ICIC Express Letters
Volume **11**, Number **11**, November **2017**

ICIC International ©2017 ISSN 1881-803X
pp. 1593–1598

THE CLASS OF STRONGLY STABILIZABLE PLANTS

Tatsuya Hoshikawa[1], Jinghui Li[1], Yuuko Tatsumi[1]
Takaaki Suzuki[2] and Kou Yamada[2]

[1]Graduate School of Science and Technology
[2]Division of Mechanical Science and Technology
Gunma University
1-5-1 Tenjincho, Kiryu 376-8515, Japan
{ t10801250; t161b070; t08302053; suzuki.taka; yamada }@gunma-u.ac.jp

Received May 2017; accepted August 2017

Abstract. *When the control system is stabilized by a stable controller, the controller is said to be a strongly stabilizing controller. There exist many design methods of stabilizing controller, but most of the proposed design methods do not consider the stability of stabilizing controllers. Since the instability of stabilizing controller occurs to make the closed-loop system very sensitive to disturbances and reduce the tracking performance to reference inputs, it is required in practice to use the stable stabilizing controller whenever it is possible. Youla et al. showed that the plant is strongly stabilizable if and only if the plant satisfies the parity interlacing property condition and examined a design procedure of stable stabilizing controller. However, they do not clarify the class of strongly stabilizable plants. If the class of strongly stabilizable plants is clarified, we can obtain the parameterization of all stable stabilizing controllers. In addition, we have a possibility to clarify the characteristic of strongly stabilizable plants. From this viewpoint, it is desirable to clarify the class of strongly stabilizable plants. The purpose of this paper is to clarify the class of strongly stabilizable plants.*

Keywords: Strong stabilization, Strongly stabilizable plants, Closed-loop systems

1. **Introduction.** In this paper, we examine the class of plants which could be stabilized by a stable controller. That is, we clarify the class of strongly stabilizable plants, which is equivalently called the parameterization of all strongly stabilizable plants. The parameterization problem is to find all stabilizing controllers for plants [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] are sought. Hagiwara et al. clarify the parameterization of all plants stabilized by proportional controllers [11]. Since this parameterization can successfully search for all proper stabilizing controllers, it is used as a tool for many control problems.

For an unstable plant, the parameterization of all stabilizing controllers is solved by Youla et al. [1, 2]. The structure of a parameterization of all stabilizing controllers for unstable plants has full-order state feedback, including a full-order observer [3]. Glaria and Goodwin [4] gave a simple parameterization for single-input/single-output minimum-phase systems. However, two difficulties remained. One is that the parameterization of all stabilizing controllers generally includes improper controllers. In practice, the controller is required to be proper. The other one is that they do not give the parameterization of all internally stabilizing controllers. Yamada overcame these problems and proposed the parameterization of all proper internally stabilizing controllers for single-input/single-output minimum-phase systems [5].

For a stable plant, the parameterization of all stabilizing controllers has a structure identical to that of internal model control which has advantages as closed-loop stability is assured simply by choosing a stable internal model controller parameter and closed-loop performance characteristics are related directly to controller parameters. It makes on-line tuning of the internal model controller very convenient. Morari and Zafiriou examined the parameterization of all stabilizing internal model controllers for unstable plants [6]. However, their internal model is not necessarily proper. In addition, their parameterization includes improper internal model controllers. To overcome these problems, Chen et al. proposed the simple parameterization of all proper stabilizing internal model controllers for minimum-phase unstable plants [12]. Zhang et al. [13] proposed a new parameterization, which need not the coprime factorization. In this way, the parameterization of all stabilizing controllers is advanced.

Using unstable stabilizing controllers, unstable poles of stabilizing controller make the closed-loop transfer function have zeros in right half plane. It makes the closed-loop system very sensitive to disturbances and reduces the tracking performance to reference inputs [8, 9]. In addition, if the feedback-loop of feedback control system is broken down, that is, the feedback control system becomes feed-forward control system, the unstable pole of stabilizing controller becomes the unstable pole of the control system. Thus, the control system becomes unstable even if the plant is stable. From above reasons, it is desirable in practice that the control system is stabilized by stable stabilizing controller [9]. Therefore, several design methods of a stable stabilizing controller, which is referred as a strongly stabilizing controller, have been considered [8, 9, 10, 14, 15, 16, 17, 18].

