ICIC Express Letters
Volume **11**, Number **1**, January **2017**
 ISSN 1881-803X
pp. 87–93

AN IMPROVED GENETIC ALGORITHM BASED ON A LÉVY DISTRIBUTION SELECTION OPERATOR

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Received June 2016; accepted September 2016

Abstract. *A genetic algorithm (GA) is a classic evolutionary algorithm, whose performance is dramatically influenced by convergence speed and solving precision. During GA selection operation, the Lévy distribution with different parameters in sampling for different sub-populations was used to generate new individuals with similar characteristics. In addition, based on the population-based algorithm portfolios (PAP) strategy, genetic operations were performed for multiple populations by integrating the advantages of Lévy distributions with different parameters, thereby improving the algorithm capability of searching and solving various problems. Simulation results on different numerical optimization functions show that the algorithm can effectively avoid the problem of premature convergence, thus resulting in high-quality solutions.*
Keywords: Genetic algorithm, Lévy distribution, Numerical optimization problems, Population-based algorithm portfolios

1. **Introduction.** As a global optimal algorithm, the genetic algorithm (GA) [1] has been widely applied in scientific and industrial fields. The standard genetic algorithm (SGA) is a global search algorithm, where the search should focus on the whole search space in the early stage to avoid the premature convergence, while the strong local searching ability is required in the later stage to refine the search. Therefore, crossover, and mutation operations need to be improved [2-9]. Yao et al. [6] adopted the Cauchy distribution during the mutation stage of evolutionary programming and obtained promising results for most real value optimization problems. However, merely using the Cauchy distribution is not beneficial for algorithm convergence during late stages. Peng et al. [7,8] proposed the Lévy distribution-based operator for evolutionary problems and observed significantly improved performance. He and Yang [9] developed a differential evolution algorithm based on adaptive mutation and the Lévy distribution, which demonstrated marked increase in convergence speed. Peng et al. [10] reported the population-based algorithm portfolios (PAP) strategy, which effectively incorporates the advantages of different sub-populations and showed promising solving ability for different types of problems. Here, we propose a Lévy distribution-based selection operator that allows independent evolution of multiple sub-populations. The PAP strategy is used to integrate the advantages of Lévy distributions with different parameters, with the expectation that new individuals with population characteristics and high fitness scores would be generated during the selection stage, thereby avoiding the premature problem in the early stage and accelerating convergence in the late stage.

The rest of the paper is organized as follows. Section 2 introduces some basics about the Lévy distribution. The detail of the proposed algorithm is given in Section 3. Section 4 presents the simulation results and analyses. Finally, conclusions are drawn and some future work is given in the last section.

2. **Lévy Distribution.** The Lévy distribution is a stable distribution proposed by Lévy in the 1930s [11]. It is expressed as:

$$L_{\alpha,\gamma}(y) = \frac{1}{\pi} \int_0^\infty e^{-\gamma q^\alpha} \cos(qy) dq \tag{1}$$

In the equation, $e = 2.718$, $y \in R$, $\gamma > 0$ is the scale parameter, $0 < \alpha \leq 2$ is the distribution index, and $q > 0$ is a variable real value. The Lévy distribution is a collection of distributions of the same type, and the common Gaussian distribution and Cauchy distribution are two special cases: $\alpha = 2$ depicts a Gaussian distribution; $\alpha = 1$ indicates a Cauchy distribution.

Figure 1 shows the curves of probability density for a Lévy distribution [X $\sim$ Lévy(γ, δ)], Gaussian distribution [X $\sim$ N(μ, σ^2)], and Cauchy distribution [X $\sim$ Cauchy(γ, δ)]. Herein, γ is the scale parameter, δ is the position parameter, μ is the average value, and σ is standard deviation. It can be observed that despite similar curves, the distribution ranges of different probability density curves are distinct. The demanded random variables could be generated by adjusting the relative parameters of the Lévy distribution.

