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T-S FUZZY MODEL-BASED ADAPTIVE CONTROLLER DESIGN FOR UAV WITH ACTUATOR SATURATION

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Abstract. *In this paper, the adaptive fault tolerant tracking control problem is studied for an unmanned aerial vehicle (UAV) with actuator saturations and unknown external disturbances. Firstly, the Takagi-Sugeno fuzzy models are established to represent the nonlinear flight control systems of a UAV with actuator saturation and unknown disturbances. Then, an adaptive normal tracking controller is designed using an online estimator, in which a compensation control term is introduced so as to reduce the effect of actuator saturation. Based on the normal tracking controller, a novel adaptive fault tolerant control (FTC) scheme is presented in case of actuator loss-of-effectiveness fault. Compared with the existing work, the FTC approach proposed in this paper does not rely on any fault diagnosis unit and is easily applied in aerospace engineering. Finally, simulation results show the efficiency of the presented FTC scheme.*

Keywords: Unmanned aerial vehicle, Adaptive controller, Flight control systems, Fuzzy modelling

1. **Introduction.** Unmanned aerial vehicle (UAV) is a class of aircraft without a human pilot onboard. The flight of a UAV, depending on flight control systems (FCS) that include actuator, sensor and so on, is controlled either autonomously by onboard computers or by the remote control of a pilot [1]. It is a quickly time-varying and strong coupling characteristic of complex nonlinear system [2]. Because the flight control systems of UAV are nonlinear systems, the traditional linear control method is not appropriate. However, Takagi-Sugeno (T-S) fuzzy modelling is an effective tool which connects linear control system with nonlinear one [3], which has been an active research topic. It can utilize a series of local linearized models to realize the global approximation of an arbitrary nonlinear smooth system function, and then the complex analysis and application of nonlinear control system are greatly simplified, which is also its main merit. For the above reason, a T-S fuzzy system describing nonlinear FCS of UAV is introduced in this paper. Although some results about adaptive controller for T-S fuzzy systems have been achieved in recent years, some public problems still exist, which are needed to solve, for example, the controller design for T-S fuzzy system under unknown external disturbance and actuator saturation simultaneously is a challenging public problem now. In [4-7], the actuator saturation is not discussed, which certainly exists and needs solving in actual FCS, so the fault accommodation results obtained above have some limitations in dealing with two kinds of actual actuator faults simultaneously. In [8], the authors address the problem of actuator saturation and loss of effectiveness faults for singular T-S fuzzy control systems; however, the unknown disturbance is not considered, which is inevitable in the

actual FCS of an unmanned aerial vehicle. So fault tolerant tracking controller design problems of nonlinear system expressed by fuzzy models have not been thoroughly solved, which is still a great challenge.

In this study, the results obtained in this paper can be regarded as the complements of previous research. When actuator saturation occurs in an actual UAV, the adaptive control scheme developed in this paper guarantees the asymptotical tracking of FCS. Finally, simulation results show that our design approach has the favorable and robust ability.

2. Problem Statement and Preliminaries.

The flight control system model of UAV is given by [1]:

$$\dot{x} = f(x) + g(x)u, \qquad y = Cx \tag{1}$$

where $x \in R^n$ denotes a state vector, $u \in R^m$ denotes the control input vector, and the output vector is $y \in R^r$.

The nonlinear system is linearized locally, which expresses the input-output relation of the original system, and then Takagi and Sugeno theory is used to establish the fuzzy dynamic model. Consider a T-S fuzzy model which is composed of many fuzzy implications, where every implication is equal to a linear state-space model. The ith fuzzy rule of the T-S model of FCS is written as follows.

Plant Rule i: IF $z_1(t)$ is M_{i1} and $z_q(t)$ is M_{iq} THEN

$$\dot{x} = A_i x + B_i \delta + B_d w, \quad y = C_i x$$

where $i = 1, \ldots, N$, the fuzzy rule number is defined as N, the fuzzy set is M_{ij} ($j = 1, \ldots, q$), $z(t) = [z_1(t) \ldots z_q(t)]^T$ are the given variable, $A_i \in R^{n \times n}$, $B_i \in R^{n \times m}$ and $C_i \in R^{r \times n}$.

The overall fuzzy FCS for UAV could be deduced as follows,

$$\dot{x} = \sum_{i=1}^{N} \pi_i(z)(A_i x + B_i \delta + B_d w), \quad y = \sum_{i=1}^{N} \pi_i(z) C_i x$$

where

$$\pi_i(z) = \prod_{i=1}^{k} M_{ij}(z) \Big/ \sum_{i=1}^{N} \prod_{i=1}^{k} M_{ij}(z), \quad 0 \le \pi_i(z) \le 1, \quad \sum_{i=1}^{N} \pi_i(z) = 1$$

with $M_{ij}(z)$ being the grade of membership of $z_j(t)$ among M_{ij} and $\prod_{i=1}^{k} M_{ij}(z) \ge 0$.