Youla et al. showed that the plant is strongly stabilizable if and only if the plant satisfies the parity interlacing property condition and examined a design procedure of stable stabilizing controller [10]. Wakaiki et al. studied the sensitivity reduction problem with stable controllers for the linear time-invariant multi-input/multi-output distributed parameter system [17]. Wakaiki et al. considered the strong and robust stabilization problem that a class of plants have finitely many simple unstable zeros but possibly infinitely many unstable poles stabilized by a stable controller in the linear time-invariant single-input/single-output infinitely dimensional system [18]. However, they do not clarify the class of strongly stabilizable plants. If the class of strongly stabilizable plants is clarified, we have a possibility to obtain the parameterization of all stable stabilizing controllers. In addition, we have a possibility to clarify the characteristic of strongly stabilizable plants. From this viewpoint, it is desirable to clarify the class of strongly stabilizable plants.

In this paper, we clarify the class of strongly stabilizable plants, that is, the parameterization of all strongly stabilizable plants is clarified. This paper is organized as follows. In Section 2, we show the problem considered in this paper. In Section 3, we propose the class of all strongly stabilizable plants. In Section 4, we show a numerical example to illustrate that the plant included in the class clarified in Section 3 could be strongly stabilized. Section 5 gives concluding remarks.

Notations

R	The set of real numbers.
$R(s)$	The set of real rational functions with s.
RH_∞	The set of stable proper real rational functions.
$\mathcal{U}$	The set of unimodular functions on RH_∞. That is, $U(s) \in \mathcal{U}$ implies both $U(s) \in RH_\infty$ and $U^{-1}(s) \in RH_\infty$.

2. **Problem Formulation.** Consider the control system in

$$\begin{cases} y(s) = G(s)u(s) + d(s) \\ u(s) = C(s)\left(r(s) - y(s)\right) \end{cases}, \tag{1}$$

where $G(s) \in R(s)$ is the plant, $C(s) \in R(s)$ is the controller, $y(s) \in R(s)$ is the output, $u(s) \in R(s)$ is the control input, $d(s) \in R(s)$ is the disturbance and $r(s) \in R(s)$ is the reference input. The strong stabilization is the control method that makes the plant stable by using the stable stabilizing controller. Therefore, if the plant $G(s)$ in (1) can be stabilized by the stable controller $C(s)$, we call this plant $G(s)$ the strongly stabilizable plant.

The problem considered in this paper is to clarify the class of strongly stabilizable plants.

3. **The Class of Strongly Stabilizable Plants.** In this section, we clarify the class of strongly stabilizable plants $G(s)$, that is, the parameterization of all strongly stabilizable plants $G(s)$ is shown.

The class of strongly stabilizable plants is summarized in the following theorem.

Theorem 3.1. *$G(s)$ is assumed to be coprime. The plant $G(s)$ is strongly stabilizable if and only if $G(s)$ is written by the form in*

$$G(s) = \frac{Q_1(s)}{1 - Q_1(s)Q_2(s)}, \tag{2}$$

where $Q_1(s) \in RH_\infty$ and $Q_2(s) \in RH_\infty$ are any functions.

Proof: First, the necessity is shown. That is, we show that if the stable controller $C(s)$ makes $G(s)$ stable, then $G(s)$ takes the form in (2). From the assumption that $C(s)$ makes $G(s)$ in (1) stable, $1/(1 + C(s)G(s))$, $C(s)/(1 + C(s)G(s))$, $G(s)/(1 + C(s)G(s))$ and $C(s)G(s)/(1 + C(s)G(s))$ are all included in RH_∞. Therefore, using $Q_1(s) \in RH_\infty$, $G(s)/(1 + C(s)G(s)) \in RH_\infty$ can be rewritten as

$$\frac{G(s)}{1 + C(s)G(s)} = Q_1(s). \tag{3}$$

From simple manipulation, we have

$$G(s) = \frac{Q_1(s)}{1 - Q_1(s)C(s)}. \tag{4}$$

Since $C(s)$ is stable, using $Q_2(s) \in RH_\infty$, let $C(s)$ be

$$C(s) = Q_2(s), \tag{5}$$

(4) is rewritten as

$$G(s) = \frac{Q_1(s)}{1 - Q_1(s)Q_2(s)}. \tag{6}$$

Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, if $G(s)$ in (1) takes the form in (2), then the stable controller $C(s)$ makes $G(s)$ stable. When we set $C(s)$ as

$$C(s) = Q_2(s), \tag{7}$$

then $C(s) \in RH_\infty$ because of $Q_2(s) \in RH_\infty$. Then transfer functions $C(s)G(s)/(1 + C(s)G(s))$, $C(s)/(1+C(s)G(s))$, $G(s)/(1+C(s)G(s))$ and $1/(1+C(s)G(s))$ are rewritten as

$$\frac{C(s)G(s)}{1 + C(s)G(s)} = Q_1(s)Q_2(s), \tag{8}$$

$$\frac{C(s)}{1 + C(s)G(s)} = (1 - Q_1(s)Q_2(s))\, Q_2(s), \tag{9}$$

$$\frac{G(s)}{1 + C(s)G(s)} = Q_1(s), \tag{10}$$

and

$$\frac{1}{1 + C(s)G(s)} = 1 - Q_1(s)Q_2(s). \tag{11}$$

Since $Q_1(s) \in RH_\infty$ and $Q_2(s) \in RH_\infty$, (8), (9), (10) and (11) are stable. Thus, the sufficiency has been shown.

