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PARAMETER ESTIMATION FOR PERMANENT MAGNET DC MACHINE BY LEAST SQUARE METHOD

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Abstract. *Parameter recognition for the DC machine is playing an importance role in the industrial field. In this paper, the data of time-varying voltage, current and speed under the DC machine in operation are used to calculate the parameters of equivalent circuit and mechanical system of the DC machine. The sampling signals are processed by the polynomial fitting, which can filter noise and retain the curve characteristics. The relationship of voltage, current and speed of the DC machine can be expressed as an equation. In this paper, the equivalent circuit parameters of the DC machine are estimated by the least square method, so that the error between the estimated value and the experimental data is the smallest. From the set of parameters, output torque in the dynamic model can be established. By the curves of output torque and speed, the inertia and friction coefficient of the mechanical system can be found. In this paper, the polynomial regression is used to estimate the above parameters, which has the advantages of high computational efficiency and simple procedure.*

Keywords: Parameter recognition, DC machine, Least square method

1. **Introduction.** In the precision motion system, the system parameters have great influence on the control performance. The system parameters must be accurately identified, and then the controller for improving the performance of the motion system can be effectively designed.

For the DC motor to effectively improve the performance; clearly, controlling the parameters of the DC machine plays an important role. The design and analysis of the DC machine are based on the equivalent model of the equipment. Generally, DC machine parameters include armature resistance, armature inductance, back-emf constant, moment of inertia and friction coefficient. If a user needs to know the related messages, they must evaluate the DC motor by a series of tests.

The IEEE standard test is currently a common method for DC parameter testing [1]. This test includes measured output method, acceleration method, input method, and direct measurement method [2]. The measured output method takes DC generator as the DC motor's load; the output power plus the loss of the generator is the output power of the motor [3,4]. The acceleration method starts a DC motor from the quiescent state to a steady state under load-free case, which can find the rotor inertia. The input method subtracts losses from the motor input, which can find the motor torque [5,6]. The direct measurement method couples DC motor with a torque meter, which can measure output torque directly. Johnson and Lorenz use the feedforward control theory to calculate the motor inertia and the friction coefficient [7]. MEA motor test company installs a flywheel to DC motor to estimate the inertia of the motor [8]. The sub-identification method was proposed to find motor electrical parameters and mechanical parameters [9].

The sampling signal contains many noises for the commutation of DC machine, which will seriously interfere with the results of the calculation. If the parameters are estimated from a single operating point, it is very easy to produce errors. In this paper, the least square method is used to overcome the above shortcomings, so that the results can be as realistic as possible [10]. It minimizes the sum of the squares of the vertical distances of all data points to the curve, and finds the optimum value by a single calculation, which eliminates the need for a large number of iterative steps.

In this paper, the time-varying voltage, current and speed under DC machine starting are used to calculate parameters of equivalent circuit and mechanical system. Sampling signals are regressed with polynomials, as each sampling signal is expressed as a set of polynomial equations. The relation amount voltage, current and speed can be expressed as an equation. In this paper, the equivalent circuit parameters of the DC machine are estimated by the least square method. From the above parameters, the output torque in the dynamic model can be established. By the output torque and speed, the inertia and friction coefficient of the mechanical system can also be found.

2. **Theory.**

2.1. **Transient model of the DC machine.**

The model of DC machines can be divided into two categories, transient model and steady-state model. In the transient model, the voltage, current and speed will change under the transient period for the energy storage components. These energy storage components include armature inductance of the equivalent circuit and inertia of the mechanical system. Therefore, the transient model of the DC machine is shown in Figure 1.

In the figure,

v: Terminal voltage (V)
i: Armature current (A)
R_a: Armature resistance (Ω)
L_a: Armature inductance (H)
E: Back-emf (V)
K_E: Back-emf constant (V/rps)
ω: Speed (rps)
K_T: Torque constant (Kg·m/A)
T: Torque (Kg·m)
J: Inertia (g·m^2)
B: Friction coefficient (g·m/rps)

The parameters to be estimated in this paper include the armature resistance, armature inductance, back-emf constant in the equivalent circuit, the moment of inertia, and the coefficient of friction in the mechanical system.