Since there are different mechanical and physical restrictions on the control surfaces or input amplitude, the output of actuator is denoted by the following $\mathrm{sat}(\delta)$

$$\mathrm{sat}(\delta_s) = \begin{cases} \delta_{s\min}, & \delta_s < \delta_{s\min} \\ \delta_s, & \delta_{s\min} \le \delta_s \le \delta_{s\max} \\ \delta_{s\max}, & \delta_s > \delta_{s\max} \end{cases} \tag{2}$$

where $\mathrm{sat}(\delta)$ is the actual output of actuator with saturation constraints, $\delta_{s\min}$ ($s = 1, \ldots, m$) and $\delta_{s\max}$ ($s = 1, \ldots, m$) are the minimum saturation level and maximum one of the output of actuator, which is decided in advance, and δ_s is the control input, namely output of controller, which will be designed.

Obviously, there is a difference between a real output of actuator provided and desired control input, and the saturation error produced by actuator saturation is expressed as

$$\Delta\delta = \mathrm{sat}(\delta) - \delta \tag{3}$$

In an actual control system, the saturation error $\Delta\delta$ between an ideal input δ and a final output for actuator $\mathrm{sat}(\delta)$ provided is bounded. Thus, it is assumed that the following inequality holds.

$$||\Delta\delta|| \le h(x)\sigma^* \tag{4}$$

where $h(x) \in R^{1\times m}$ is a continuous known function and $\sigma^* \in R^{m\times 1}$ is an unknown parameter.

The overall fuzzy FCS for healthy UAV with unknown disturbances and actuator saturation are represented as follows:

$$\dot{x} = \sum_{i=1}^{N} \pi_i(z)(A_i x + B_i \mathrm{sat}(\delta) + B_d w), \quad y = \sum_{i=1}^{N} \pi_i(z) C_i x \tag{5}$$

where $\mathrm{sat}(\delta)$ defined as (2) denotes actual output of actuator of the healthy FCS and $\mathrm{sat}(\delta) \in R^m$.

Considering actuator faults, rewrite the faulty FCS for UAV with unknown disturbances and actuator saturation as

$$\dot{x} = \sum_{i=1}^{N} \pi_i(z) \left(A_i x + B_i \mathrm{sat}\left(\delta^F\right) + B_d w\right), \quad y = \sum_{i=1}^{N} \pi_i(z) C_i x \tag{6}$$

where δ^F is the input of a faulty FCS, $\mathrm{sat}(\delta^F)$ denotes an actual output of actuator of the faulty FCS, and $\delta^F = \left[\delta_1^F, \delta_2^F, \ldots, \delta_m^F\right]^T$.

To formulate FTC design, the actuator fault model is introduced as follows,

$$\delta^F = \rho\delta \tag{7}$$

In this paper, the actuator fault is set to be a loss of effectiveness (LOE) of control surface, $\rho \in R^{m\times m}$ is the unknown diagonal fault matrix with $\rho_1, \rho_2, \ldots, \rho_m$ and $0 < \rho_s \le 1$.

From property of (3), the following equation is obtained,

$$\mathrm{sat}\left(\delta_s^F\right) = \delta_s^F + \Delta\delta_s^F = \rho_s\delta_s + \Delta\delta_s^F, \quad \rho_s \in \left[\underline{\rho_s}, \overline{\rho_s}\right], \quad 0 < \underline{\rho_s} \le 1, \ \overline{\rho_s} \ge 1 \tag{8}$$

where δ_s^F is control input, $\Delta\delta_s^F$ $(s = 1, 2, \ldots, m)$ denotes a deviation between an actual output of actuator and ideal control input. ρ_s is an unknown constant modeling the s_{th} control effectiveness element of m control surfaces or actuators. $\overline{\rho_s}$, $\underline{\rho_s}$ denote the known upper of ρ_s bound and lower one, respectively. It is worth noting that when $\underline{\rho_s} = \overline{\rho_s} = 1$, the s_{th} actuator fault does not occur.