We have thus proved Theorem 3.1. □

Lemma 3.1. *When the plant $G(s)$ is written by the form in (2), one of stable controllers to stabilize the plant $G(s)$ is given by*

$$C(s) = Q_2(s). \tag{12}$$

Proof: It is obvious from the sufficiency of proof of Theorem 3.1. □

4. **Numerical Example.**

In this section, a numerical example is illustrated to show that the plant written by the form in (2) can be stabilized by using a stable controller.

Consider the problem to make the control system in (1) stable using stable controller, where the plant $G(s)$ is written as

$$G(s) = \frac{(s-7)(s+1)}{(s-1)(s+3)(s+5)}. \tag{13}$$

$G(s)$ in (13) is rewritten as

$$G(s) = \frac{\dfrac{s-7}{(s+2)(s+3)}}{1 + \dfrac{s-7}{(s+2)(s+3)}\dfrac{s+3}{s+1}}. \tag{14}$$

Since $G(s)$ in (13) written by the form in (2), $G(s)$ in (13) is strongly stabilizable, where

$$Q_1(s) = \frac{s-7}{(s+2)(s+3)}, \tag{15}$$

and

$$Q_2(s) = -\frac{s+3}{s+1}. \tag{16}$$

From Lemma 3.1, a stable controller $C(s)$ to make the control system in (1) stable is given by

$$C(s) = -\frac{s+3}{s+1}. \tag{17}$$

Using the stable stabilizing controller $C(s)$ in (17), the response of the output $y(t)$ of the control system in (1) for the step reference input $r(t) = 1$ is shown in Figure 1. Figure 1 shows that the control system in (1) is stabilized by using a stable controller $C(s)$ in (17).

In this way, we find that if the plant $G(s)$ is written by the form in (2), the plant is strongly stabilizable.

5. **Conclusions.**

In this paper, we clarified the class of strongly stabilizable plants. That is, we showed that if the plant $G(s)$ is written by the form in (2), the plant can be stabilized by stable controllers. In addition, we showed a numerical example to illustrate that the plant written by the form in (2) can be stabilized by using a stable stabilizing controller. Using the result in the present paper, we will clarify the parameterization of all stable stabilizing controllers in another article.

