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LOW COMPLEXITY ZF DETECTION ALGORITHM FOR MASSIVE MIMO SYSTEMS

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Abstract. *In Massive multiple-input multiple-output (MIMO) systems, zero-forcing (ZF) detection algorithm has attracted attention again for its low computational complexity. However, when the number of antennas is great or infinite, the inversion of the high order number of the channel matrix will bring great computational complexity to the ZF detection algorithm. In this paper, we propose a low complexity ZF algorithm based on Neumann series approximation, which converts the multiplication of large matrices into the multiplication of the diagonal matrix and the hollow matrix. And a method to simplify the optimization factor is proposed to improve the convergence speed and reduce the delay effectively. The simulation results show that the performance of the improved algorithm is approaching to the traditional ZF algorithm with the increase of the receiving antennas, and the computational complexity is reduced from $O(K^3)$ to $O(K^2)$, where K is the number of users.*

Keywords: Massive MIMO, ZF detection, Neumann series approximation, Optimization factor

1. **Introduction.** Wireless communication technology has entered the era of 4G/5G communication, and people's demand for wireless communication system to transmit data at a higher rate is increasing. Massive MIMO System, which equips large-scale antenna arrays with hundreds of antennas at the base station (BS) can significantly improve the capacity [1,2], spectrum utilization and system performance of communication systems without increasing system bandwidth. So that, Massive MIMO has been considered as one of the key technologies in modern wireless communication [3].

With the increasing number of antennas BS and the number of users, the complexity of the system becomes one of the key problems that affect the realization of Massive MIMO system [4]. As the optimal detector, complexity of maximum likelihood (ML) detector increases exponentially with the increase of the modulation orders and the transmission antenna number. The RZF (Regular ZF) precoding or linear MMSE (minimum mean-square error) detection is used to obtain the performance of the approximation capacity when the system is equipped with an order magnitude lower number of antennas in [5]. However, the linear detection algorithm involves complex matrix inversion, and its computational complexity is still high. To simplify the matrix inversion operation, scholars put forward the Richardson method is utilized to avoid the matrix inversion in [6]. A method based on the Neumann series approximation algorithm to convert the inverse of the matrix into truncated polynomial summation, which reduces the complexity is proposed in [7]. However, the large matrix multiplication brings a high degree of complexity and computational optimization factor also leads to the system delay, especially when the number of receiving antennas is far more than the number of users [8]. And [9] proposes a low complexity algorithm for beamforming based on ZF.

In order to further reduce the computational complexity, the improved ZF algorithm based on the Neumann series expansion [10] is proposed in this paper, which converts the large ZF filter matrix [11] into the sum of the diagonal matrix and the hollow matrix. In addition, we propose a simplified expression of the optimization factor to accelerate the convergence rate. The simulation results show that the proposed algorithm can reduce the computational complexity with the better performance.

2. **System Model.** We consider the typical uplink transmission of a Massive MIMO system, which employs M antennas at BS to communicate with K single-antenna users ($M >> K$). Meanwhile, let $\mathbf{H}$ denote the channel matrix, where the first (i, j) element h_{ji} represents the channel gain between the ith user and the jth receiving antenna, $i = 1, 2, \ldots, K$; $j = 1, 2, \ldots, M$. The user transmission signal and the corresponding reception signal are denoted as $\mathbf{x} = [x_1, x_2, \ldots, x_K]^T$ and $\mathbf{y} = [y_1, y_2, \ldots, y_M]^T$, where x_i and y_j represent the transmitting signal of the ith user and the receiving signal of the jth receiving antenna, respectively. n_j represents the additive Gauss white noise of the jth receiving antenna with a variance of δ_n^2, then the $N \times K$ MIMO system can be represented as:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} = h_1 x_1 + h_2 x_2 + \cdots + h_K x_K + \mathbf{n} \tag{1}$$

where $\mathbf{n} = [n_1, n_2, \ldots, n_M]^T$, h_i denotes the ith column vector of the channel matrix $\mathbf{H}$. $h_i = [h_{1i}, h_{2i}, \ldots, h_{Mi}]^T$, $i = 1, 2, \ldots, K$; $j = 1, 2, \ldots, M$.