Between the minimum and maximum bounds $\left[\underline{\rho_s}, \overline{\rho_s}\right]$, a set is defined as follows:

$$N_\rho = \left\{\rho : \rho = \mathrm{diag}\left[\rho_1, \ldots, \rho_m\right], \rho_s = \underline{\rho_s} \text{ or } \rho_s = \overline{\rho_s}, \ s = 1, \ldots, m\right\}$$

$$\mathrm{sat}\left(\delta_s^F\right) = \begin{cases} \delta_{s\min}, & \delta_s^F < \delta_{s\min} \\ \delta_s^F, & \delta_{s\min} \le \delta_s^F \le \delta_{s\max} \\ \delta_{s\max}, & \delta_s^F > \delta_{s\max} \end{cases}$$

It is widely accepted that the steady-state tracking error is accommodated by an integral function of a controller. To design an adaptive controller with integral $\eta(t) = \int_0^t (y_r(s) - y(s))ds$, combining (6) and $\eta(t)$, we obtain the following augmented system:

$$\begin{bmatrix} \dot{\eta} \\ \dot{x} \end{bmatrix} = \sum_{i=1}^{N} \pi_i(z) \left\{ \begin{bmatrix} 0 & -C_i \\ 0 & A_i \end{bmatrix} \begin{bmatrix} \eta \\ x \end{bmatrix} + \begin{bmatrix} 0 \\ B_i \end{bmatrix} \mathrm{sat}\left(\delta^F\right) + \begin{bmatrix} I & 0 \\ 0 & B_d \end{bmatrix} \begin{bmatrix} y_r \\ w \end{bmatrix} \right\} \tag{9}$$

Let $\bar{x} = \left[\eta^T, x^T\right]^T$, and the above faulty system (9) can be rewritten as

$$\dot{\bar{x}} = \sum_{i=1}^{N} \pi_i(z) \left(\mathbb{A}_i \bar{x} + \mathbb{B}_i \mathrm{sat}\left(\delta^F\right)\right) + \mathbb{B}_d d \tag{10}$$

Note that if the failure parameter $\rho = I$ (I is an identity matrix), then no fault happens and $\mathrm{sat}(\delta) = \mathrm{sat}\left(\delta^F\right)$.

The required output y tracks the reference command y_r under the condition of no steady state error, namely

$$\lim_{t\to\infty} \varepsilon = 0,\ \varepsilon = y_r - y,\ \eta = \int_0^t \varepsilon(s)\ ds \tag{11}$$

Besides, the robust tracking control performance index γ for all $d(t)$ is satisfied:

$$\int_0^t \eta^T(s)\eta(s)ds \le \gamma^2 \int_0^t d^T(s)d(s)ds \tag{12}$$

To proceed with the design of robust FTC for a faulty UAV with disturbances and saturation constraints, two assumptions, in turn, are as follows.

Assumption 2.1. *The loss of effectiveness (LOE) of the actuator is bounded; moreover, there exists a positive scalar $\varpi > 0$ such that $0 < ||\rho|| < \varpi$ holds.*

Assumption 2.2. *The unknown disturbance w is bounded, namely, there exists a positive scalar $\varsigma > 0$ such that $||w|| < \varsigma$ holds.*

3. Main Results.

3.1. Normal control law design for a UAV with input saturation.

For the healthy flight control systems for UAV, one considers the following controller including an adaptive variable u_2:

$$\delta = \sum_{j=1}^{N} \pi_j(z)\mathbb{K}_j\bar{x} + u_2 \tag{13}$$

where the normal control input is δ, the feedback gain matrix is $\mathbb{K}_j \in R^{m\times(k+n)}$ to be determined and $\mathbb{K}_j = [K_{j\eta}\ K_{jx}]$, and $u_2 \in R^m$ is used to compensate for the actuator saturation.

Choose the following compensation control law

$$u_2(t) = -\frac{\mathbb{B}_i^T P\bar{x}(t)}{||\bar{x}^T(t)P\mathbb{B}_i||}h(x)\hat{\sigma}, \quad \dot{\hat{\sigma}}_s = -\sum_{i=1}^{N} \pi_i(z)\vartheta_s h_s^T(x)||\bar{x}^T P\mathbb{B}_i|| \tag{14}$$

where $\hat{\sigma}$ is an estimation of σ^*, $\vartheta_s > 0$ is a learning coefficient determined by $\hat{\sigma}$, and $h(x) = [h_1(x), \dots, h_m(x)]$.

Substituting (13) into (10), the controlled fuzzy system is described as

$$\dot{\bar{x}} = \sum_{i=1}^{N}\sum_{j=1}^{N} \pi_i(z)\pi_j(z)\left(\mathbb{A}_i + \mathbb{B}_i\mathbb{K}_j\right)\bar{x} + \sum_{i=1}^{N} \pi_i(z)\left[\mathbb{B}_i\Delta\delta + \mathbb{B}_i u_2\right] + \mathbb{B}_d d \tag{15}$$

3.2. Adaptive FTC design.

Here, an FTC law δ^F utilizing adaptive fault compensation controller is proposed

$$\delta^F = \delta + \delta^C \tag{16}$$

where δ is the normal control input term presented in (13), and δ^C is an adaptive fault compensation factor so it is zero or does not lie on whether actuator faults occur.

