{x}_i - \hat{x}_{i+1}$, $i = 1, \cdots, N-1$, are the first $N-1$ components of new state $\bar{x}$, and the sum variable $\bar{x}_N = \sum_{i=1}^N \hat{x}_i$ is the last component.

Substituting (9) into (6), system (6) is rewritten as follows:

$$\dot{\bar{x}} = PMP^{-1}\bar{x} + P(I_N \otimes (A - BK_1 + BK_2))h. \tag{10}$$

According to the definition of linear state transformation, $\bar{x}$ can be divided into two parts, i.e., $\bar{x} = \left[y^T, z^T\right]^T$, where $y = \left[\bar{x}_1^T, \cdots, \bar{x}_{N-1}^T\right]^T \in \mathbf{R}^{(N-1)n}$, $z = \bar{x}_N \in \mathbf{R}^n$. System (10) can be written into two equations:

$$\begin{cases} \dot{y} = \bar{A}y + R_y, \\ \dot{z} = \bar{C}y + \bar{D}z + R_z, \end{cases} \tag{11}$$

where

$$\begin{aligned} \bar{A} &= \left(\tilde{P}_0 \otimes I_n\right) M \left(\hat{P}_0 \otimes I_n\right) = I_{N-1} \otimes (A - BK_1) - \left(\tilde{P}_0 L \hat{P}_0\right) \otimes (BW), \\ \bar{C} &= \left(\mathbf{1}_N^T \otimes I_n\right) M \left(\hat{P}_0 \otimes I_n\right) = -\left(\mathbf{1}_N^T L \hat{P}_0\right) \otimes (BW), \\ \bar{D} &= \left(\mathbf{1}_N^T \otimes I_n\right) M \left(N^{-1}\mathbf{1}_N \otimes I_n\right) = A - BK, \\ R_y &= \left(\tilde{P}_0 \otimes (A - BK_1 + BK_2)\right) h, \\ R_z &= \left(\mathbf{1}_N^T \otimes (A - BK_1 + BK_2)\right) h. \end{aligned}$$

Thus, the formation control problem of muti-agent system (1) is equivalently transformed into a stability problem of system (11) with respect to the partial variable y. Thus, the theorem is deduced as follows.

Theorem 3.1. *Under the time-invariant communication topology $\{N_i : i = 1, \cdots, N\}$ and given formation vector h, the MAS (1) achieves the desired formation via the protocol (2) if and only if*

1) $A - BK_1 + BK_2 = 0$,

2) the matrix $\bar{A}$ is Hurwize stable.
Moreover, the formation trajectory function is expressed by

$$\xi(t;x(0)) = N^{-1}\left(\left[\begin{array}{cc} 0_{1\times(N-1)} & 1 \end{array}\right]\otimes I_n\right)e^{PMP^{-1}t}P(x(0)-h). \tag{12}$$

4. **Design of Gain Matrices.** In this section, we further consider the design problem of gain matrices in the formation control protocol (2). According to stability theory, if the matrix $\bar{A}$ is Hurwize stable, then there exists a positive definite matrix H that satisfies the following Lyapunov inequality:

$$\bar{A}^T H + H\bar{A} < 0. \tag{13}$$

As mentioned in Section 2, three parts in the formation control protocol (2) play different roles and can be chosen mutually independently. First, the feedback gain matrix K_1 can be designed in advance by some methods, e.g., the pole placement. Then K_2 can be calculated according to the formation criterion. Assume that W is given; thus, the matrix Inequality (13) becomes a BMI with respect to the matrices H and L, i.e., H and a_{ij}. Since the BMI is not convex and has multiple local solutions, the computational complexity for solving the BMI is much higher than ordinary Linear Matrix Inequality (LMI). Motivated by [14], a homotopy method is adopted to solve the BMI.

First of all, a real variable μ ranging from 0 to 1, i.e., $\mu \in [0,1]$, is introduced and a new function is constructed:

$$F(H,L,\mu) = (1-\mu)F_1(H) + \mu F_2(H,L), \tag{14}$$

where

$$\begin{aligned} F_1(H) &= \left(I_{N-1}\otimes(A-BK_1)-\lambda I_{(N-1)n}\right)^T H + H\left(I_{N-1}\otimes(A-BK_1)-\lambda I_{(N-1)n}\right), \\ F_2(H,L) &= \bar{A}^T H + H\bar{A}. \end{aligned}$$

The scalar λ is chosen such that $(A - BK_1 - \lambda I_n)$ is stable. Generally speaking, λ can be set to a larger number than the largest real part of eigenvalues of $A - BK_1$, i.e., $\lambda > \max\{Re\lambda(A-BK_1)\}$.

