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HYBRID APPROACH FOR SINGLE-VENDOR-SINGLE-BUYER INTEGRATION INVENTORY POLICY WITH SPACE LIMIT

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Abstract. *Integrated inventory is one of the promising cooperation policies among companies to upgrade companies' competitiveness. An integrated inventory policy between single vendor and single buyer with space limit constraint, a realistic constraint, is considered. Since space limit does not allow a large size of shipping batch, a hybrid approach that makes shipping batch size increase at the front part of order cycle and makes it fixed at the back part of order cycle is presented. A solution procedure based on Lagrangian relaxation approach is developed to provide a good solution. An example is introduced to compare hybrid approach with other existing approaches. Statistical analysis shows that the hybrid approach is better than the existing approaches such as the approach for equal batch shipment and increasing batch shipment.*
Keywords: Integrated inventory, Space limit constraint, Supply chain management

1. **Introduction.** Nowadays, the cooperation among companies focuses on saving cost and upgrading companies' competitiveness. Integrated inventory among members in supply chain is one of the promising cooperation policies. AlDurgam et al. [1] consider single-manufacturer single-vendor integrated inventory model with a variable production rate. They show that controlling manufacturer's production rate is sometimes beneficial only to manufacturer. Das et al. [4] develop three solution procedures for an integrated inventory model with defective item. The manufacturer offers a credit period to the buyer to settle account because the buyer's losses due to defectiveness are required to be compensated.

Hill [7], Ben-Daya and Hariga [2] propose single-vendor single-buyer integration inventory policy assuming that shipping batch sizes are equal. Glock [5] proposes the approach of this policy assuming that shipping batch sizes are increasing.

To make integrated inventory policy more realistic, various constraints such as service level [8], capacity constraint [10], system availability and budget constraint [3] are adopted. Especially, Lee [9] considers space limit constraint for integrated inventory policy assuming that batch size is increasing.

The existing researches consider either equal shipping batch size or increasing shipping batch size, but a hybrid approach that mixes equal shipping batch size with increasing shipping batch size may be required to reduce total cost.

The contribution of this paper is to develop a hybrid approach for single-vendor-single-buyer integration inventory policy with space limit constraint. Furthermore, the performance of the proposed approach is compared with those of other existing approaches.

The rest of this paper is organized as follows. The next section introduces notations and mathematical formulation. Section 3 shows the development of the proposed approach and algorithms. In Section 4, numerical examples are considered and statistical analyses are done to show the performance of the proposed approach. Section 5 presents future research directions and concludes the paper.

2. **Problem Definition.** Buyer's demand is assumed to be probabilistic and follows normal distribution, and vendor's inventory policy is a continuous inventory review (Q, r) policy [4]. If buyer places order, then vendor begins to produce items and transfer items to the buyer several times. If shipping batch sizes are always equal, then it is called *equal shipping batch size*. If shipping batch size keeps increasing with a fixed factor, then it is called *increasing shipping batch size*. In the hybrid approach, the front part of order cycle uses increasing shipping batch size, and the back part of order cycle uses equal shipping batch size.

Suppose that shipping batches increase with a fixed factor $\alpha(\geq 1)$. The size of shipping batch in the jth shipment can be given as

$$q_j = q_1\alpha^{j-1}$$

where q_1 is the first shipping batch size.

Assuming that lead time is proportional to the sum of vendor's lot size and a fixed delayed time (b), lead time can be given as

$$LT(q_j) = pq_j + b$$

Therefore, the hybrid approach can be expressed as follows. Firstly, buyer places the number of $\sum_{j=1}^{n_1} q_j + n_2q_1\alpha^{n_1-1}$ of the item, and vendor begins to produce $\sum_{j=1}^{n} q_j$ of that item with annual production rate $1/p$. Secondly, the buyer receives at n_1 times of increasing shipping batches and then at n_2 times of equal batches from vendor. In the front part of order cycle, the shipping batch size in the jth shipment is $q_1\alpha^{j-1}$ $(0 \leq j \leq n_1)$. In the back part of order cycle, shipping batch size is fixed as $q_1\alpha^{n_1-1}$ $(n_1+1 \leq j \leq n_1+n_2)$. Thirdly, the buyer orders if his on hand inventory level reaches at a reorder point r, right after receiving the $n_1 + n_2$th shipment. This procedure keeps repeated.

The following notations are introduced.