3. **Neumann Series Approximation.** The most important computational complexity of the detection algorithm lies in the matrix inverse operation. Specifically, if the inverse matrix of the $K \times K$-dimensional matrix $\mathbf{Z}$ is required the $O(K^3)$ operand, then the complexity of inverse operation will increase rapidly with the growing number of K. Therefore, an effective method of inversion is essential to pursue the cost-effectiveness of hardware implementation. Stewart proposed the Neumann series expansion algorithm [10].

The Neumann series expansion of matrix $\mathbf{Z}^{-1}$ can be:

$$\mathbf{Z}^{-1} = \sum_{n=0}^{\infty} \left(\mathbf{X}^{-1}(\mathbf{X} - \mathbf{Z})\right)^n \mathbf{X}^{-1} \tag{2}$$

where the conditions need to be satisfied with $\lim_{n\to\infty} (\mathbf{I} - \mathbf{X}^{-1}\mathbf{Z})^n = 0$ or $\lim_{n\to\infty} \left(\mathbf{I} - \mathbf{Z}\mathbf{X}^{-1}\right)^n = 0$ and $\mathbf{X}$ is an invertible matrix.

Take the first L items to approximate $\mathbf{Z}^{-1}$ in (2):

$$\mathbf{Z}_L^{-1} = \sum_{n=0}^{L-1} \left(\mathbf{X}^{-1}(\mathbf{X} - \mathbf{Z})\right)^n \mathbf{X}^{-1} \tag{3}$$

Then, the matrix $\mathbf{Z}$ is converted into the sum of the diagonal matrix $\mathbf{D}$ and the hollow matrix $\mathbf{E}$ (all the elements on the diagonal are zero), that is $\mathbf{Z} = \mathbf{D} + \mathbf{E}$, $\mathbf{X} = \mathbf{D}$ which applies into Formula (3):

$$\mathbf{Z}_L^{-1} = \sum_{n=0}^{L-1} \left(-\mathbf{D}^{-1}\mathbf{E}\right)^n \mathbf{D}^{-1} \tag{4}$$

When $L = 2$,

$$\mathbf{Z}_2^{-1} = \mathbf{D}^{-1} - \mathbf{D}^{-1}\mathbf{E}\mathbf{D}^{-1} \tag{5}$$

It needs $O(K^2)$ operation in (5), where K is the dimension of matrix. Compared with the complexity $O(K^3)$ of the traditional matrix direct inversion, the complexity of the Neumann series approximate inversion algorithm is greatly reduced.

4. **The Proposed Improved ZF-opt Detection Algorithm.** The filtering matrix of ZF detection algorithm is [11]:

$$\mathbf{W}_{ZF} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H \tag{6}$$

where $(\cdot)^H$ denotes the conjugate transpose. The complexity of ZF detection algorithm is $O(K^3)$, and we apply the Neumann approximation algorithm in reducing the complexity of the ZF detection algorithm.

$\mathbf{W}_{ZF}$ is simplified by using Neumann series approximation [12,13]:

$$\mathbf{W}_{ZF} \approx \left[\sum_{n=0}^{L-1}\alpha_n\left(\mathbf{H}^H\mathbf{H}\right)^n\right]\mathbf{H}^H \tag{7}$$

where L is the term of the approximation, $\boldsymbol{\alpha} = [\alpha_0, \alpha_1, \ldots, \alpha_{L-1}]$ is optimization factor.