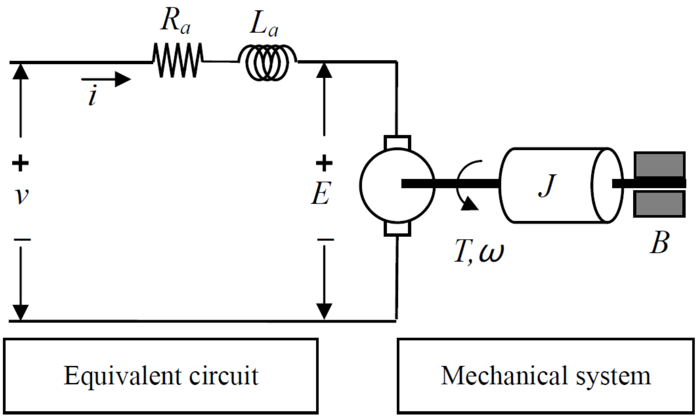

Figure 1. Transient model of the DC machine

From Figure 1, the relationship amount parameters can be established as Equation (1).

$$v = R_a i + L_a \frac{di}{dt} + E \tag{1}$$

Since the back-emf constant is proportional to the speed, Equation (1) can be rewritten as

$$v = R_a i + L_a \frac{di}{dt} + K_E \omega \tag{2}$$

The relationship of the mechanical system is shown in Equation (3)

$$T = J \frac{d\omega}{dt} + B\omega \tag{3}$$

2.2. **Noise filter.** For the commutation of DC machine, the voltage, current, and speed signal are interference with noise [11]. This interference will make the analysis results extremely unstable. Figure 2 is a typical example; in addition to voltage, current, and speed curves, the signals also contain very high noises. This method will filter out these noises first, and retain the curve characteristics. The work of noise filtering can be done by a polynomial regression method.

The estimated values can be expressed as a polynomial

$$y_n = a_m n^m + a_{m-1} n^{m-1} + \cdots + a_1 n + a_0, \ n = 1, \ldots, N \tag{4}$$

where y_n represents the terminal voltage v_n, armature current i_n, or speed ω_n of the sampled signal.