π: Buyer's shortage cost per item for buyer
h_v: Vendor's inventory holding cost per item
h_b: Buyer's inventory holding cost per item
A_v: Vendor's setup cost per order
A_b: Buyer's setup cost per order
F: Buyer's transportation cost per shipment
D: Annual demand
W_v: Vendor's maximum allowable inventory level
W_b: Buyer's maximum allowable inventory level
Z: Random variable for standard normal distribution, $N(0, 1)$
X: Random variable for lead time demand, $X \sim N(\mu, \sigma^2)$
$L(r)$: Expected shortage demand for reorder point r, $L(r) = \int_r^\infty (x - r)f(x)dx$ where $f(x)$ is probability density function for random variable X

Decision variables are in the following. q_1 is size of the 1st shipment from the vendor to the buyer, z_1 is safety factor for the buyer in the first shipment, n_1 is the number of increasing batch shipments, n_2 is the number of equal batch shipments, and α is fixed factor for shipping batch increase.

Buyer's cost (BC) is

$$\begin{aligned} BC = {} & (A_b + (n_1+n_2)F)\frac{D}{q_1\sum_{i=1}^{n_1}\alpha^{i-1} + n_2q_1\alpha^{n_1-1}} \\ & + h_b\left(u_1\frac{q_1\sum_{i=1}^{n_1}\alpha^{2i-2}}{2\sum_{j=1}^{n_1}\alpha^{j-1}} + u_2\frac{q_1\alpha^{n_1-1}}{2} + z_1\sigma\sqrt{pq_1+b}\right) \end{aligned}$$

$$+\pi\frac{D\sigma}{q_1\sum_{i=1}^{n_1}\alpha^{i-1}+n_2q_1\alpha^{n_1-1}}\sum_{i=1}^{n_1+n_2}\sqrt{pq_i+b}L(z_i)$$

where

$$u_1=\frac{q_1\sum_{i=1}^{n_1}\alpha^{i-1}}{q_1\sum_{i=1}^{n_1}\alpha^{i-1}+n_2q_1\alpha^{n_1-1}}$$

$$u_2=\frac{n_2q_1\alpha^{n_1-1}}{q_1\sum_{i=1}^{n_1}\alpha^{i-1}+n_2q_1\alpha^{n_1-1}}$$

and $z_i = z_{n_1}$ for $i = n_1, n_1+1, \ldots, n_2$.

Vendor's cost (VC) is

$$VC=A_v\frac{D}{q_1\sum_{i=1}^{n_1}\alpha^{i-1}+n_2q_1\alpha^{n_1-1}}+\left(\begin{array}{c}Dpq_1+\frac{(1-Dp)}{2}\left(q_1\sum_{i=1}^{n_1}\alpha^{i-1}+n_2q_1\alpha^{n_1-1}\right)\\ -u_1\frac{q_1\sum_{j=1}^{n_1}\alpha^{2j-2}}{2\sum_{j=1}^{n_1}\alpha^{j-1}}-u_2\frac{q_1\alpha^{n_1-1}}{2}\end{array}\right)h_v$$

Therefore, total cost (TC) is the sum of BC and VC.

$$\begin{aligned}&TC(q_1,z_1,n_1,n_2,\alpha)\\ =\;&\frac{q_1}{2}\left[2Dp+(1-Dp)\left(\sum_{i=1}^{n_1}\alpha^{i-1}+n_2\alpha^{n_1-1}\right)h_v\right.\\ &\left.+\left(u_1\frac{\sum_{i=1}^{n_1}\alpha^{2i-2}}{\sum_{j=1}^{n_1}\alpha^{j-1}}+u_2\alpha^{n_1-1}\right)(h_b-h_v)\right]+z_1\sigma\sqrt{pq_1+b}h_b\\ &+\left[(A_b+A_v+(n_1+n_2)F)+\pi\sigma\sum_{i=1}^{n_1+n_2}\sqrt{pq_1\alpha^{i-1}+b}L(z_i)\right]\\ &\times\frac{D}{q_1\sum_{i=1}^{n_1}\alpha^{i-1}+n_2q_1\alpha^{n_1-1}}\end{aligned}\tag{1}$$

Mathematical formulation for this problem is in the following.