Thus, the problem of solving the matrix inequality (13) is embedded in the family of problems:

$$F(H,L,\mu) < 0, \quad \mu \in [0,1]. \tag{15}$$

The solving procedure is as follows.

Step 1: Initialize a positive integer η, and let $\mu = k/\eta$ where the integer $k \in [0,\eta]$.

Step 2: Let $k = 0$, $\mu = k/\eta$, and solve $F(H,L,\mu) < 0$, i.e., $F_1(H) < 0$, to obtain the positive definite matrix H.

Step 3: Let $k = k+1$, $\mu = k/\eta$, and solve $F(H,L,\mu) < 0$ to obtain the matrices W_{ij} where H is the calculated result in above step. If the LMI $F(H,L,\mu) < 0$ is feasible, continue to calculate the matrix H such that $F(H,L,\mu) < 0$ with the calculated L and the same μ in this step, and then go to Step 5. If the LMI is not feasible, go to Step 4.

Step 4: Let $\eta = 2\eta$, $k = 2(k-1)$, and go to Step 3. If the number η is large enough, it means this procedure does not converge.

Step 5: If $\mu < 1$, go to Step 3. When $\mu = 1$, the obtained L, i.e., the solution of $F_2(H,L) < 0$, is the desired solution.

Many formation researches, e.g., [1, 4, 5], do not consider the design problem. This section provides a feasible method to design the gain matrices in the protocol (2). Through the iteration procedure, the bilinear matrix inequality is transformed into a group of linear matrix inequalities. These simple linear matrix inequalities reduce the total computational complexity.

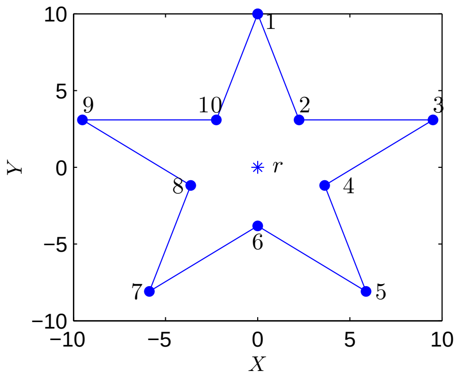

FIGURE 2. The five-pointed star formation

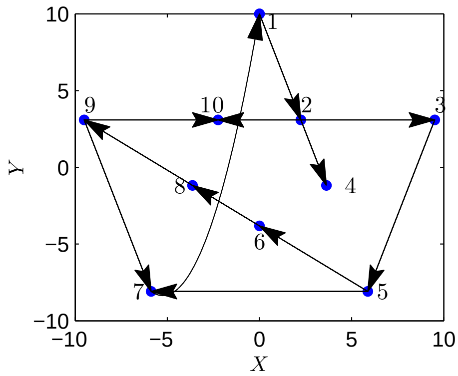

FIGURE 3. The communication topology

5. **Simulation.** Consider a linear MAS consisting of 10 agents, where the parameter matrices in system (1) are of the following form, respectively,

$$A = \begin{bmatrix} 0.24 & 1.45 \\ -0.65 & 0.66 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 & 0.2 \\ 0.7 & 0.3 \end{bmatrix}. \tag{16}$$

As shown in Figure 2, these agents assemble into a five-pointed star formation whose radius of the circumscribed circle is 10, and origin of coordinates is their common reference point. Figure 3 shows the commmunication topology. The corresponding formation vectors and given gain matrices are given as follows:

$$\begin{aligned} &h_1 = \begin{bmatrix} 0 \\ 10 \end{bmatrix}, \ h_2 = \begin{bmatrix} 2.2451 \\ 3.0902 \end{bmatrix}, \ h_3 = \begin{bmatrix} 9.5106 \\ 3.0902 \end{bmatrix}, \ h_4 = \begin{bmatrix} 3.6327 \\ -1.1803 \end{bmatrix}, \\ &h_5 = \begin{bmatrix} 5.8779 \\ -8.0902 \end{bmatrix}, \ h_6 = \begin{bmatrix} 0 \\ -3.8197 \end{bmatrix}, \ h_7 = \begin{bmatrix} -5.8779 \\ -8.0902 \end{bmatrix}, \ h_8 = \begin{bmatrix} -3.6327 \\ -1.1803 \end{bmatrix}, \\ &h_9 = \begin{bmatrix} -9.5106 \\ 3.0902 \end{bmatrix}, \ h_{10} = \begin{bmatrix} -2.2451 \\ 3.0902 \end{bmatrix}, \ K_1 = \begin{bmatrix} 0.2 & 0.3 \\ 0.7 & 1.5 \end{bmatrix}, \\ &K_2 = \begin{bmatrix} -20.0 & -30.0 \\ 50.0 & 70.0 \end{bmatrix}, \ W = \begin{bmatrix} 1.2 & 0.5 \\ 0.1 & 1.0 \end{bmatrix}. \end{aligned} \tag{17}$$