4.1. **Based on diagonal matrix decomposition.** When the number of users K is large, the large matrix $(\mathbf{H}^H\mathbf{H})$ multiplication still brings greater computational complexity. In order to further reduce the complexity of the detection algorithm, we decompose the large matrix $\mathbf{H}^H\mathbf{H}$. Let $\mathbf{R}_{ZF} = \mathbf{H}^H\mathbf{H}$, where $\mathbf{H}$ is independent with Gaussian distribution. In the Massive MIMO system, the number of antennas at base station (BS) is greater than the number of users $M > K$, and the matrix $\mathbf{R}_{ZF}$ becomes a diagonally dominant matrix [14]. $\mathbf{R}_{ZF}$ can be decomposed as:

$$\begin{aligned}\mathbf{R}_{ZF} &= \mathbf{D}_{ZF} + \mathbf{E}_{ZF}\\ &= \begin{bmatrix} h_1^Hh_1 & 0 & \cdots & 0\\ 0 & h_2^Hh_2 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0_2 & \cdots & h_K^Hh_K\end{bmatrix} + \begin{bmatrix} 0 & h_1^Hh_2 & \cdots & h_1^Hh_K\\ h_2^Hh_1 & 0 & \cdots & h_2^Hh_K\\ \vdots & \vdots & \ddots & \vdots\\ h_K^Hh_1 & h_K^Hh_2 & \cdots & 0\end{bmatrix}\end{aligned} \tag{8}$$

where $\mathbf{D}_{ZF}$ is the main diagonal matrix of the matrix $\mathbf{R}_{ZF}$, $\mathbf{E}_{ZF}$ is the hollow matrix with all the elements zero on the diagonal. Neumann series approximation of $\mathbf{R}_{ZF}^{-1}$ is derived as:

$$\mathbf{R}_{ZF}^{-1} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1} = \left(\mathbf{D}_{ZF} + \mathbf{E}_{ZF}\right)^{-1} = \sum_{n=0}^{\infty}\left(-\mathbf{D}_{ZF}^{-1}\mathbf{E}_{ZF}\right)^n\mathbf{D}_{ZF}^{-1} \tag{9}$$

Substituting Formula (9) into Formula (6):

$$\begin{aligned}\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H &= \left[\sum_{n=0}^{\infty}\left(-\mathbf{D}_{ZF}^{-1}\mathbf{E}_{ZF}\right)^n\mathbf{D}_{ZF}^{-1}\right]\mathbf{H}^H\\ &\approx \left[\sum_{n=0}^{L-1}\alpha_n\left(-\mathbf{D}_{ZF}^{-1}\mathbf{E}_{ZF}\right)^n\mathbf{D}_{ZF}^{-1}\right]\mathbf{H}^H\end{aligned} \tag{10}$$

Formula (10) shows that the inverse of the large matrix $(\mathbf{H}^H\mathbf{H})^{-1}$ is transformed into the sum of the product of $(-\mathbf{D}_{ZF}^{-1}\mathbf{E}_{ZF})$, and the inverse of the diagonal matrix requires only the reciprocal of the elements on the diagonal, so the Neumann approximation algorithm can reduce the inverse computational complexity effectively. And the appropriate optimization factor α_n in the next section will improve the accuracy of the approximation algorithm and accelerates the convergence rate of the algorithm.

4.2. **Optimization factor.** This section discusses the value of the optimization factor $\boldsymbol{\alpha} = [\alpha_0, \alpha_1, \ldots, \alpha_{L-1}]$, which can improve the detection performance on the basis of fast convergence. Let:

$$\left[\sum_{n=0}^{L-1}\alpha_n\left(-\mathbf{D}_{ZF}^{-1}\mathbf{E}_{ZF}\right)^n\right]\left(\mathbf{D}_{ZF} + \mathbf{E}_{ZF}\right)^{-1} \approx \mathbf{I} \tag{11}$$

where $\mathbf{I}$ is identity matrix. Let $\mathbf{A}_{ZF} = -\mathbf{D}_{ZF}^{-1}\mathbf{E}_{ZF}$, and the left side of Formula (11) is:

$$\begin{aligned}&\left[\sum_{n=0}^{L-1}\alpha_n\left(-\mathbf{D}_{ZF}^{-1}\mathbf{E}_{ZF}\right)^n\right]\left(\mathbf{D}_{ZF}+\mathbf{E}_{ZF}\right)^{-1}\\ &=\sum_{n=0}^{L-1}\alpha_n\left(-\mathbf{D}_{ZF}^{-1}\mathbf{E}_{ZF}\right)^n-\sum_{n=0}^{L-1}\alpha_n\left(-\mathbf{D}_{ZF}^{-1}\mathbf{E}_{ZF}\right)^{n+1}=\sum_{n=0}^{L-1}\alpha_n\left(\mathbf{A}_{ZF}^n-\mathbf{A}_{ZF}^{n+1}\right)\end{aligned} \tag{12}$$

For the lower order plays a main role in the convergence, it will be fast convergence if $|\alpha_n (\mathbf{A}_{ZF}^n - \mathbf{A}_{ZF}^{n+1})|$ close to $1/L$, as:

$$\alpha_n \approx \frac{1}{L\times\left|\left(\mathbf{A}_{ZF}^n-\mathbf{A}_{ZF}^{n+1}\right)\right|}=\frac{1}{L\times\left|\mathbf{A}_{ZF}^n\right|\left|\left(\mathbf{I}+\mathbf{D}_{ZF}^{-1}\mathbf{E}_{ZF}\right)\right|} \tag{13}$$

The matrix $(\mathbf{I}+\mathbf{D}_{ZF}^{-1}\mathbf{E}_{ZF})$ denotes with $\mathbf{H} = [h_1, h_2, \ldots, h_K]$ as:

$$\mathbf{I}+\mathbf{D}_{ZF}^{-1}\mathbf{E}_{ZF}=\begin{bmatrix} 1 & \dfrac{h_1^H h_2}{h_1^H h_1} & \cdots & \dfrac{h_1^H h_K}{h_1^H h_1}\\ \dfrac{h_2^H h_1}{h_2^H h_2} & 1 & \cdots & \dfrac{h_2^H h_K}{h_2^H h_2}\\ \vdots & \vdots & \ddots & \vdots\\ \dfrac{h_K^H h_1}{h_K^H h_K} & \dfrac{h_K^H h_2}{h_K^H h_K} & \cdots & 1\end{bmatrix} \tag{14}$$

As $M \to \infty$, $(h_i^H h_j / h_i^H h_i) \to 0$, where $i, j = 1, 2, \ldots, K$ $(i \neq j)$, then Formula (14) $\to \mathbf{I}_K$.

Let $\mathbf{A}_{ZF} = -\mathbf{D}_{ZF}^{-1}\mathbf{E}_{ZF}$, then the determinant of $\mathbf{A}_{ZF}^n$ is $|\mathbf{A}_{ZF}^n| = (-1)^{Kn}|\mathbf{D}_{ZF}^{-1}|^n|\mathbf{E}_{ZF}|^n$. According to Marchenko-Pastur's law [15], as M, $K \to \infty$, the maximum and minimum eigenvalues of the matrix $\mathbf{E}_{ZF}$ are:

$$\lambda_{\max}\left(\mathbf{E}_{ZF}\right)\to\frac{1}{\beta}+\frac{2}{\sqrt{\beta}},\quad \lambda_{\min}\left(\mathbf{E}_{ZF}\right)\to\frac{1}{\beta}-\frac{2}{\sqrt{\beta}} \tag{15}$$

where $\beta = M/K$. In order to reduce the influence of the higher order $(-\mathbf{D}_{ZF}^{-1}\mathbf{E}_{ZF})^n$, it needs to accelerate the convergence of Formula (10), and we choose the maximum eigenvalue $\lambda_{\max}(\mathbf{E}_{ZF})$ of the matrix $\mathbf{E}_{ZF}$, we can obtain that:

$$|\mathbf{E}_{ZF}|^n=\left|\frac{1}{\beta}+\frac{2}{\sqrt{\beta}}\right|^{Kn} \tag{16}$$

$$\alpha_n=\frac{1}{L\left(1/\beta+2/\sqrt{\beta}\right)^{Kn}} \tag{17}$$

For the dimensions K and M in the actual Massive MIMO system are finite, the approximation of Equation (17) is not very accurate. Therefore, it is very necessary to amend the factor α_n. When n: $0 \to L-1$, the lower order part of $(-\mathbf{D}_{ZF}^{-1}\mathbf{E}_{ZF})^n$ contains the vast majority information of the channel matrix $\mathbf{H}$, and the higher order part of $(-\mathbf{D}_{ZF}^{-1}\mathbf{E}_{ZF})^n$ is mainly used to modify the approximate precision of the algorithm, so the modifying factors need to converge quickly, which meets the following. (1) Let $\alpha_0 = 1$ to ensure that the first order contains all the information. (2) As the increasing number of n, the value of α_n $(n \geq 1)$ decreases gradually. Formula (17) is simplified as:

$$\alpha_n=\frac{1}{\left(\left(1/\beta+2/\sqrt{\beta}\right)n+1\right)^n} \tag{18}$$

The complex inverse operation was converted into the Neumann approximation to reduce the complexity of the detection and the method of optimization factor was proposed to further reduce the complexity by ensuring the detection performance.

4.3. **The specific description of the proposed algorithm.**

Algorithm 1 The proposed improved ZF-opt detection algorithm

1. Get the filtering matrix of ZF detection from [11].
$\mathbf{W}_{ZF} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H$
2. Let $\mathbf{R}_{ZF} = \mathbf{H}^H\mathbf{H}$, then decompose $\mathbf{R}_{ZF}$ based on diagonal matrix decomposition
$\mathbf{R}_{ZF} = \mathbf{D}_{ZF} + \mathbf{E}_{ZF}$
3. Neumann series approximation of $\mathbf{R}_{ZF}^{-1}$ is derived as
$\mathbf{R}_{ZF}^{-1} = (\mathbf{D}_{ZF} + \mathbf{E}_{ZF})^{-1} = \sum_{n=0}^{\infty}\left(-\mathbf{D}_{ZF}^{-1}\mathbf{E}_{ZF}\right)^n\mathbf{D}_{ZF}^{-1}$
4. Use Neumann series approximation, get
$\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H \approx \left[\sum_{n=0}^{L-1}\alpha_n\left(-\mathbf{D}_{ZF}^{-1}\mathbf{E}_{ZF}\right)^n\mathbf{D}_{ZF}^{-1}\right]\mathbf{H}^H$
5. Improve the optimization factor
$\alpha_n = \dfrac{1}{\left(\left(1/\beta + 2/\sqrt{\beta}\right)n + 1\right)^n}$
6. Use the improved α_n to solve $\mathbf{W}_{ZF}$.

4.4. **Complexity analysis.** The complexity of the algorithm can usually be measured by the number of multipliers, dividers, and adders. The complexity of the ZF detection algorithm is the inverse operation of the matrix and the main sources of the inverse computational complexity are multipliers and adders. Therefore, the complexity of the algorithm is compared with the Cholesky decomposition [16] based on the exact matrix inversion operation. And the number of real multipliers (A multiplicative multiplier is equivalent to four real multipliers) and real adders (A complex number of adder is equivalent to two real adders) of low complexity ZF detection algorithm is proposed in this paper. The statistical results are shown in Table 1.

Table 1. Complexity comparison

Algorithms	Multiplications	Additions
$L = 2$ Approximate	$4K^2 - 4K$	$2K^2 - 2K$
$L = 3$ Approximate	$8K^3 + 4K^2 - 2K$	$4K^3 + 2K^2 - K$
Cholesky Decomposition	$16K^3 + 4K^2 + K$	$8K^3 + 3K^2 + 2K$

As Table 1 shows that the complexity of the improved algorithm is $O(K^2)$ as $L = 2$, and when $L = 3$, the complexity of the improved algorithm and the matrix exact inversion based on Cholesky decomposition are all $O(K^3)$ as $L = 3$. As we choose $L = 2$, the complexity of the improved ZF algorithm is far lower than the exact inverse method with a small loss of performance.