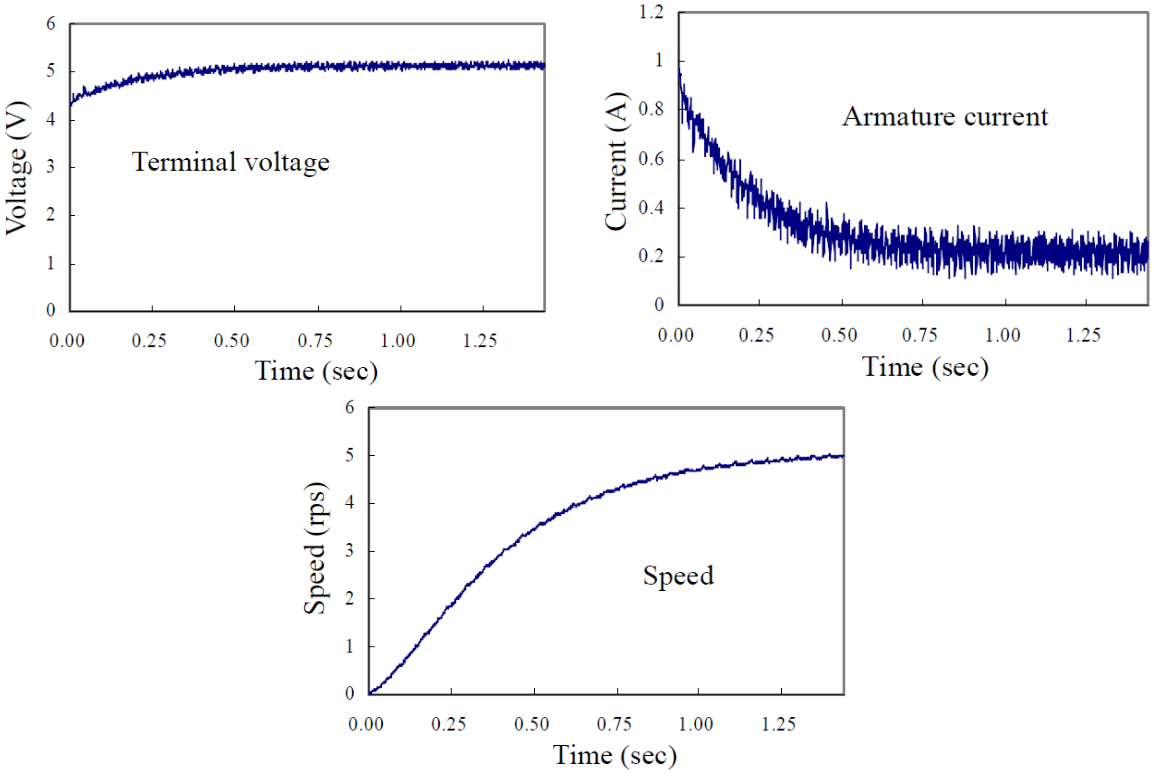

Figure 2. Sampled signals

To obtain the minimum error of the estimated value and the sampled data, the polynomial coefficient can be found as [12].

$$\begin{bmatrix} a_m \\ a_{m-1} \\ \vdots \\ a_0 \end{bmatrix} = \begin{bmatrix} \sum_{n=1}^{N} n^{2m} & \sum_{n=1}^{N} n^{2m-1} & \cdots & \sum_{n=1}^{N} n^{m} \\ \sum_{n=1}^{N} n^{2m-1} & \sum_{n=1}^{N} n^{2m-2} & \cdots & \sum_{n=1}^{N} n^{m-1} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{n=1}^{N} n^{m} & \sum_{n=1}^{N} n^{m-1} & \cdots & \sum_{n=1}^{N} 1 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{n=1}^{N} y_n n^{m} \\ \sum_{n=1}^{N} y_n n^{m-1} \\ \vdots \\ \sum_{n=1}^{N} y_n \end{bmatrix} \tag{5}$$

Polynomial regression can indeed preserve the transient characteristics of the signal and completely filter out the interference of the noise, and then the calculation of the parameters can be accurately obtained.

2.3. **Equivalent circuit parameters.** In this paper, the equivalent circuit parameters of the DC machine are estimated by the least square method. The error between the estimated value and the experimental data is the smallest, so that the results can be close to the actual values. A series of data can be obtained from the sample

$$\left(v_n, i_n, \frac{di_n}{dt}, \omega_n\right), \ n = 1, \ldots, N \tag{6}$$

where the voltage, current, and speed are obtained from (4), which are the filtered data. di/dt is the change rate of current, and its numerical form can be expressed as

$$\frac{di_n}{dt} = (i_n - i_{n-1}) * f_s \tag{7}$$

where f_s (Hz) is the sampling rate.

According to the equivalent circuit model, the relationship between these four data can be expressed as (1). R_a, L_a, and K_E are the parameters that this paper intends to find. The equivalent circuit parameters can be obtained

$$\begin{bmatrix} R_a \\ L_a \\ K_E \end{bmatrix} = \begin{bmatrix} \sum_{n=1}^{N} i_n i_n & \sum_{n=1}^{N} \frac{di_n}{dt} i_n & \sum_{n=1}^{N} \omega_n i_n \\ \sum_{n=1}^{N} i_n \frac{di_n}{dt} & \sum_{n=1}^{N} \frac{di_n}{dt}\frac{di_n}{dt} & \sum_{n=1}^{N} \omega_n \frac{di_n}{dt} \\ \sum_{n=1}^{N} i_n \omega_n & \sum_{n=1}^{N} \frac{di_n}{dt} \omega_n & \sum_{n=1}^{N} \omega_n \omega_n \end{bmatrix}^{-1} \begin{bmatrix} \sum_{n=1}^{N} v_n i_n \\ \sum_{n=1}^{N} v_n \frac{di_n}{dt} \\ \sum_{n=1}^{N} v_n \omega_n \end{bmatrix} \tag{8}$$

2.4. **Mechanical system parameters.** The parameters of the equivalent circuit can be obtained by the aforementioned method. The output torque can be further obtained

$$T = K_T i \tag{9}$$

According to the theory, the torque constant K_T will be equal to the back-emf constant K_E. The angular acceleration can also be obtained according to the speed. Its numerical form can be expressed as

$$\frac{d\omega_n}{dt} = (\omega_n - \omega_{n-1}) * f_s \tag{10}$$

So a series of data can be obtained

$$\left(\omega_n, \frac{d\omega_n}{dt}, T_n\right), \ n = 1, \ldots, N \tag{11}$$

Consider the system is linear, the inertia and friction coefficient are constant. According to the mechanical system model, the relationship among these three data can be expressed as (3). The inertia J and the friction coefficient B are the parameters that this paper intends to find.

