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ANALYSIS OF THE REAL GRAVITATIONAL WAVE DATA GW150914 WITH THE HILBERT-HUANG TRANSFORM

Kazuki Sakai[1], Ken-ichi Oohara[2], Masato Kaneyama[3]
and Hirotaka Takahashi[4,5]

[1]Department of Information Science and Control Engineering
[4]Department of Information and Management Systems Engineering
Nagaoka University of Technology
1603-1, Kamitomioka, Nagaoka, Niigata 940-2188, Japan
k_sakai@stn.nagaokaut.ac.jp; hirotaka@kjs.nagaokaut.ac.jp

[2]Graduate School of Science and Technology
Niigata University
8050, Ikarashi 2-no-cho, Nishi-ku, Niigata 950-2181, Japan
oohara@astro.sc.niigata-u.ac.jp

[3]Department of Physics
Graduate School of Science
Osaka City University
3-3-138, Sugimoto, Sumiyoshi-ku, Osaka 558-8585, Japan
kaneyama@yukimura.hep.osaka-cu.ac.jp

[5]Earthquake Research Institute
The University of Tokyo
1-1-1, Yayoi, Bunkyo-Ku, Tokyo 113-0032, Japan

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Abstract. *On September 14, 2015, the first direct detection of gravitational waves was achieved by the laser interferometer gravitational wave observatory (LIGO) in the USA. Since the amplitude of the gravitational wave is too small compared to detector noises, only noise components appear in a plot of raw observed data. We analyzed the observed gravitational wave data to reveal a signal structure contained in the data by means of the Hilbert-Huang transform (HHT). We preliminarily applied some filters to attenuating noise components, and applied the HHT to the output. As a result, resolution of the time-frequency maps obtained by the HHT is higher than that of the maps by the wavelet transform. In particular, we could extract a specific time-frequency structure, which is not emerging in results by the wavelet transform, so that the frequency of the main mode becomes constant.*

Keywords: Signal processing, Time-frequency analysis, Hilbert-Huang transform, Gravitational wave

1. **Introduction.** Gravitational waves are ripples of spacetime, which propagate at the same speed as light. Since interaction of gravitational waves with matter is weak, they are not absorbed and scattered while they are traveling through the universe. It means that gravitational waves possibly provide information that no other ways, such as electromagnetic waves, X rays and neutrinos, can provide. The existence of gravitational waves was first predicted by Einstein in 1916, and the first direct detection of them was achieved by the laser interferometer gravitational wave observatory (LIGO) in the USA on September 14, 2015 [1]. This first detection event was named GW150914.

GW150914 was emitted from a coalescence of black hole binary with an estimated luminosity distance of 410^{+160}_{-180} Mpc from the earth. Figure 1 shows time-frequency maps (T-F maps) of the event plotted by LIGO, which is obtained by means of the wavelet transform. The left panel of Figure 1 is of the data from the LIGO Hanford Observatory

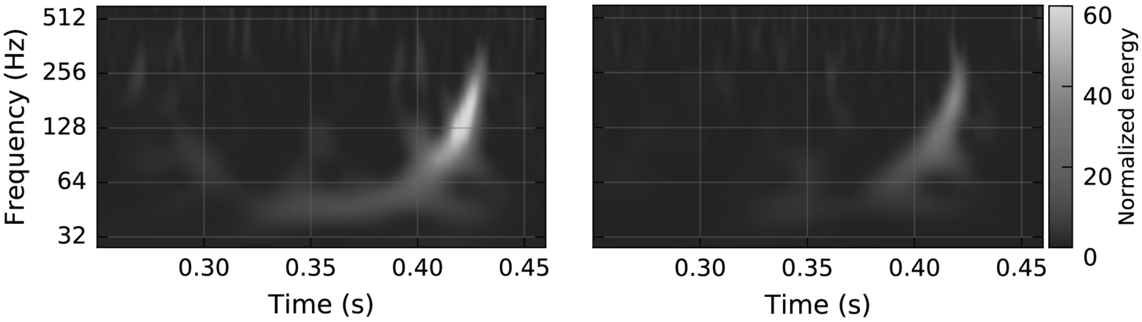

Figure 1. T-F maps of the GW150914 by means of the wavelet transform, plotted by LIGO [1]. Times are relative times to 2015-09-14 09:50:45 (UTC). A specific structure is contained in both plots around 0.4 s.

(hereinafter referred to Hanford), and the right panel is of the data from the LIGO Livingston Observatory (Livingston). To make the T-F maps, a 30-350 Hz bandpass filter is applied to the raw data and some bandstop filters to remove strong line noises (strong narrow-band noises) of instrumental origin. The T-F maps indicate that a specific structured signal is contained around 0.4 s.