Next, to acquire a necessary estimation of actuator faults, a target model is given:

$$\dot{\hat{x}} = \sum_{i=1}^{N} \pi_i(z)[A_i\hat{x} + B_i\hat{\rho}r + B_i u_3], \quad \hat{y} = \sum_{i=1}^{N} \pi_i(z)C_i\hat{x} \tag{17}$$

where $\hat{\rho} = \mathrm{diag}\,[\hat{\rho}_1, \dots, \hat{\rho}_m]$ expresses the estimation of remaining effectiveness factor. To implement given control objective, the input $r \in R^m$ and $u_3 \in R^m$ are determined later.

To design the suitable input δ^F, the output y of system (10) with actuator saturation (4), unknown disturbances and actuator faults (7), can track the trajectory y_r asymptotically, and the augmented target model can be introduced

$$\begin{bmatrix} \dot{\hat{\eta}} \\ \dot{\hat{x}} \end{bmatrix} = \sum_{i=1}^{N} \pi_i(z) \left\{ \begin{bmatrix} 0 & -C_i \\ 0 & A_i \end{bmatrix} \begin{bmatrix} \hat{\eta} \\ \hat{x} \end{bmatrix} + \begin{bmatrix} 0 \\ B_i \end{bmatrix} \hat{\rho} r + \begin{bmatrix} 0 \\ B_i \end{bmatrix} u_3 + \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_r \\ w \end{bmatrix} \right\} \tag{18}$$

which can be expressed as the following form,

$$\dot{\hat{\bar{x}}} = \sum_{i=1}^{N} \pi_i(z) \left[\mathbb{A}_i \hat{\bar{x}} + \mathbb{B}_i \left(\hat{\rho} r + u_3 \right) \right] + \mathbb{D}_1 d \tag{19}$$

If one defines the state error vector of the augmented system as $e = \bar{x} - \hat{\bar{x}}$ and assumes a fault tolerant control law

$$\delta^F = r + \sum_{j=1}^{N} \mathbb{F}_j e \tag{20}$$

From (10) and (19), the following equality can be obtained,

$$\dot{e} = \sum_{i=1}^{N} \sum_{j=1}^{N} \pi_i(z) \pi_j(z) \left[\left(\mathbb{A}_i + \mathbb{B}_i \rho \mathbb{F}_j \right) e + \mathbb{B}_i \tilde{\rho} r + \mathbb{B}_i \left(\Delta \delta^F - u_3 \right) + \tilde{\mathbb{B}}_d d \right] \tag{21}$$

where $\tilde{\rho}_s = \rho_s - \hat{\rho}_s$ $(s = 1, 2, \ldots, m)$, $\tilde{\mathbb{B}}_d = \mathbb{B}_d - \mathbb{D}_1$, $\mathbb{F}_j$ $(j = 1, \ldots, m)$ is a difference control gain, which is designed to stabilize a T-S fuzzy system (21).

Let $r = [r_1, \ldots, r_m]^T$, $\mathbb{B}_i = [b_{i1}, \ldots, b_{im}]$, and the augmented system (21) is described by

$$\dot{e} = \sum_{i=1}^{N} \sum_{j=1}^{N} \pi_i(z) \mu_j(z) \left[\left(\mathbb{A}_i + \mathbb{B}_i \rho \mathbb{F}_j \right) e + \sum_{s=1}^{m} b_{is} \tilde{\rho}_s r_s + \mathbb{B}_i \left(\Delta \delta^F - u_3 \right) + \tilde{\mathbb{B}}_d d \right] \tag{22}$$

The control term u_3 is a compensation controller to accommodate the effect of actuator saturation, which is given by

$$u_3 = -\frac{\mathbb{B}_i^T G e}{||e^T G \mathbb{B}_i||} h(x) \hat{\xi} \tag{23}$$

where $\hat{\xi}$ is the estimate of the unknown constant vector ξ.

Theorem 3.1. *The augmented T-S fuzzy system (22) with γ-disturbance attenuation is asymptotically stable, as long as there exist real matrices $\mathbb{W} \in R^{m \times (n+k)}$, $\mathbb{W}_j > 0$, $j = 1, \ldots, N$, symmetric positive matrices $\mathbb{Q} \in R^{(n+k) \times (n+k)}$ such that the conditions hold as follows*

$$H_{ii} < 0, \quad \frac{2}{N-1} H_{ii} + H_{ij} + H_{ji} < 0, \quad 1 \leq i \neq j \leq N \tag{24}$$

with

$$H_{ij} = \begin{bmatrix} \mathbb{A}_i \mathbb{Q} + \mathbb{B}_i \rho \mathbb{W}_j + \left(\mathbb{A}_i \mathbb{Q} + \mathbb{B}_i \rho \mathbb{W}_j \right)^T & \mathbb{B}_d & \mathbb{Q} \\ \mathbb{B}_d^T & -\gamma^2 I & 0 \\ \mathbb{Q} & 0 & -I \end{bmatrix}$$