The parameters can be obtained

$$\begin{bmatrix} J \\ B \end{bmatrix} = \begin{bmatrix} \sum_{n=1}^{N} \left(\frac{d\omega_n}{dt}\right)^2 & \sum_{n=1}^{N} \omega_n \frac{d\omega_n}{dt} \\ \sum_{n=1}^{N} \omega_n \frac{d\omega_n}{dt} & \sum_{n=1}^{N} (\omega_n)^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{n=1}^{N} T_n \frac{d\omega_n}{dt} \\ \sum_{n=1}^{N} T_n \omega_n \end{bmatrix} \tag{12}$$

2.5. **Analysis procedure.** This section concludes the above theory as a complete procedure.

Step 1: Signal capture. Obtain the instantaneous signal of voltage, current and speed when the permanent magnet DC machine is started.

Step 2: Filter out the noise. According to (4) and (5), filter out the voltage, current, and speed of noise.

Step 3: Equivalent circuit parameter. Using (8) can find the equivalent circuit parameters.

Step 4: Mechanical system parameters. Moment of inertia and friction coefficient can be obtained according to Equation (12).

Step 5: Calculation finished.

3. **Result and Discussion.** This paper uses the personal computer to complete the monitoring system with the A/D converter, the sensor, the DC motor, the control circuit, the servo driver and the man-machine interface. The system can perform tests on different operating conditions as required. This section analyzes a 150W DC machine as an example to illustrate the practical application of this method [13,14]. The first part describes the results of interference filtering, which verifies the capacity of noise handling. The second part discusses the estimation results for the equivalent circuit parameters. The third part describes the analysis results for the mechanical parameters. In order to evaluate the accuracy, this paper establishes three error indices.

Error of polynomial fitting to the sampling signal is

$$E_S = \sum_{n=1}^{N} (y_n - \hat{y}_n)^2 \Bigg/ \sum_{n=1}^{N} (y_n)^2 \tag{13}$$

where y_n is the original sampling data, and $\hat{y}_n$ is the polynomial value.

Error between estimated terminal voltage and the sampling voltage is

$$E_E = \sum_{n=1}^{N} (v_n - \hat{v}_n)^2 \Bigg/ \sum_{n=1}^{N} (v_n)^2 \tag{14}$$

where v_n is the sampling signal, and $\hat{v}_n$ is the estimation signal.

Error between estimated speed and the sampling speed is

$$E_T = \sum_{n=1}^{N} (\omega_n - \hat{\omega}_n)^2 \Bigg/ \sum_{n=1}^{N} (\omega_n)^2 \tag{15}$$

where ω_n is the sampling speed, and $\hat{\omega}_n$ is the estimation speed.

3.1. **Interference filter results.** The interferences on sampling signals by noises are shown in Figure 2. In this paper, polynomial regression is used to filter out the noise in the signal. The order of the polynomials needs to be determined when polynomials fit a sampling signal. In this case, the polynomials and their resulting errors are shown in Table 1, which illustrates that the polynomials can obtain satisfactory results. Noises in voltage, current, and speed can be completely filtered and the signal characteristics can be retained. The fitting results are fairly stable and close to the actual values, which confirms the feasibility of this step.

Table 1. Results of polynomial fitting

Order	Signal	Polynomial									E_S
		n^8	n^7	n^6	n^5	n^4	n^3	n^2	n^1	n^0	$(*10^{-6})$
5	Voltage				0.75	−3.87	7.89	−8.20	4.46	4.08	77.3
	Current				−0.71	3.59	−7.28	7.53	−4.08	1.17	15,000
	Speed				−7.75	33.13	−50.37	28.86	0.83	−0.12	117.0
6	Voltage			−0.59	3.44	−8.48	11.62	−9.62	4.68	4.07	77.8
	Current			0.39	−2.48	6.63	−9.74	8.46	−4.22	1.18	15,000
	Speed			−0.59	3.44	−8.48	11.62	−9.62	4.68	4.07	39.2
7	Voltage		−0.73	3.29	−4.70	0.09	6.89	−8.33	4.53	4.07	77.8
	Current		0.83	−4.03	6.80	−3.14	−4.34	6.98	−4.06	1.17	15,000
	Speed		−9.77	62.21	−163.71	228.88	−179.20	71.01	−4.89	0.07	28.9
8	Voltage	3.63	−22.76	57.83	−75.57	51.75	−14.00	−4.00	4.15	4.08	77.8
	Current	−3.35	21.17	−54.38	72.23	−50.83	14.94	2.99	−3.71	1.17	15,000
	Speed	4.88	−39.36	135.46	−258.88	298.27	−207.27	76.82	−5.39	0.09	29.3