Time-frequency analysis is useful to investigate a signal structure of target data and to analyze physical dynamics generating the signal. Hilbert-Huang transform (HHT) is a relatively new approach for time-frequency analysis [3], which was constructed with the intention to apply to non-stationary and non-linear system. In the HHT, target data is decomposed into finite intrinsic mode functions (IMFs) by the empirical mode decomposition (EMD), and then instantaneous amplitude (IA) and instantaneous frequency (IF) of each IMF are derived by the Hilbert spectral analysis (HSA). By combining time evolutions of IA and IF, we can plot a time-frequency structure of the data. In original EMD, 'mode mixing' frequently occurs, which is an undesirable result that two or more different modes are decomposed into the same IMF. Wu and Huang [4] proposed the ensemble EMD (EEMD) as an effective solution to the mode mixing problem. A detailed review of the algorithm of the HHT including the EEMD is found in [5].

The effectiveness of the HHT in analysis of gravitational wave data has been investigated based on simulations [5, 6, 7, 8, 9]. However, the effectiveness for real observed data is also needed to be investigated. It is known that analysis of real observed data is more complicated than that of simulated data, due to non-Gaussianity and non-stationarity of real noise and the existence of line noises. In particular, we know that the existence of line noises has bad influence on the performance of the EMD. For this reason, through analyzing the GW150914 data, we examine some digital filters as preprocessing to attenuate the noise components of real observed data, and investigate the effectiveness of the HHT in analysis of the preprocessed data.

The rest of the paper is organized as follows. In Section 2, we present the setup and the result of the preprocessing. Section 3 provides the analyzing results of the preprocessed GW150914 data by the HHT. After that, Section 4 summarizes the paper.

2. **Preprocessing.** Figure 2 shows the strain of the observed data from Hanford; lower frequency components than 10 Hz of it have been removed because the band was not calibrated properly. Since the amplitude of the gravitational wave signal is too small compared to detector noises, only noises appear in the graph. Figure 3 shows the estimated spectrum densities of detector noise, or detector sensitivity, which are estimated by means of Welch method from real observed data for 32 seconds. As these indicate, real observed data from ground based detectors of gravitational waves contain noises of various origins such as seismic noise, thermal noises of mirrors, and quantum noise of laser. The intensity

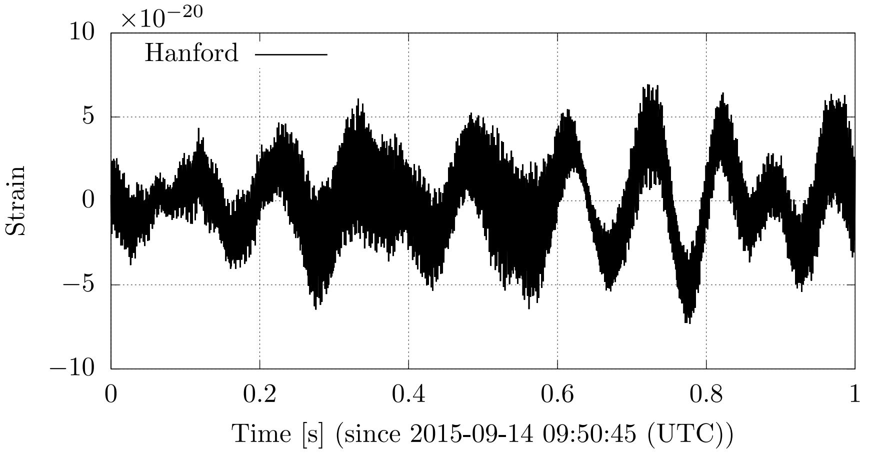

FIGURE 2. Strain of the observed data. Lower frequency components than 10 Hz have been removed because the band was not calibrated properly.