$$\text{P: Min } TC(q_1,z_1,n_1,n_2,\alpha)\tag{2}$$

subject to

$$q_1\alpha^{n_1-1}+z_1\sigma\sqrt{pq_1+b}\le W_b\tag{3}$$

$$q_1\alpha^{n_1-1}\le W_v\tag{4}$$

$$q_1,z_1\ge 0\tag{5}$$

$$n_1,n_2\in N\tag{6}$$

Objective function (2) is to minimize total cost. Constraint (3) is the buyer's space limit constraint that prevents buyer's maximum inventory level from exceeding buyer's maximum allowable inventory level (W_b). $q_1\alpha^{n_1-1}$ is the largest shipment and the last shipment during the increasing batch shipment cycle. Therefore, the buyer's maximum inventory level is the sum of the largest shipment and safety stock. Moreover, constraint (4) is the vendor's space limit constraint that prohibits vendor's maximum inventory level from exceeding vendor's maximum allowable inventory level (W_v). Since vendor's maximum inventory level is hard to derive, the largest shipment is used. Equation (5) ensures that decision variables q_1, z_1 are non-negative. Equation (6) ensures that decision variables n_1, n_2 are positive integers.

3. **The Proposed Method.** Lagrangian dual problem for primal problem (P) with relaxing space restriction constraints for vendor and buyer (3), (4) can be formulated as:

$$\begin{aligned}\text{LD: Max}_{\lambda_v,\lambda_b}\ \text{Min } L &= TC(q_1, z_1, n_1, n_2, \alpha) - \lambda_v(W_v - q_1\alpha^{n_1-1}) \\ &\quad -\lambda_b\left(W_b - q_1\alpha^{n_1-1} - z_1\sigma\sqrt{pq_1+b}\right)\end{aligned} \tag{7}$$

Subject to (5)

λ_v, λ_b are Lagrange multipliers corresponding space limit constraints. Further, n_1, n_2 are nonnegative real numbers. The optimal solution for LD can be used as the greatest lower bound of P. Suppose that λ_v, λ_b are obtained. The Lagrangian relaxed problem is given as

$$\begin{aligned}\text{LR: Min } L &= \frac{q_1}{2}\left[2Dp + (1-Dp)\left(\textstyle\sum_{i=1}^{n_1}\alpha^{i-1} + n_2\alpha^{n_1-1}\right)h_v\right. \\ &\quad \left. + \left(u_1\frac{\sum_{i=1}^{n_1}\alpha^{2i-2}}{\sum_{j=1}^{n_1}\alpha^{j-1}} + u_2\alpha^{n_1-1}\right)(h_b - h_v)\right] + z_1\sigma\sqrt{pq_1+b}h_b \\ &\quad + \left[(A_b + A_v + (n_1+n_2)F) + \pi\sigma\textstyle\sum_{i=1}^{n_1+n_2}\sqrt{pq_1\alpha^{i-1}+b}L(z_i)\right] \\ &\quad \times\frac{D}{q_1\sum_{i=1}^{n_1}\alpha^{i-1} + n_2q_1\alpha^{n_1-1}} - \lambda_v(W_v - q_1\alpha^{n_1-1}) \\ &\quad -\lambda_b(W_b - q_1\alpha^{n_1-1} - z_1\sigma\sqrt{pq_1+b})\end{aligned} \tag{8}$$

Subject to (7)

All the optimal solutions for a problem must satisfy the first order necessary conditions. The first order necessary conditions for LR can be derived by differentiating (8) with respect to q_1, z_1, λ_v, λ_b, and setting those to 0. Therefore, the first order necessary conditions (9)-(12) are derived as follows:

$$\begin{aligned}\frac{\partial L}{\partial q_1} &= \frac{C}{2} - \frac{D(A_b + A_v + (n_1+n_2)F)}{q_1^2\left(\sum_{i=1}^{n_1}\alpha^{i-1} + n_2\alpha^{n_1-1}\right)} + \frac{p\sigma h_b z_1}{2\sqrt{pq_1+b}} \\ &\quad +\pi\sigma\textstyle\sum_{i=1}^{n_1+n_2}\left(\frac{Dp\alpha^{i-1}L(z_i)}{2q_1\left(\sum_{i=1}^{n_1}\alpha^{i-1} + n_2\alpha^{n_1-1}\right)\sqrt{pq_1\alpha^{i-1}+b}}\right. \\ &\quad \left.-\frac{D\sqrt{pq_1\alpha^{i-1}+b}L(z_i)}{q_1^2\left(\sum_{i=1}^{n_1}\alpha^{i-1} + n_2\alpha^{n_1-1}\right)}\right) + \lambda_v\alpha^{n_1-1} + \lambda_b\alpha^{n_1-1} + \frac{\lambda_b p\sigma z_1}{2\sqrt{pq_1+b}} \\ &= 0\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{\partial L}{\partial z_1} &= \sigma\sqrt{pq_1+b}(h_b + \lambda_b) \\ &\quad - \left[\pi\sigma\frac{D}{q_1\left(\sum_{i=1}^{n_1}\alpha^{i-1} + n_2\alpha^{n_1-1}\right)}\textstyle\sum_{i=1}^{n_1+n_2}\sqrt{pq_1\alpha^{i-1}+b}(1 - F(z_i))\right] \\ &= 0\end{aligned} \tag{10}$$