and $\hat{\rho}_s$, $(s = 1, \ldots, m)$ are determined on the basis of an adaptive estimation algorithm as follows:

$$\begin{aligned} \dot{\hat{\rho}}_s &= Proj_{[\underline{\rho_s}, \bar{\rho}_s]} \left\{ l_s e^T G b_{is} r_s \right\} \\ &= \begin{cases} 0, & \text{if } \hat{\rho}_s = \bar{\rho}_s, l_s e^T G b_{is} r_s \geq 0 \\ & \text{or } \hat{\rho}_s = \underline{\rho_s}, l_s e^T G b_{is} r_s \leq 0 \\ \sum_{i=1}^{N} \pi_i(z) l_s e^T G b_{is} r_s, & \text{otherwise} \end{cases} \end{aligned} \tag{25}$$

where $l_s > 0$ denotes a learning parameter to be designed by the minimum bound of fault and maximum one $\left(\bar{\rho}_s, \underline{\rho_s}\right)$, the projection operator is expressed by Proj$\{\cdot\}$, which is to project an estimation $\hat{\rho}_s$ in the range $\left[\bar{\rho}_s, \underline{\rho_s}\right]$, and an error control gain $\mathbb{F}_j$ is deduced by $\mathbb{F}_j = \mathbb{W}_j\mathbb{Q}^{-1}$.

Proof: Select the following Lyapunov function,

$$V = V_1 + V_2,\ V_1 = e^TGe + \sum_{s=1}^{m}\frac{\tilde{\rho}_s^2}{l_s}, \quad V_2(t) = \sum_{s=1}^{m}\frac{\tilde{\xi}_s^T\tilde{\xi}_s}{\tau_s} \tag{26}$$

where $G = \mathbb{Q}^{-1} > 0$, $\tilde{\xi}_s = \xi_s - \hat{\xi}_s$ $(s = 1, \ldots, m)$. $\tau_s > 0$ is an adaptive gain.

Taking the derivative of V_1 along the trajectory of the augmented system (22), it can be deduced as

$$\begin{aligned}\dot{V}_1 = & \sum_{i=1}^{N}\sum_{j=1}^{N}\pi_i(z)\pi_j(z)e^T\left[G(\mathbb{A}_i + \mathbb{B}_i\rho\mathbb{F}_j) + (\mathbb{A}_i + \mathbb{B}_i\rho\mathbb{F}_j)^T G\right]e \\ & + e^TG\mathbb{B}_d d + d^T\mathbb{B}_d^TGe + 2\sum_{i=1}^{N}\pi_i(z)e^TG\mathbb{B}_i(\Delta\delta^F - u_3) \\ & + 2\sum_{s=1}^{m}\sum_{i=1}^{N}\pi_i(z)\tilde{\rho}_s e^TGb_{is}r_s + 2\sum_{s=1}^{m}\frac{\tilde{\rho}_s\dot{\tilde{\rho}}_s}{l_s}\end{aligned} \tag{27}$$

Considering that ρ_s is an unknown scalar, one can easily obtain that $\dot{\hat{\rho}}_s = -\dot{\tilde{\rho}}_s$. Based on (25), it is easily known that

$$\frac{\tilde{\rho}_s\dot{\hat{\rho}}_s}{l_s} = \sum_{i=1}^{N}\pi_i(z)\tilde{\rho}_s e^TGb_{is}r_s \tag{28}$$

Meanwhile, choose the adaptive parameter update law

$$\dot{\hat{\xi}}_s = -\sum_{i=1}^{N}\pi_i(z)\tau_s||e^TG\mathbb{B}_i||h_s^T(x) \tag{29}$$

Substituting (28) and (29) into (27), the following inequality is obtained,

$$\dot{V} \leq \sum_{i=1}^{N}\sum_{j=1}^{N}\pi_i(z)\pi_j(z)e^T\left[G\left(\mathbb{A}_i + \mathbb{B}_i\rho\mathbb{F}_j\right)\left(\mathbb{A}_i + \mathbb{B}_i\rho\mathbb{F}_j\right)^T G\right]e + e^TG\mathbb{B}_d d + d^T\mathbb{B}_d^TGe \tag{30}$$

Here, the H-infinity tracking performance index γ is considered, and we have

$$\dot{V}(t) + e^Te - \gamma^2 d^Td = \sum_{i=1}^{N}\sum_{j=1}^{N}\pi_i(z)\pi_j(z)$$

$$\begin{bmatrix} e \\ d \end{bmatrix}^T \begin{bmatrix} G(\mathbb{A}_i + \mathbb{B}_i\rho\mathbb{F}_j) + (\mathbb{A}_i + \mathbb{B}_i\rho\mathbb{F}_j)^TG + I & G\tilde{B}_d \\ \tilde{B}_d^TG & -\gamma^2 I \end{bmatrix}\begin{bmatrix} e \\ d \end{bmatrix} \tag{31}$$