3.2. **Analysis results for equivalent circuit parameters.** In this section, the different order polynomials are used to analyze the equivalent circuit parameters, as shown in Table 2. The 6-order polynomial gets K_E as a negative value, which does not conform to the actual characteristics. However, 5, 7, and 8-order polynomials can find reasonable values, and reduce errors to a very low level. Figure 3 shows a comparison of estimated values by 5-order polynomial with the actual ones. It can be found that the estimated values from obtained parameters are very close to the actual characteristic curve; these are quite satisfactory results.

Table 2. Analyzed results of equivalent circuit parameters

Order Parameter	5	6	7	8
R_a	21.99	25.45	23.99	20.84
L_a	4.81	5.88	5.4	4.4
K_E	0.124	−0.039	0.02	0.17
$E_E(*10^{-3})$	1.58	0.34	0.97	1.25

3.3. **Analysis results for mechanical system parameters.** From the results of the previous section, the torque can be further obtained. According to (12), inertia J and friction coefficient B can be found as Table 3. The result can be a very low error. Figure 4 shows the comparison of the simulated values by 5-order polynomial with the actual ones. It can be found that the estimated values from obtained parameters fit well with the actual characteristic curve of the DC machine. This proves that the method can get accurate DC machine parameters.

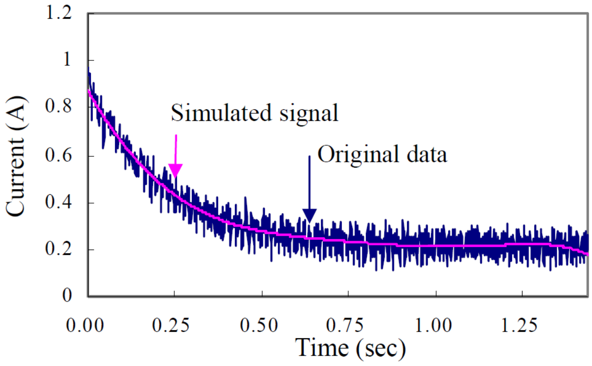

Figure 3. Comparison of currents

Table 3. Analyzed results of mechanical system parameters

Order / Parameter	5	7	8
J	7.4	1.2	10.3
B	3.4	0.5	4.3
$E_T(*10^{-3})$	5.68	5.37	5.32

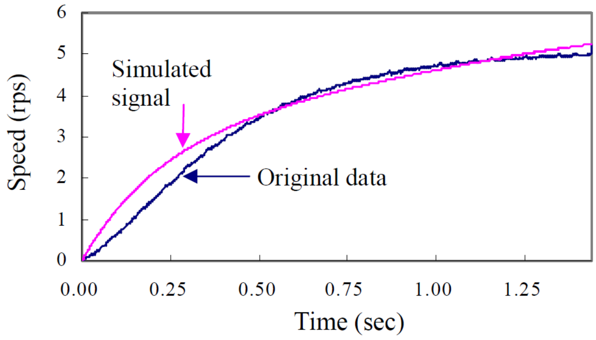

Figure 4. Comparison of speeds

4. **Conclusions.** In this paper, the data of time-varying voltage, current and speed under the DC machine initiation are used to calculate the parameters of equivalent circuit and mechanical system of the DC machine. The sampling signals are processed by the polynomial fitting, which can filter noise and retain the curve characteristics. The relationship of voltage, current and speed of the DC machine can be expressed as an equation. In this paper, the equivalent circuit parameters are estimated by the least square method, so that the error between the estimated value and the experimental data is the smallest. By the curves of output torque and speed, the inertia and friction coefficient of the mechanical system can be found. The results show that the simulated results are very close to the measured signals. This confirms the accuracy and reliability of the method.

The behavior of the DC machine is often expressed as a differential equation, and the solution of the differential equation forms an exponential form. When fitting an exponential curve with polynomial, the results will be different from the different order selection. These differences will affect results of the subsequent parameter estimation. It is worthy of follow-up study to overcome the difference.

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