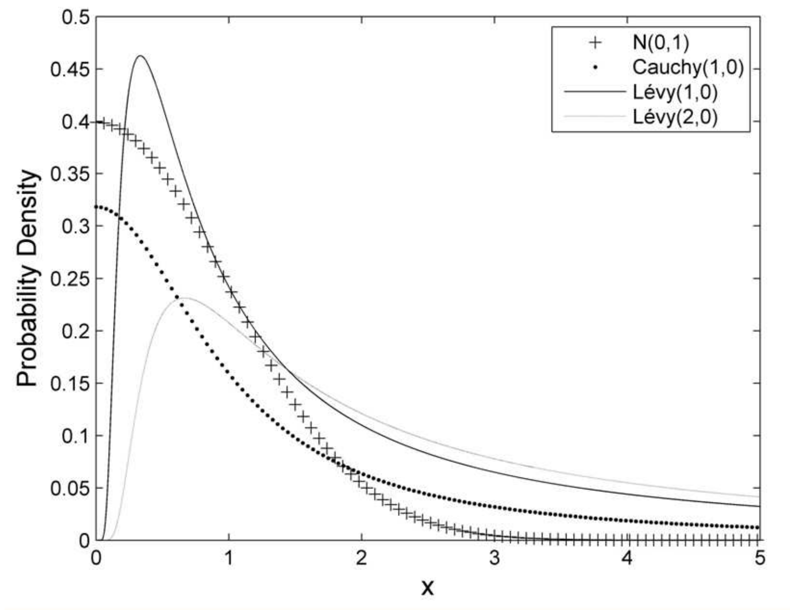


FIGURE 1. Comparison of probability density curves

3. **Improved GA.** As earlier mentioned, GA requires fast convergence, as well as population diversity. In addition, different optimization problems require different parameters. A GA was proposed based on a Lévy distribution probability operator, and the PAP strategy was adopted to integrate the advantages of a Lévy distribution with different control parameters, thereby improving algorithm performance in solving different optimization problems. Table 1 shows the pseudo-codes of the algorithm.

The key focus of the present study was improvement of the selection operator. A GA generally falls within the local optimum due to a lack of excellent genetic types. Based on this particular issue, the Lévy distribution was used in the selection stage. A Lévy distribution has a probability density function that varies with different parameters, thus ensuring that the genetic represents the characteristics of the current population, as well as maintains the population diversity for different optimization problems, thereby accelerating convergence and avoiding premature problems. The pseudo-codes of an improved selection operator are presented in Table 2.

Table 1. Pseudo-codes of the improved genetic algorithm

1. Initialize N sub-populations with the size of M.
2. While meeting termination condition, do
3. Assess the fitness scores of all individuals in N sub-populations
4. For $i = 1 : N$
5. Use Lévy distribution-based GA for the ith sub-population, $\alpha = \alpha_i$
6. End for
7. If the number of specified genetic generations has been reached,
8. For $i = 1 : N$
9. Except for sub-population i, choose t excellent individuals from each of the rest $N - 1$ sub-populations
10. Combine all individuals of sub-population i with the above $(N - 1) \times t$ individuals to form a new sub-population.
11. Select M excellent individuals from the new sub-population to form the new sub-population i.
12. End for
13. End if
14. End while
15. Compare the optimal solutions of N sub-populations, and take the maximum value as the optimal solution.

Table 2. Pseudo-codes of the improved selection operator

1. For $i = 1 : N$
2. Sort the M individuals in the ith sub-population according to fitness scores.
3. Divide the ith sub-population into K sub-groups.
4. For the first S subgroups of the ith sub-population, new individuals are generated according to the Lévy distribution with specified parameters.
5. K candidate pools are generated for the ith sub-population
6. Roulette is performed for the K subgroups of the ith sub-population
7. End for

4. **Function Testing.** In the present study, 15 real value optimization functions [6,13] were used to test the performance of improved GAs (Table 3). The value of n equals 30. The SGA, Gaussian, and Cauchy selection-based genetic algorithm (GCSGA) and Lévy distribution-based genetic algorithm were compared. The genetic parameters and number of assessments were essentially the same. For SGA, the number of individuals was 200, the crossover probability was 0.7, mutation probability was 0.07, and number of assessments was 2000000. For GCSGA, the number of individuals in the two sub-populations was both 100. For LSGA, the number of individuals in the four sub-populations was all 50. The α value of the Lévy distribution was set to 1.0, 1.3, 1.7, and 2.0 [8], respectively. During selection, four sub-populations were divided into 25 groups, the individuals in the first four groups [12] showed probability distribution disturbances. The sub-populations communicate with each other every 500 generations [10] with respect to the maximal value.

Table 4 shows the mean values and standard deviations of the 15 real value optimization functions obtained by SGA, GCSGA, and LSGA, which were calculated based on 50 tests. The numbers in bold indicate a small mean value or standard deviation. A small mean value means preferable algorithm performance, whereas a small standard deviation indicates that the algorithm is relatively stable.