$$\frac{\partial L}{\partial \lambda_v} = q_1\alpha^{n_1-1} - W_v = 0 \tag{11}$$

$$\frac{\partial L}{\partial \lambda_b} = q_1\alpha^{n_1-1} + z_1\sigma\sqrt{pq_1+b} - W_b = 0 \tag{12}$$

Equation (9) can be rearranged to get q_1 as

$$q_1 = \sqrt{\frac{Q_N}{Q_D}} \tag{13}$$

where $Q_N = 2D\left[(A_b + A_v + (n_1+n_2)F) + \pi\sigma\sum_{i=1}^{n}\sqrt{pq_1\alpha^{i-1}+b}L(z_i)\right]$

$$Q_D = \left(\sum_{i=1}^{n_1}\alpha^{i-1} + n_2\alpha^{n_1-1}\right)\Bigg[C + \frac{p\sigma z_1}{\sqrt{pq_1+b}}(h_b+\lambda_b) + \pi\sigma\sum_{i=1}^{n_1+n_2}\left(\frac{Dp\alpha^{i-1}L(z_i)}{q_1\left(\sum_{i=1}^{n_1}\alpha^{i-1}+n_2\alpha^{n_1-1}\right)\sqrt{pq_1\alpha^{i-1}+b}}\right) + 2\alpha^{n_1-1}(\lambda_v+\lambda_b)\Bigg]$$

Let $C = \left[2Dp + (1-Dp)(\sum_{i=1}^{n_1}\alpha^{i-1} + n_2\alpha^{n_1-1})h_v + \left(u_1\frac{\sum_{i=1}^{n_1}\alpha^{2i-2}}{\sum_{j=1}^{n_1}\alpha^{j-1}} + u_2\alpha^{n_1-1}\right)(h_b - h_v)\right]$.

Equqtion (10) can be rearranged as

$$\frac{q_1(h_b+\lambda_b)}{\pi D} = \left[\frac{1}{\left(\sum_{i=1}^{n_1}\alpha^{i-1} + n_2\alpha^{n_1-1}\right)}\sum_{i=1}^{n_1+n_2}\sqrt{\frac{pq_1\alpha^{i-1}+b}{pq_1+b}}(1-F(z_i))\right] \tag{14}$$

Note that Equations (11) and (12) are sensitive to the value of λ_v, λ_b. Space limit constraints (3) and (4) can be the functions of λ_v, λ_b in the following.

$$g_v(\lambda_v) = q_1\alpha^{n_1-1} - W_v \tag{15}$$

$$g_b(\lambda_b) = q_1\alpha^{n_1-1} + z_1\sigma\sqrt{pq_1+b} - W_b \tag{16}$$

If g_v, $g_b \leq 0$, then constraints (5) and (6) hold.

The proposed procedure for LR to get decision variables given λ_v, λ_b is shown in the following.

Algorithm A. The algorithm for getting q_1^*, z_i^*, n_1^*, n_2^*, α^* for given λ_v, λ_b,

Step 1. For given λ_v, λ_b, let $L(z_i) = 0$, $n_1 = n_2 = 1$.

Step 2. Set $\alpha = 1$.

Step 3. Perform a) and b) iteratively until q_1 and z_i converge.

a) Obtain q_1 by inserting z_i into (13)

b) Obtain z_i by inserting q_1 into (14)

Step 4. Obtain α by golden section search. If α is changed, then return to Step 3. Otherwise, go to Step 5. Set maximum value of α to pD.

Step 5. If TC decreases, then set $n_1 = n_1 + 1$ and return to Step 2. Otherwise, set $n_1 = 1$ and go to Step 6.

Step 6. If TC decreases, then let $q_1^* = q_1$, $z_i^* = z_i$, $n_1^* = n_1$, $n_2^* = n_2$, $\alpha^* = \alpha$ and set $n_2 = n_2 + 1$ and return to Step 2. Otherwise, stop.