If $\mathbb{F}_j = \mathbb{W}_j\mathbb{Q}^{-1}, \mathbb{Q} = G^{-1}$, (31) < 0 is equivalent to the following

$$\begin{bmatrix} (\mathbb{A}_i + \mathbb{B}_i\rho\mathbb{F}_j)\mathbb{Q} + \mathbb{Q}^T(\mathbb{A}_i + \mathbb{B}_i\rho\mathbb{F}_j)^T & \tilde{B}_d & \mathbb{Q} \\ \tilde{B}_d^T & -\gamma^2 I & 0 \\ \mathbb{Q} & 0 & -I \end{bmatrix} < 0 \tag{32}$$

From (24) in Theorem 3.1, it is known that (32) < 0 is right, and the proof is completed.

For Lyapunov stability theory, when actuator faults in (7) and actuator saturation of (2) occur, it is obvious from (31) that the augmented T-S fuzzy system (22) with disturbance attenuation level γ is asymptotically stable using the adaptive estimation law (25). Then, a new adaptive FTC algorithm could be obtained

$$\delta^F = r + \sum_{j=1}^{N} \pi_j(z)\mathbb{F}_j e = -\frac{\mathbb{B}_i^T P\bar{x}}{||\bar{x}^T P\mathbb{B}_i||}h(x)\hat{\sigma} + \sum_{j=1}^{N} \pi_j(z)\left[\hat{\rho}^{-1}\mathbb{K}_j\hat{x} + \mathbb{F}_j e\right] = \delta + \delta^C$$

where $r = -\frac{\mathbb{B}_i^T P\bar{x}}{||\bar{x}^T P\mathbb{B}_i||}h(x)\hat{\sigma} + \sum_{j=1}^{N}\pi_j(z)\hat{\rho}^{-1}\mathbb{K}_j\hat{x}$, $\delta = -\frac{\mathbb{B}_i^T P\bar{x}}{||\bar{x}^T P\mathbb{B}_i||}h(x)\hat{\sigma} + \sum_{j=1}^{N}\pi_j(z)\mathbb{K}_j\bar{x}$ and $\delta^C = \sum_{j=1}^{N}\pi_j(z)\left[(\mathbb{F}_j - \mathbb{K}_j)e + \hat{\rho}^{-1}(I-\hat{\rho})\mathbb{K}_j\hat{\bar{x}}\right]$.

4. **Numerical Example.** In this section, the efficiency of the presented controller is demonstrated. The nonlinear flight control systems for UAV refer to [1], where $x = [\phi, \theta, V, \alpha, \beta, p, q, r, h]^T$ is state vector, which includes roll angle, pitch angle, airspeed, attack angle, sideslip angle, roll rate, pitch rate, yaw rate and altitude, respectively. $\delta = [\delta_x, \delta_{ar}, \delta_{al}, \delta_{fr}, \delta_{fl}, \delta_{er}, \delta_{el}]^T$ is input vector, which is the throttle, the right aileron and left one, similarly, followed by flaps, the elevators. $y = [\phi, \theta, \beta]^T$ is defined as the output vector, which denotes roll angle, pitch angle, sideslip angle. $\rho = \text{diag}[\rho_1, \ldots, \rho_7]$ is the unknown scalar modeling the remaining control effectiveness of seven actuator channels. Let the throttle and left flap actuator channels show faults simultaneously, and $t_f = 5$s is the occurrence time of faults.

The unknown disturbance $w(t) = [0.01\cos t, 0, 0, -0.02\sin t, 0, 0.015\cos t]^T$ and required output command $y_r = [1, 1, 1]^T$. Consider that the nonlinearity of UAV flight control systems mainly comes from airspeed V and altitude h. In the airspeed range $V \in [15, 50]$, we assume that V has two related fuzzy sets $\{V = 15\}$ and $\{V = 50\}$, and h has three related fuzzy sets $\{h = 200\}$, $\{h = 1500\}$ and $\{h = 3000\}$. The similar corresponding membership functions are obtained as [9]. We choose six operating points: $[V, h] \in \{[15, 200], [15, 1500], [15, 3000], [50, 200], [50, 1500], [50, 3000]\}$. Under the membership functions and the six operating points, six plant rules and six control rules can be defined. All A_i and B_i can be obtained by substituting the six operating points to $f(x)$, $g(x)$.