TABLE 3. Testing function

Test Function	*Domain*	$f_{\min}$
$f_1(x) = \sum_{i=1}^n x_i^2$	$[-100, 100]^n$	0
$f_2(x) = \sum_{i=1}^n \lvert x_i \rvert + \prod_{i=1}^n \lvert x_i \rvert$	$[-10, 10]^n$	0
$f_3(x) = \sum_{i=1}^n \left(\sum_{j=1}^i x_j\right)^2$	$[-100, 100]^n$	0
$f_4(x) = \sum_{i=1}^{n-1} \left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2\right]$	$[-30, 30]^n$	0
$f_5(x) = \sum_{i=1}^n (\lfloor x_i + 0.5 \rfloor)^2$	$[-100, 100]^n$	0
$f_6(x) = \sum_{i=1}^n i x_i^4 + random[0, 1)$	$[-1.28, 1.28]^n$	0
$f_7(x) = \sqrt{\sum_{i=1}^n \lvert x_i \rvert^{2+4\frac{i-1}{n-1}}} + f_7^*$	$[-100, 100]^n$	0
$f_8(x) = \sum_{i=1}^n -x_i \sin\left(\sqrt{\lvert x_i \rvert}\right)$	$[-500, 500]^n$	-12569.5
$f_9(x) = \sum_{i=1}^n [x_i^2 - 10\cos(2\pi x_i) + 10]$	$[-5.12, 5.12]^n$	0
$f_{10}(x) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^n x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^n \cos 2\pi x_i\right) + 20 + e$	$[-32, 32]^n$	0
$f_{11}(x) = \frac{1}{4000}\sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	$[-600, 600]^n$	0
$f_{12}(x) = \frac{\pi}{n}\left\{10\sin^2(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2\left[1 + 10\sin^2(\pi y_{i+1})\right] + (y_n - 1)^2\right\} + \sum_{i=1}^n \mu(x_i, 10, 100, 4)$	$[-50, 50]^n$	0
$f_{13}(x) = 0.1\left\{\sin^2(3\pi x_1) + \sum_{i=1}^{n-1}(x_i - 1)^2\left[1 + \sin^2(3\pi x_{i+1})\right] + (x_n - 1)^2[1 + \sin^2(2\pi x_n)]\right\} + \sum_{i=1}^n \mu(x_i, 5, 100, 4)$	$[-50, 50]^n$	0
$f_{14}(x) = \min\left(\sum_{i=1}^n (\widehat{x}_i - \mu_0)^2, dn + s\sum_{i=1}^n (\widehat{x}_i - \mu_1)^2\right) + 10\left(n - \sum_{i=1}^n \cos(2\pi \widehat{z}_i)\right) + f_{14}^*$	$[-100, 100]^n$	0
$f_{15}(x) = g_1(g_2(z_1, z_2)) + g_1(g_2(z_2, z_3)) + \ldots + g_1(g_2(z_{n-1}, z_n)) + g_1(g_2(z_n, z_1)) + f_{15}^*$	$[-100, 100]^n$	0

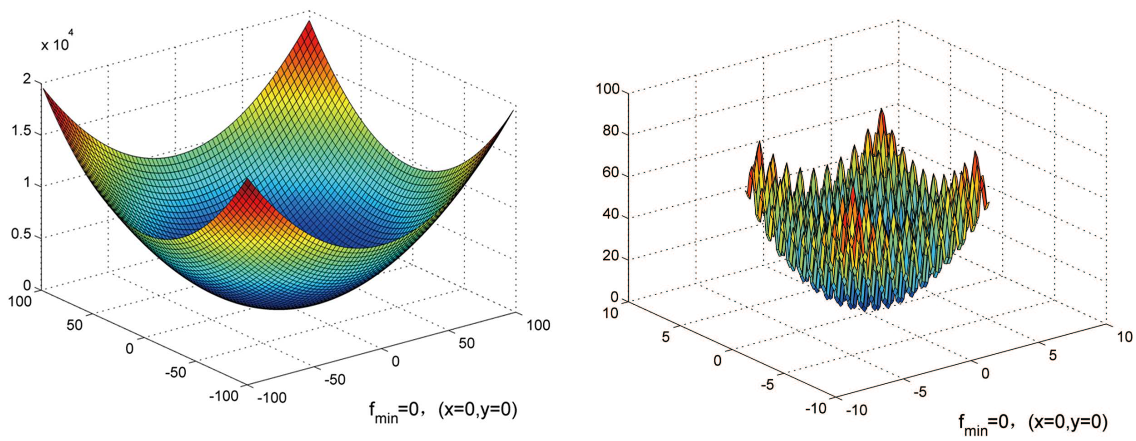
Based on function properties, it can be seen that functions $f_1(x) \sim f_7(x)$ were unimodal. Figure 2(a) shows the two-dimensional structure of function $f_5(x)$, which is a wave trough. Thus, large scope searching in the solution space during the early stage of assessment could speed up the solving process, whereas searching the solution space close to optimal solutions in the late stage could effectively avoid the premature phenomena. Figures 3(a)