Solutions q_1^*, z_i^*, n_1^*, n_2^*, α^* obtained by Algorithm A may violate space limit constrains. If the solution violates those constraints, q_1 should be decreased until both of space limit constraints (3) and (4) hold.

Let $q_1\alpha^{n-1} = W_v$. Then

$$Q_v = \frac{W_v}{\alpha^{n-1}} \tag{17}$$

Let $q_1\alpha^{n-1} + z_1\sigma\sqrt{pq_1+b} = W_b$. Then

$$Q_b = \frac{W_b - z_1\sigma\sqrt{pq_1+b}}{\alpha^{n-1}} \tag{18}$$

The proposed procedure to construct a feasible solution by decreasing q_1 is in the following.

Algorithm B. The algorithm for making a feasible solution

Step 1. Recall an infeasible solution from Algorithm A.

Step 2. If $g_v(\lambda_v) > 0$, then calculate Q_v using Equation (17). Otherwise, set $Q_v = q_1$.

Step 3. If $g_v(\lambda_b) > 0$, then calculate Q_b using Equation (18). Otherwise, set $Q_b = q_1$.

Step 4. Set $Q^* = \min[Q_v, Q_b]$. Space limit constraints are satisfied by using Q^* instead of q_1.

To find optimal λ_v and λ_b, the subgradient method is used. The subgradient of d and step size s are shown in the following.

$$d = |g_v(\lambda_v)| + |g_b(\lambda_b)|$$

$$s = \theta \frac{TC^* - LB}{d^2}$$

where TC^* is the total cost using Equation (1), LB is lower bound using Equation (8), and θ is a scalar chosen from 0 to 2.

4. **Numerical Example and Result.** To show the performance of the hybrid approach and sensitivity analysis for space limit of vendor and buyer, the numerical example is used in Table 1.

The results applied by the proposed approach and the existing approach are given in Table 2.

As W_v and W_b are decreased by 15, z_1 and n_1 are decreased but n_2 is increased. If space limits are low, safety stock level is small, the number of increasing batch sizes is reduced, and the number of equal batch sizes is increased. Total costs for all approaches are increased. Especially, total cost of the approach for increasing batch size keeps increasing because increment of shipping batch size is directly affected by space limit. On the other hand, total cost of the approach for equal batch size is not increased much because the fixed batch size is not affected much by space limit. It seems that α does not have a certain pattern and q_1 is closely related to n_1. When n_1 decreases from 3 to 2 at data 8, q_1 increases from 14.91 to 34.24. That is, the size of the first batch shipment (q_1) rises if the number of increasing batch shipments (n_1) drops. The number of equal batch sizes

TABLE 1. Data for numerical example

Notation	Value	Notation	Value
D	1000	A_v	\$400
p	1/2900	A_b	\$100
σ	10	h_v	\$4
F	\$25	h_b	\$6
π	\$100	b	0.01

TABLE 2. The values of α, q_1, z_1, n_1, n_2 for hybrid approach and total cost for hybrid approach, increasing batch, equal batch by data if W_b and W_v are changed from 230 to 95 and from 215 to 80 by 15, respectively

Data	α	q_1	z_1	n_1	n_2	TC_{hyb}	TC_{inc}	TC_{equ}
1	2.90	17.20	4.01	3	3	2051.55	2087.91	2143.18
2	2.90	17.20	4.01	3	3	2051.55	2088.12	2143.18
3	2.90	17.20	4.01	3	3	2051.55	2089.57	2143.18
4	2.90	17.20	4.01	3	3	2051.55	2092.42	2143.18
5	2.90	17.20	4.01	3	3	2051.55	2097.56	2143.18
6	2.75	18.46	3.94	3	3	2052.75	2105.82	2143.18
7	2.82	14.91	3.83	3	4	2061.97	2114.78	2143.18
8	2.83	34.24	3.26	2	5	2079.71	2123.31	2143.18
9	2.89	29.96	3.24	2	6	2094.92	2139.88	2146.57
10	2.90	26.17	3.21	2	7	2119.40	2156.44	2158.81

(n_2) keeps increasing because space limit prevents from increasing of n_1 and promotes the increase of n_2.

Statistical analysis is performed to compare the hybrid approach with the existing approaches such as the approach for increasing batch shipments and the approach for equal batch shipment. A paired t-test is done to test the discrepancy between two population means because a comparison of two different approaches is applied to the data with equal parameters settings [2]. If sample size is over 30, then z-test can be performed. However, t-test should be performed because the sample size is 10 in this example.