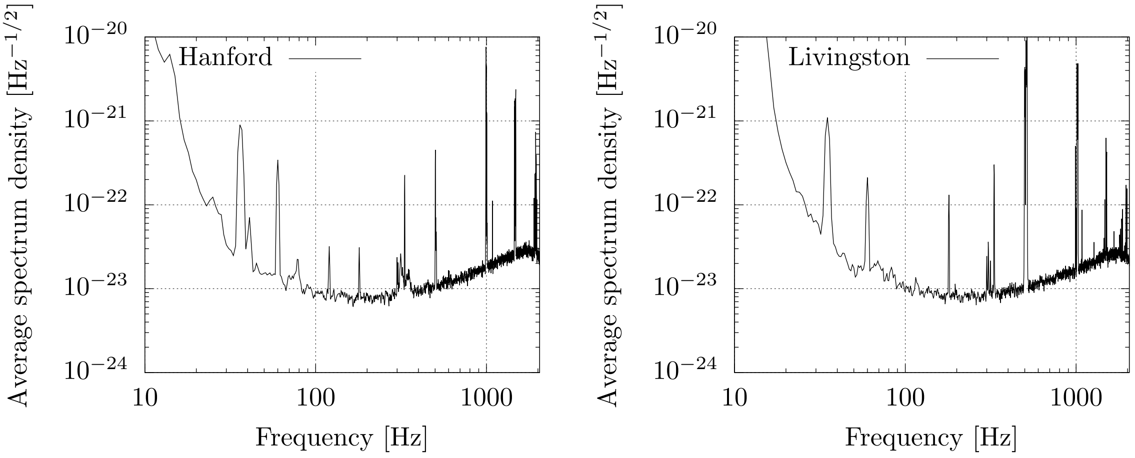

FIGURE 3. Detector sensitivities of the two observatories of LIGO. They are estimated by means of the Welch method from observed data for 32 s.

of the noise spectrum is relatively weak in the band between about 100 Hz and 300 Hz, in other words the detectors are relatively sensitive in the band, hence gravitational waves whose frequency components lay in the band are targets of the detectors, and the frequency components of the detected gravitational wave are laid in the band as shown in Figure 1. Figure 3 also shows the observed data contains strong narrow-band noise components, or line noises. The line noises arise from mirror suspension resonances, some main power harmonics, and injected signals for calibration.

To investigate a signal structure of observed gravitational wave data with the HHT, we applied a band-pass filter and a band-stop filter to extracting the sensitive band, and some notch filters to attenuate the strong frequency components by the line noises. Table 1 and Table 2 list the properties of the filters we applied. All the filters we applied are digital infinite impulse response filters based on Butterworth filter with filter order of four. The spectrum densities of the data are shown in Figure 4. The filtered strain data obtained from Hanford and Livingston are shown in Figure 5 with the waveform numerically simulated with the same condition of the source as the event. We can visually recognize that the observed data and the simulated waveform have similar structures. We analyze the filtered strains by the HHT.

3. **Analysis by the HHT.** We performed the EEMD to the filtered data with the stoppage threshold of EMD $\varepsilon = 10^{-4}$ and the standard deviation of added noises in

Table 1. Cutoff frequencies of filters we applied to the raw observed data to extract the sensitive band

	Cutoff frequencies [Hz]			
	Hanford		Livingston	
	lower	higher	lower	higher
Bandpass	35.0	350.0	35.0	350.0
Bandstop	330.8	671.6	330.8	671.6

Table 2. Filter properties of notch filters we applied to attenuate line noises. f_{central}, Δf_{pass} and Δf_{stop} represent central frequency, pass band width and stop band width of a notch filter respectively.

	Properties of notch filters [Hz]					
	Hanford			Livingston		
	f_{central}	Δf_{pass}	Δf_{stop}	f_{central}	Δf_{pass}	Δf_{stop}
Notch 1	36	1	0.1	35	3	0.1
Notch 2	40	1	0.1	60	4	1
Notch 3	60	1	0.1	180	5	1
Notch 4	331	10	1	325	10	1