Table 4. Comparison of testing results

Function	*SGA*		*GCSGA*		*LSGA*	
	Mean Value	*Standard Deviation*	*Mean Value*	*Standard Deviation*	*Mean Value*	*Standard Deviation*
f_1	162.92549	298.84232	0.04682	0.04936	**0.00025**	**0.00009**
f_2	1.16002	1.28994	0.37131	0.35841	**0.16256**	**0.29613**
f_3	116737.33104	201398.43343	7.63085	11.28067	**0.06991**	**0.04727**
f_4	5248.62935	13880.22732	**38.31525**	33.70466	49.39222	**30.97531**
f_5	139.42000	199.67935	1.26000	2.84119	**0.82000**	**1.26000**
f_6	0.17082	0.34834	0.01795	0.01434	**0.01215**	**0.00746**
f_7	0.016471	0.02565	0.00294	0.00737	**0.00293**	**0.00106**
f_8	−12478.01	91.65929	−12564.56	23.19129	**−12569.48**	**0.04370**
f_9	3.89380	3.91445	0.77023	**0.95800**	**0.63598**	0.96998
f_{10}	2.89543	1.92542	0.70798	0.66228	**0.01236**	**0.00265**
f_{11}	2.20201	1.87872	0.03469	0.02012	**0.00602**	**0.00817**
f_{12}	41273.94651	144407.05772	0.000008	0.000019	**0.00000026**	**0.0000001**
f_{13}	20723.81112	45419.07202	**0.00041**	**0.00215**	0.001518	0.00428
f_{14}	35.35896	4.8812	33.54111	**2.25846**	**32.63503**	2.43337
f_{15}	34.49848	74.94913	1.35058	**0.35231**	**1.29075**	0.39197

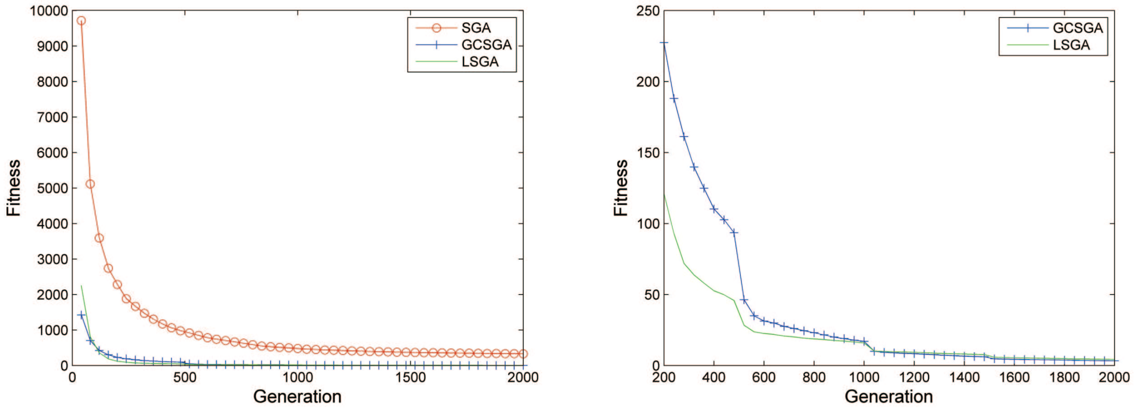
and 3(b) compared different fitness variation curves, which clarified the influences of α in the Lévy distribution on convergence speed and solving resolution.

The rest of the functions were multi-modal. Figures 2(b), 4(a), and 4(b) show the two dimensional structures of $f_9(x)$, $f_{10}(x)$, and $f_{12}(x)$. Because there were several wave troughs, these may easily fall into the premature phenomena, which is not good for improving solving precision. Therefore, different α values were used in the improved algorithm; they sequentially played a major role in different stages, thereby ensuring promising results in different stages. In particular, the two dimensional structure of $f_{12}(x)$ is flat and smooth and shows various wave troughs, and thus it is very difficult to avoid the premature phenomena with random solution space searching. The fitness curves in Figures 5(a) and 5(b) also prove that the improved algorithm searched the solution space close to optimal solutions at different scales, thereby accelerating convergence and avoiding premature.