In this study, both the normal robust controller without FTC and the adaptive fault accommodation approach developed in this paper are carried out, which are denoted by the dashed line and solid one respectively. When we do not consider actuator saturation, namely, $\rho_1 = 70\%$, $\rho_5 = 60\%$, $\triangle\delta = 0$. In Figure 1, the dashed line of normal robust controller does not track the given command. However, the solid line of FTC can asymptotically converge to 1. Moreover, the control input responses of the controller with FTC are smaller than ones of the normal controller without FTC, namely, the less energy is required by the tracking controller with FTC. To verify the adaptive capability of the proposed approach in actuator saturation case, the control surface position limits, namely actuator saturation levels, are defined as $\delta_{1\min} = 0$ (deg), $\delta_{i\min} = -5$ (deg) $(i = 2, \ldots, 7)$, and $\delta_{i\max} = 5$ (deg) $(i = 1, \ldots, 7)$. According to Figure 2, it is known that the satisfactory tracking performance of FCS with the adaptive FTC, which compensates the actuator faults and saturation, is obtained. At the same time, it is easily seen that the outputs of FCS using the normal controller without FTC are unstable.

5. **Conclusions.** This paper addresses an FTC approach for flight control systems of a UAV with actuator saturation and external disturbance. The T-S fuzzy models are employed for representing FCS of a UAV. Considering actuator saturation constraints, unknown LOE faults and unknown disturbance, a novel FTC strategy is developed by adaptive theory. On the basis of Lyapunov technique, the stability of the T-S fuzzy FCS is proved. Finally, the simulation illustrates the efficiency of the presented control scheme. Another type of actuator fault will be considered in our research work in the future.