5. **Performance Simulation.** Based on the Massive MIMO uplink system model, the QPSK modulation scheme is employed and it is assumed that the channel is a Rayleigh fading channel and the channel state information is known at the receiver; let the noise be Additive Gaussian white noise with each element assumed to be independent and identically distributed; 16, 32, 64, receiving antennas are respectively arranged at BS with 8 single antenna users. The simulation results are shown in Figure 1, where improved ZF-opt represents an improved ZF detection algorithm with optimization factor and ZF represents a traditional ZF detection algorithm.

Figure 1 shows the decoding performance simulation of the improved ZF algorithm with optimization factor (improved ZF-opt) in different M when $L = 1, 2, 3$, respectively.

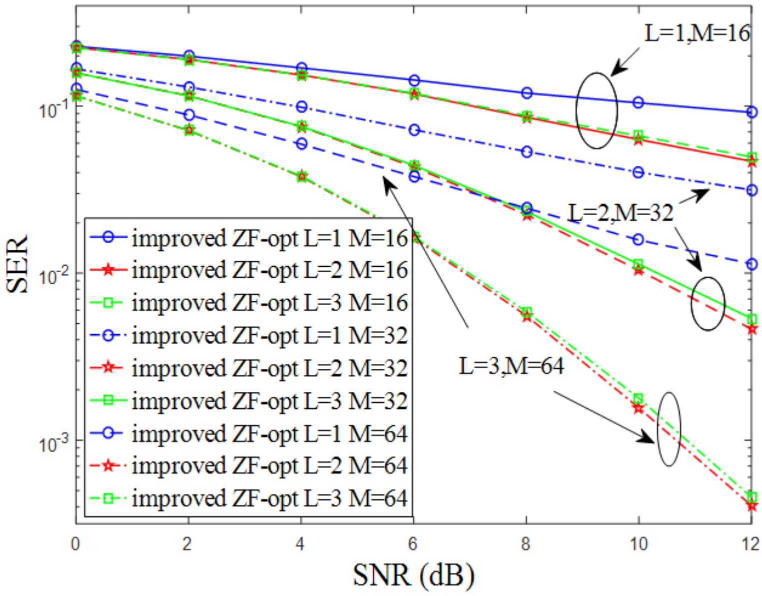

FIGURE 1. When $L = 1, 2, 3$, performance comparison of the improved ZF-opt algorithm