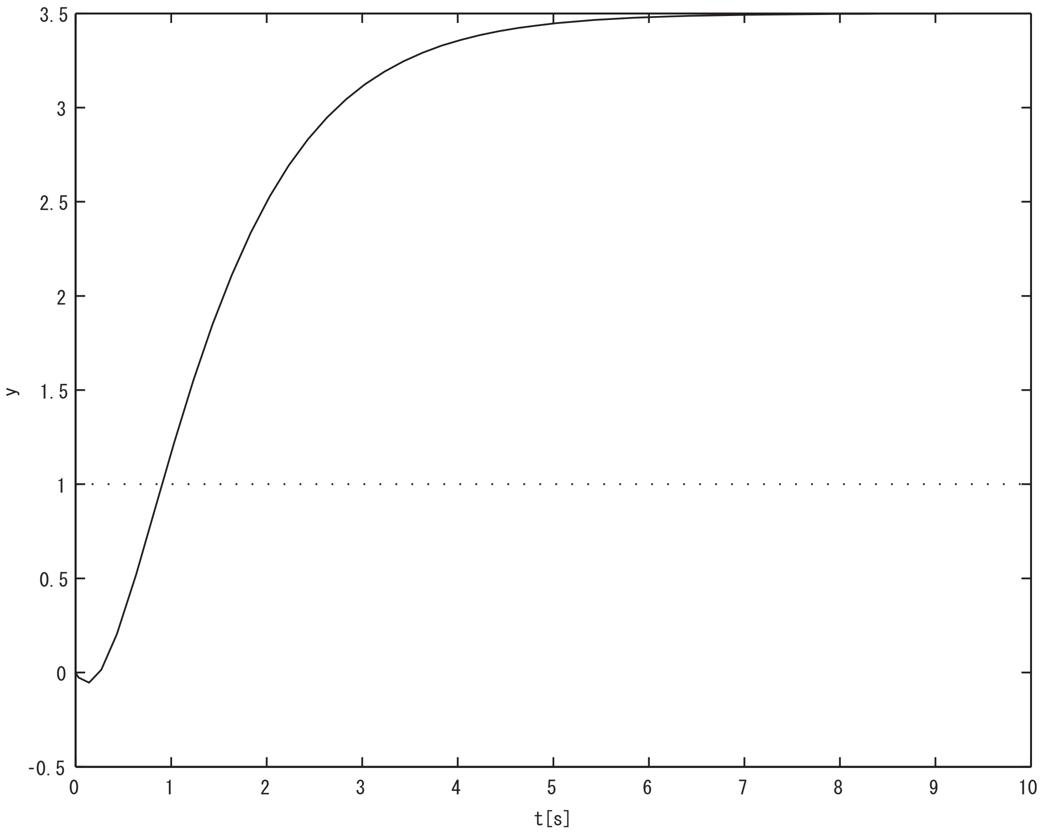

Figure 1. Response of the output $y(t)$ of the control system in (1) for the step reference input $r(t) = 1$

REFERENCES

[1] D. C. Youla, H. A. Jabr and J. J. Bongiorno, Modern Wiener-Hopf design of optimal controllers Part I: The single-input-output case, *IEEE Trans. Automatic Control*, vol.21, pp.3-13, 1976.
[2] C. A. Desoer, R. W. Liu, J. Murray and R. Saeks, Feedback system design: The fractional representation approach to analysis and synthesis, *IEEE Trans. Automatic Control*, vol.25, pp.399-412, 1980.
[3] K. Zhou, J. C. Doyle and K. Glover, *Robust and Optimal Control*, Prentice-Hall, NJ, 1996.
[4] J. J. Glaria and G. C. Goodwin, A parametrization for the class of all stabilizing controllers for linear minimum phase systems, *IEEE Trans. Automatic Control*, vol.39, pp.433-434, 1994.
[5] K. Yamada, A parameterization for the class of all proper stabilizing controllers for linear minimum phase systems, *IFAC Proceedings Volumes*, vol.34, no.8, pp.569-574, 2001.
[6] M. Morari and E. Zafiriou, *Robust Process Control*, Prentice-Hall, NJ, 1989.
[7] G. Zames, Feedback and optimal sensitivity: Model reference transformations, multiplicative seminorms and approximate inverses, *IEEE Trans. Automatic Control*, vol.26, pp.301-320, 1981.
[8] M. Vidyasagar, *Control System Synthesis: A Factorization Approach*, MIT Press, 1985.
[9] Y. S. Chou, T. Z. Wu and J. L. Leu, On strong stabilization and H_∞ strong stabilization problems, *The 42nd IEEE Conference on Decision and Control*, vol.5, pp.5155-5160, 2003.
[10] D. C. Youla, J. J. Bongiorno Jr. and C. N. Lu, Single-loop feedback-stabilization of linear multivariable dynamical plants, *Automatica*, vol.10, pp.159-173, 1974.
[11] T. Hagiwara, K. Yamada, T. Sakanushi, S. Aoyama and A. C. Hoang, The parameterization of all plants stabilized by proportional controller, *The 25th International Technical Conference on Circuit/Systems Computers and Communications CD-ROM*, pp.76-78, 2010.
[12] Z. X. Chen, K. Yamada, N. T. Mai, I. Murakami, Y. Ando, T. Hagiwara and T. Hoshikawa, A design method for internal model controllers for minimum-phase unstable plants, *ICIC Express Letters*, vol.4, no.6(A), pp.2045-2050, 2010.
[13] W. Zhang, F. Allgower and T. Liu, Controller parameterization for SISO and MIMO plants with time delay, *Systems and Control Letters*, vol.55, pp.794-802, 2006.

[14] P. Dorato, H. Park and Y. Li, An algorithm for interpolation with units in H_∞, with appplications to feedback stabilization, *Automatica*, vol.25, pp.427-430, 1989.
[15] H. Ito, H. Ohmori and A. Sano, Design of stable controllers attaining low H_∞ weighed sensitivity, *IEEE Trans. Automatic Control*, vol.38, pp.485-488, 1993.
[16] M. Zeren and H. Ozbay, On the strong stabilization and stable H_∞ controller design problems for MIMO systems, *Automatica*, vol.36, pp.1675-1684, 2000.
[17] M. Wakaiki, Y. Yamamoto and H. Ozbay, Sensitivity reduction by strongly stabilizing controllers for MIMO distributed parameter systems, *IEEE Trans. Automatic Control*, vol.57, no.8, pp.2089-2094, 2012.
[18] M. Wakaiki, Y. Yamamoto and H. Ozbay, Stable controllers for robust stabilization of systems with infinitely many unstable poles, *Systems and Control Letters*, vol.62, no.6, pp.511-516, 2013.