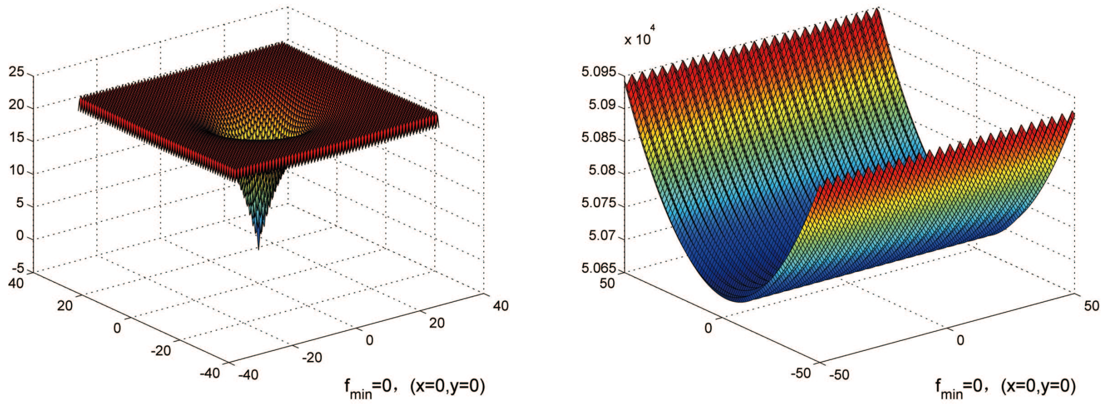
(a) (b)

Figure 2. Two dimensional structures of functions f_5 and f_9



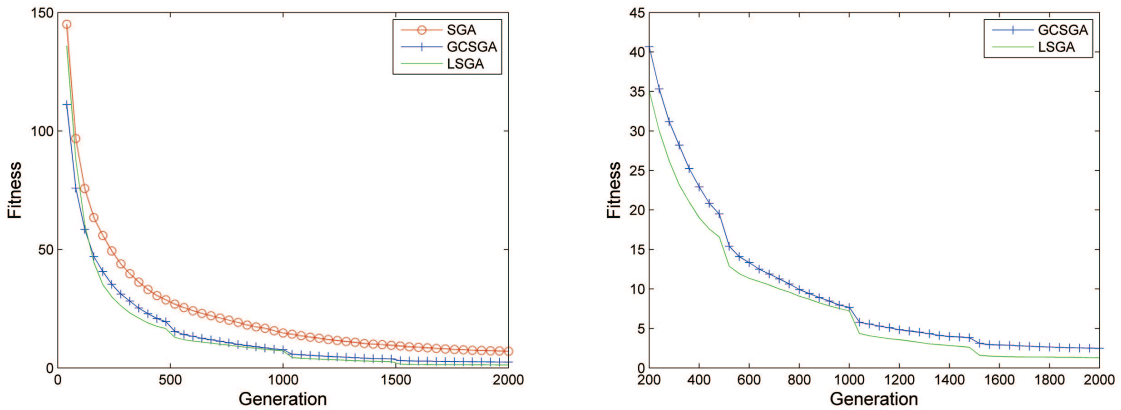
(a) (b)

FIGURE 3. Fitness variation curve of function f_5



(a) (b)

FIGURE 4. Two dimensional structures of functions f_{10} and f_{12}



(a) (b)

FIGURE 5. Fitness variation curve of function f_9

5. **Conclusions.** Using the Lévy distribution of different parameters for different subpopulations was proposed during the selection stage to direct the search in the solution space. The PAP strategy was used to integrate the advantages of different subpopulations, which not only ensured population diversity, but, to a certain extent, enhanced searching efficiency as well. By testing different types of functions, we have shown that the improved algorithm can significantly increase the convergence speed in the late stages and prevent the occurrence of premature phenomena in the early stage. The next step is to improve the performance of the improved algorithm in terms of functions with high dimensions. Additionally, parallel computer optimization will also be conducted to reduce related consumption.

Acknowledgment. This work is partially supported by the National Basic Research Program (973 Program) of China (2014CB360500, 2014CB360506). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

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