The gain matrices are given as follows. And the given weights are $a_{17} = 0.1$, $a_{21} = 0.7$, $a_{32} = 0.9$, $a_{42} = 0.2$, $a_{53} = 0.6$, $a_{65} = 0.4$, $a_{75} = 0.5$, $a_{79} = 0.1$, $a_{86} = 0.3$, $a_{98} = 0.3$, $a_{(10)9} = 0.8$, $a_{(10)2} = 0.8$. So the eigenvalues of $\bar{A}$ are $-1.0160 \pm 0.8180i$, $-0.6740 \pm 1.1739i$, $-0.5920 \pm 0.8778i$, $-0.2738 \pm 1.3120i$, $-0.2969 \pm 0.8447i$, $-0.3381 \pm 1.0548i$, $-0.1354 \pm 1.1294i$, $-0.1664 \pm 0.9542i$, $-0.1270 \pm 1.0336i$. All the eigenvalues of $\bar{A}$ have negative real parts and the matrix $\bar{A}$ is Hurwize stable. In this case, the MAS can achieve the desired formation. Figure 4 shows the state trajectories of the agents starting from the initial states $[14, 2.5]$, $[3, 2]$, $[3, 10]$, $[12, 1]$, $[1, 3]$, $[14, 4]$, $[-1, 3]$, $[5, 2]$, $[5, 7]$, $[2, 2]$, respectively. To avoid confusion, only the trajectories of the 1st and 5th agents and the five-pointed star formations in $50s$, $60s$, $70s$, $80s$ are shown. The simulation illustrates the effectiveness of the consensus criterion.

Then we consider the design of the gain matrices in protocol (2) for the MAS (16) with the same communication topology and matrices K_i, W as stated above. The number λ is set as 1.7. We start the design iteration in Section 4 with $\eta = 200$ and get

$$\begin{aligned} &a_{17} = 0.2665, \quad a_{21} = 0.3979, \quad a_{32} = 0.6004, \quad a_{42} = 0.4691, \quad a_{53} = 0.4280, \\ &a_{65} = 0.5571, \quad a_{75} = 0.4541, \quad a_{79} = 0.1598, \quad a_{86} = 0.3300, \quad a_{98} = 0.5841, \\ &a_{(10)9} = 0.5242, \quad a_{(10)2} = 0.6578. \end{aligned} \tag{18}$$

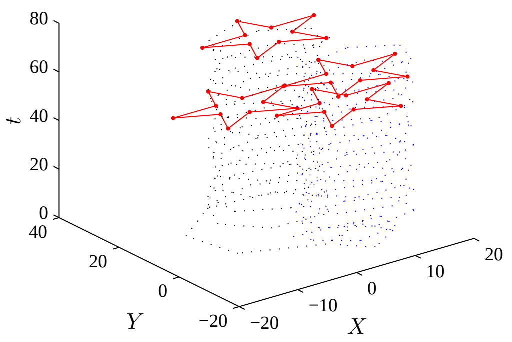

FIGURE 4. State trajectories of MAS with the prescribed gain matrices

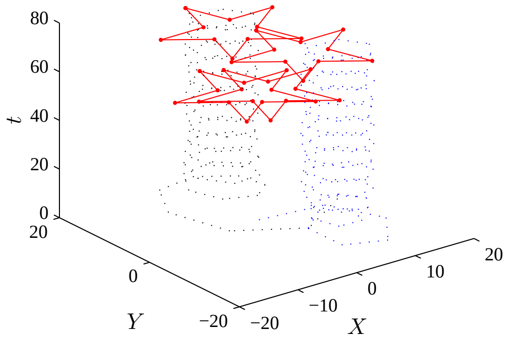

FIGURE 5. State trajectories of MAS with the designed gain matrices

Figure 5 shows the state trajectories of the agents starting from the same initial states as above stated values with the designed gain matrices. This example illustrates the effectiveness of the design procedure of gains in the protocol.

6. **Conclusion.** The paper studies the formation control problem of a general linear multi-agent system under the time-invariant and directed communication topology and with time-invariant relative position and angle constraints. With the help of consensus theory and partial stability theory, a consensus based formation control protocol is proposed and a sufficient and necessary consensus criterion is achieved. The theoretical results are helpful for practice and other formation control analyses. Future work will focus on the formation problem with more complex conditions, e.g., given trajectory, nonlinear systems, and time delay.

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