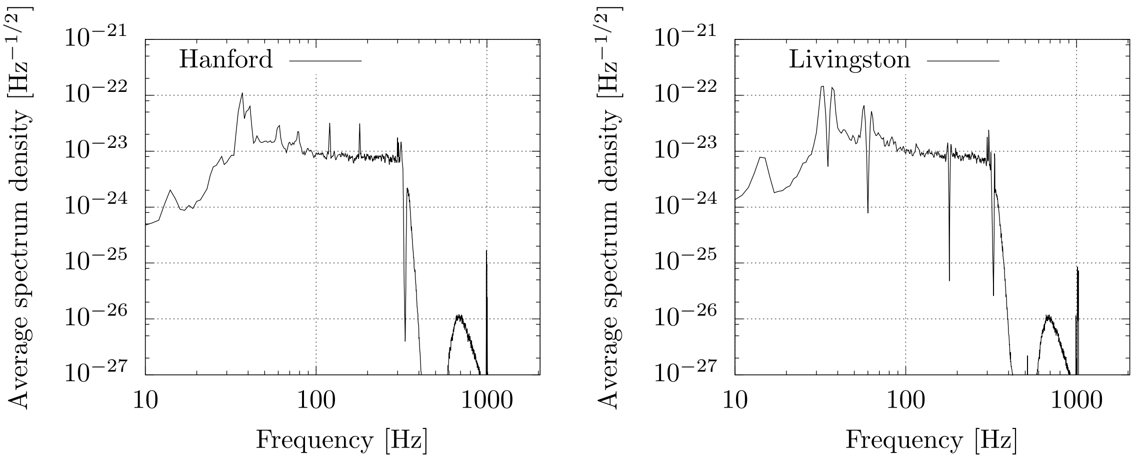

Figure 4. Average power spectrum densities of the filtered data

EEMD $\sigma_{\mathrm{e}} = 10^{-3}$. Obtained IMF1, 2, 3 are shown in Figure 6 with the plot of simulated waveform.

Time-frequency maps of the filtered strains obtained by the HHT are shown in Figure 7. The dashed lines plotted in the maps represent instantaneous frequency of the simulated waveform estimated by the HSA.

The amplitude of Livingston data in inspiral phase is smaller than that of Hanford data. It is because we notched wider bands from Livingston than Hanford, at the band signal contained.

Resolution of the map obtained by the HHT plotted in Figure 7 is higher than resolution of the time-frequency map obtained by the wavelet transform shown in Figure 1. In particular, we can see that the frequency of main mode becomes constant slightly after 0 s in Figure 7; on the other hand, we cannot see that in Figure 1.

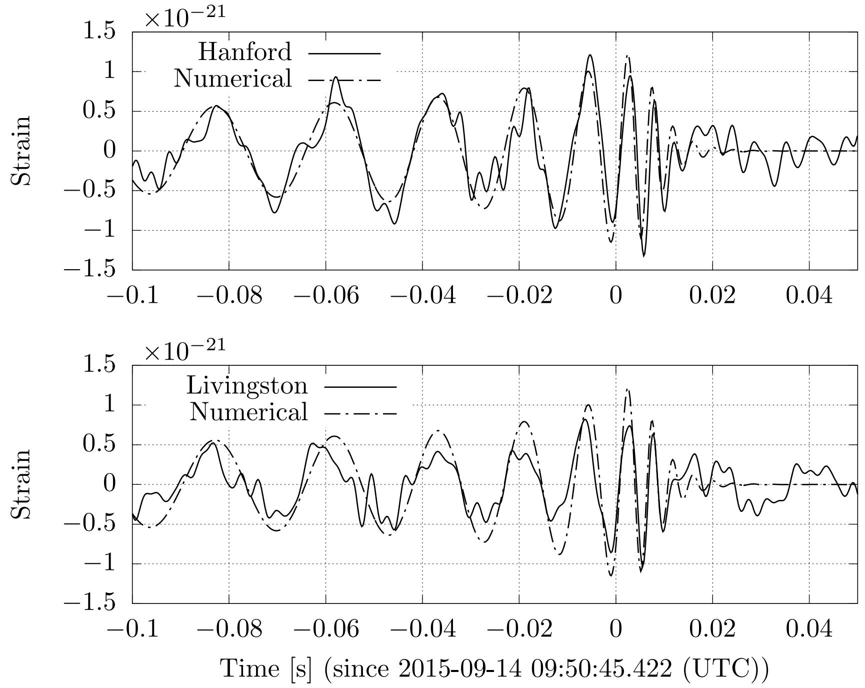

Figure 5. Filtered strains of Hanford and Livingston, with a numerically simulated waveform of a gravitational wave from a black hole binary coalescence with the same configuration as GW150914