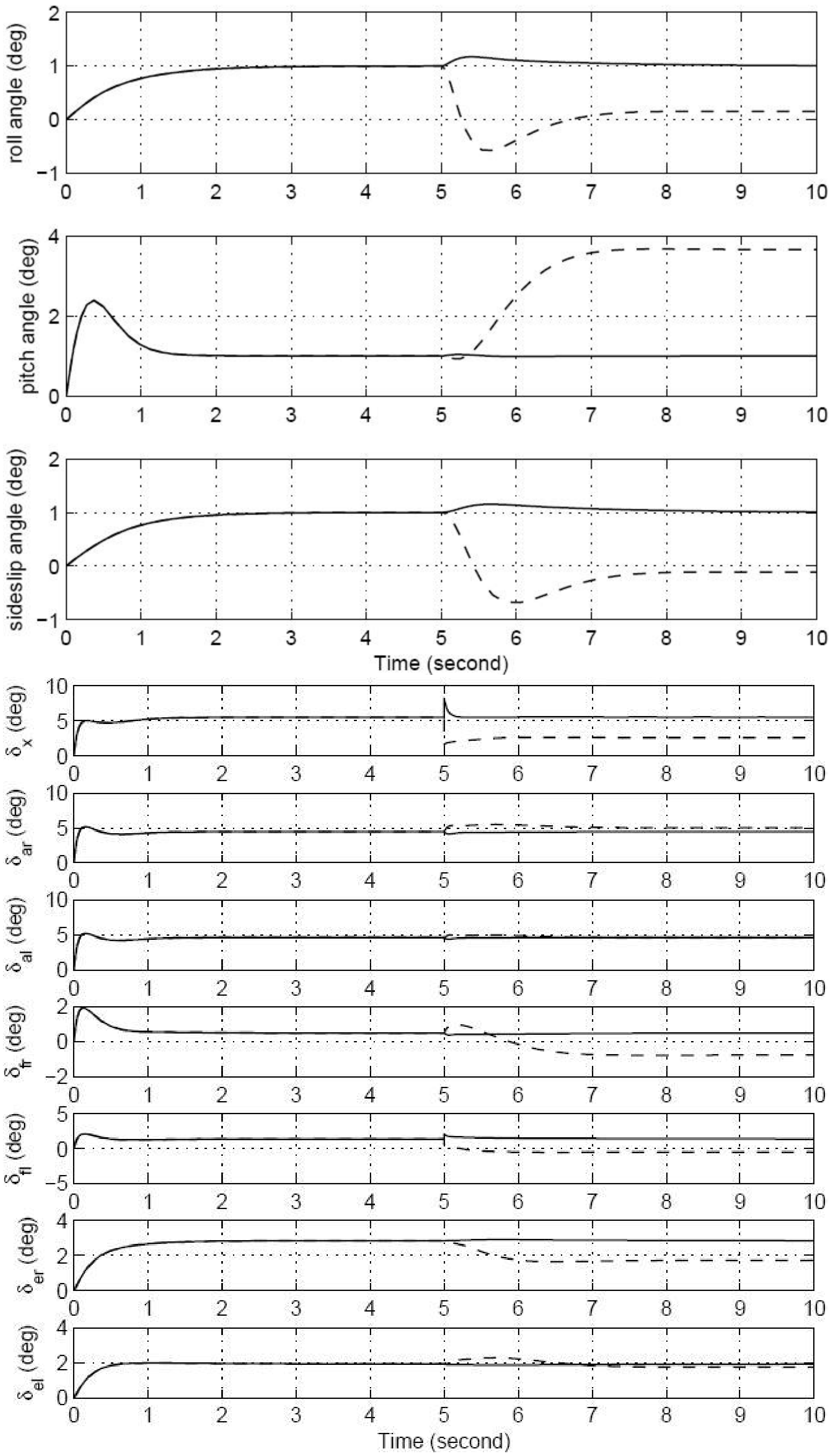

FIGURE 1. Attitude angles and control input curves in actuator faulty case (dashed line: without FTC; solid line: FTC)

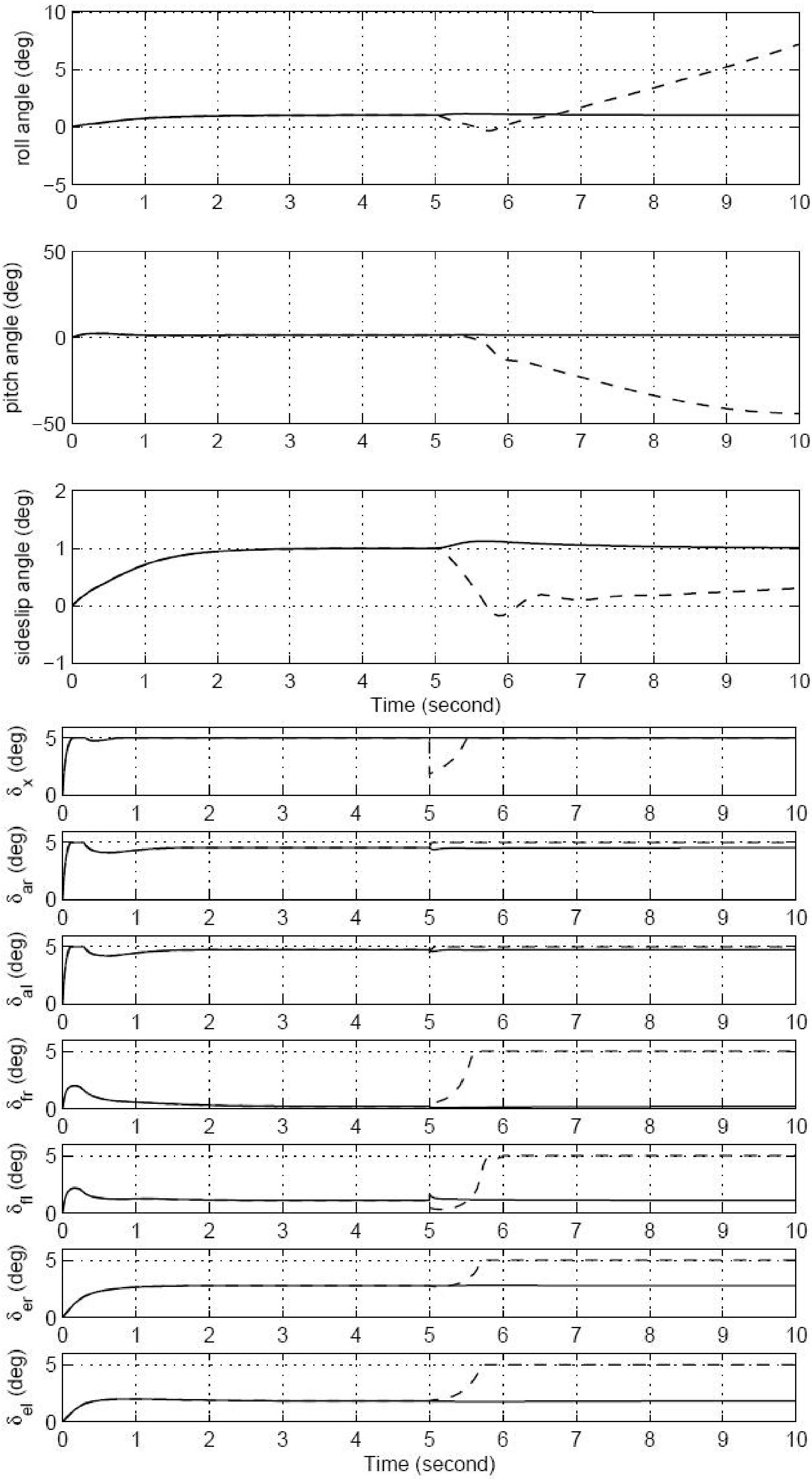

Figure 2. Attitude angles and input curves in actuator faulty and saturated case (dashed line: without FTC; solid line: FTC)

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