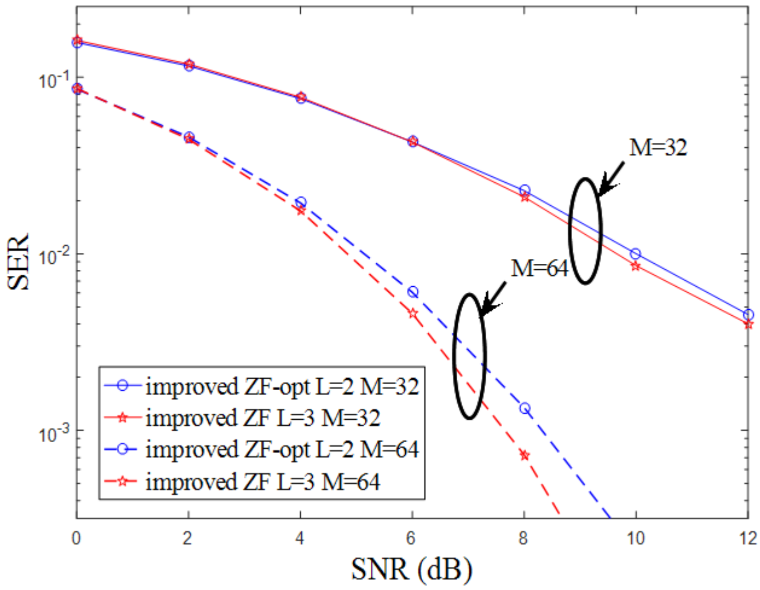

FIGURE 2. SER performance comparison for whether having optimization factor

In Figure 1, with the increase of L, the performance of the improved ZF-opt is improved at the same M. Especially in the large M, the improved ZF-opt detection algorithm as $L = 2$ has a significant improvement comparing with $L = 1$. As the increase of M, the performance of improved ZF-opt detection algorithm is also getting better at the same L.

Figure 2 shows that the performance of the improved ZF-opt detection algorithm as $L = 2$ is close to the performance of improved ZF algorithm without the optimization factor (improved ZF) as $L = 3$. It is clear that with the increased number of the receiving antennas, the SER (symbol error rate) performance of both algorithms are greatly improved. The complexity of both algorithm is $O(K^2)$ as $L = 2$, and when $L = 3$, the complexity is $O(K^3)$. Therefore, we choose $L = 2$ to obtain trade-off between detection performance and algorithm complexity in this paper.

The SER performance comparison between the proposed improved ZF-opt detection algorithm, the improved ZF algorithm and the traditional ZF detection algorithm is shown

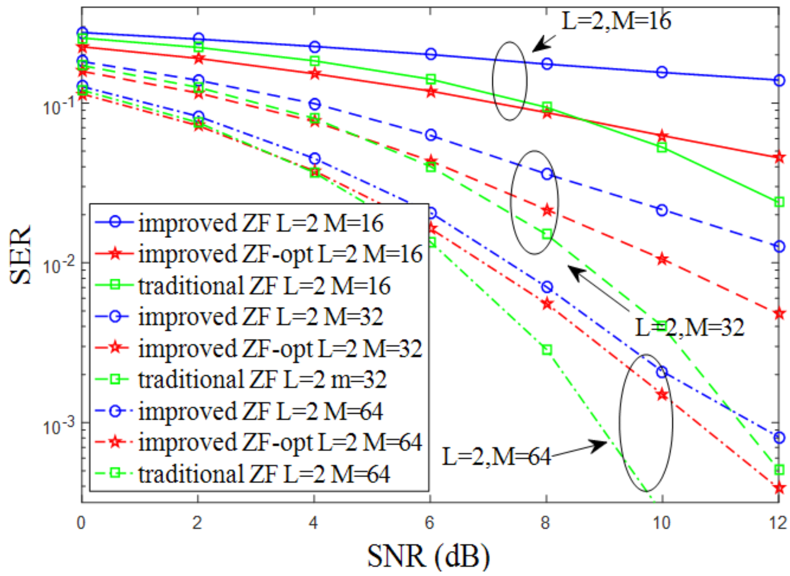

FIGURE 3. Comparison of detection performance of three algorithms

in Figure 3. It can be concluded from Figure 3 that the performance of the proposed improved ZF-opt detection algorithm is obviously better than that of the improved ZF algorithm. With the increased number of the receiving antennas, the SER performance of the improved ZF-opt algorithms becomes closer to that of the traditional ZF algorithm. The simulation results show that the optimization factor can improve the detection performance greatly.

6. **Conclusions.** In this paper, the optimization factor expression is obtained by the method of fast convergence of the improved algorithm, and a simplified form of the optimization factor is given as well, which greatly decreases the delay caused by the calculation of the optimization factor. Simulation results show that with the increasing number of receiving antennas at BS, the performance of the proposed algorithm gradually approached the traditional ZF detection algorithm while the complexity is reduced from $O(K^3)$ to $O(K^2)$. In this paper, the channel status information (CSI) is assumed perfectly known; in the future research, the case of imperfect CSI will be studied.

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