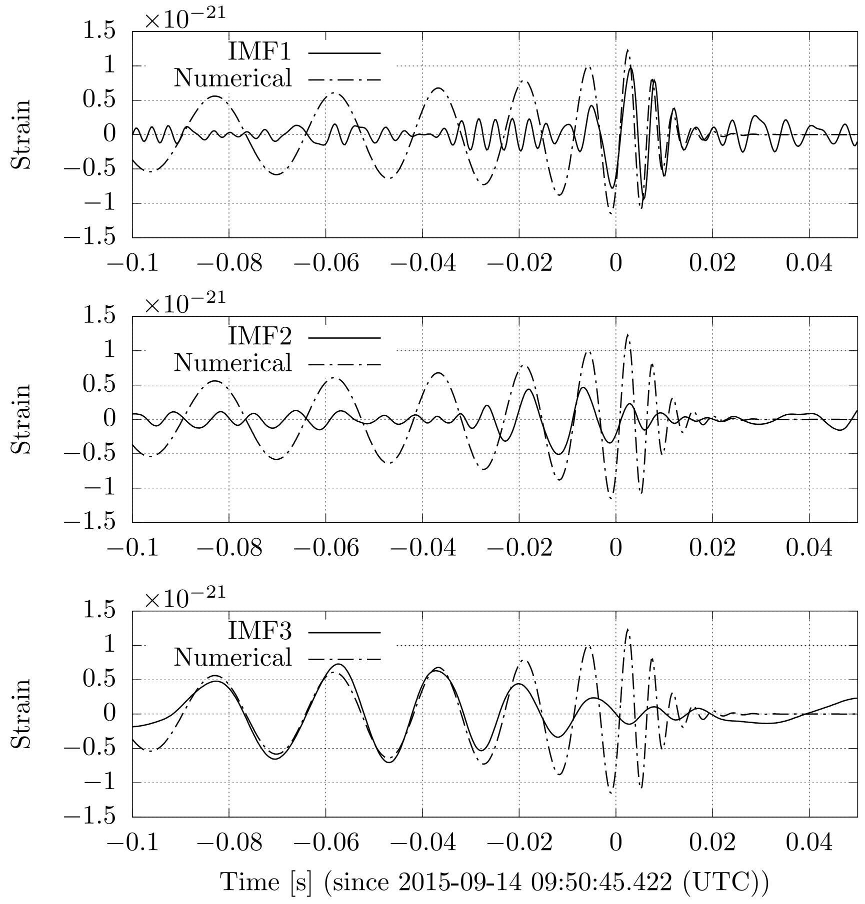

Figure 6. IMF1, 2, 3 of Hanford, and the simulated waveform

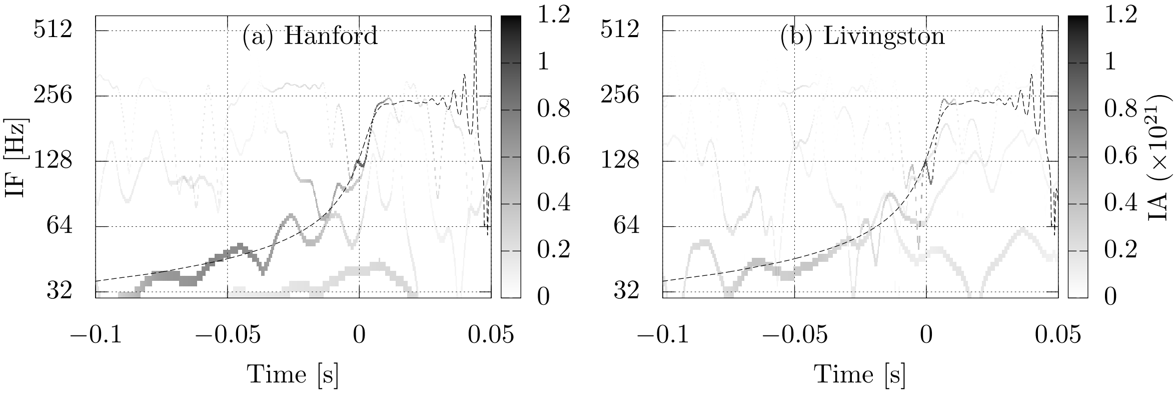

Figure 7. T-F maps by the Hilbert-Huang transform. The dashed lines represent instantaneous frequency of the simulated waveform estimated by the HSA.

However, as Figures 6 and 7 show, in the interval between about −0.03 s and 0 s, the mode of gravitational wave is separately decomposed into IMF2 and IMF3. This is one kind of 'mode splitting' problem of the HHT, which is a representative problem [10].

4. **Summary.** By means of the Hilbert-Huang transform (HHT), we analyzed the real observed gravitational wave data named GW150914, which is the first direct detection of gravitational waves observed by LIGO. We applied a bandpass filter and a bandstop filter to extracting a sensitive band of detectors and some notch filters to attenuate strong line noises, and applied the HHT to the output. As a result of a comparison of time-frequency maps obtained by the HHT with ones by the wavelet transform, the HHT can show signal structures with high resolution. In particular, the structure such that frequency of main mode becomes constant is emerging only in the result by the HHT.

Some improvements are still required on the HHT, especially to solve the mode splitting problem. We are going to tackle the improvements of the HHT.

In this paper, we reported the high resolution performance of the HHT in representing the signal structures of GW150914 on time-frequency maps. We are also planning to confirm the effectiveness of the HHT in investigating physical dynamics from the extracted signal structures.

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