Statistical tests for total costs between the hybrid approach and equal batch shipment and between the hybrid approach and increasing batch shipment are done. The null hypothesis (H_0) and alternative hypothesis (H_1) are as follows:

$$\text{Hypothesis 1: } H_0: \mu_{equ} - \mu_{hyb} = 0 \quad H_1: \mu_{equ} - \mu_{hyb} > 0$$
$$\text{Hypothesis 2: } H_0: \mu_{inc} - \mu_{hyb} = 0 \quad H_1: \mu_{inc} - \mu_{hyb} > 0$$

μ_{hyb}, μ_{equ}, μ_{inc} are means of total costs for hybrid approach, the approach for equal batch shipment, and the approach for increasing batch shipment, respectively.

Null hypotheses mean that there is no difference between the hybrid approach and the existing approaches such as the approach for equal batch shipment and the approach for increasing batch shipment. Alternative hypotheses mean that the hybrid approach is superior to the existing approaches. The means and standard deviations for the differences between total cost of the hybrid approach and the approach for equal batch shipment and between the hybrid approach and the approach for increasing batch shipment are given in Table 3.

Table 3. Means and standard deviations for difference between total cost of the approach for equal batch shipment and those of hybrid approach and between total cost of the approach for increasing batch shipment and those of hybrid approach

	$TC_{equ} - TC_{hyb}$	$TC_{inc} - TC_{hyb}$
mean ($\overline{D}$)	78.43	42.93
Standard deviation (S_D)	18.67	6.01

Test statistic is $t_0 = \frac{\overline{D}}{\frac{S_D}{\sqrt{n}}}$, where n is sample size. The results of hypothesis tests for Hypothesis 1 and Hypothesis 2 at significant level of $\alpha = 0.05$ are as follows:

$$\text{Hypothesis 1: } t_0 = 21.43 > t(9, 0.05) = 1.83$$
$$\text{Hypothesis 2: } t_0 = 12.60 > t(9, 0.05) = 1.83$$

Therefore, we reject H_0 of both Hypothesis 1 and Hypothesis 2. Statistical analysis shows that the hybrid approach is better than the existing approaches such as the approach for equal batch shipment and increasing batch shipment.

5. **Conclusions.** A hybrid approach for single-vendor-single-buyer integrated inventory model with space limit is proposed in this research. Examples are developed to compare the hybrid approach with the existing approach. Statistical analysis shows that hybrid approach is superior to the existing approach. Future research works can be done extending the model by including transportation costs, variable production rate, and defective item, etc.

REFERENCES

[1] M. AlDurgam, K. Adegbola and C. H. Glock, A single-vendor single-manufacturer integrated inventory model with stochastic demand and variable production rate, *International Journal of Production Economics*, vol.191, pp.335-350, 2017.

[2] M. Ben-Daya and M. Hariga, Integrated single vendor single buyer model with stochastic demand and variable lead time, *International Journal of Production Economics*, vol.92, pp.75-80, 2004.

[3] F. Costantino, G. D. Gravio and M. Tronci, Multi-echelon, multi-indenture spare parts inventory control subject to system availability and budget constraints, *Reliability Engineering & System Safety*, vol.119, pp.95-101, 2013.

[4] B. C. Das, B. Das and S. K. Mondal, An integrated production-inventory model with defective item dependent stochastic credit period, *Computers & Industrial Engineering*, vol.110, pp.255-263, 2017.

[5] C. H. Glock, A comment: Integrated single vendor-single buyer model with stochastic demand and variable lead time, *International Journal of Production Economics*, vol.122, pp.790-792, 2009.

[6] G. Hadley and T. M. Whitin, *Analysis of Inventory Systems*, Prentice Hall Inc., Englewood Cliffs, NJ, 1963.

[7] R. M. Hill, The single-vendor single-buyer integrated production-inventory model with a generalised policy, *European Journal of Operational Research*, vol.97, pp.493-499, 1997.

[8] J. K. Jha and K. Shanker, An integrated inventory problem with transportation in a divergent supply chain under service level constraint, *Journal of Manufacturing Systems*, vol.33, pp.462-475, 2014.

[9] D. Lee, An inventory problem with space limit and increasing shipping batch size, *ICIC Express Letters, Part B: Applications*, vol.7, no.8, pp.1827-1833, 2016.

[10] L. Ouyang, C. Ho, C. Su and C. Yang, An integrated inventory model with capacity constraint and order-size dependent trade credit, *Computers & Industrial Engineering*, vol.84, pp.133-143, 2015.