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THE PARAMETERIZATION OF ALL ROBUST STABILIZING SIMPLE REPETITIVE CONTROLLERS FOR MULTIPLE-INPUT/MULTIPLE-OUTPUT PLANTS

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Abstract. *The simple repetitive control system proposed by Yamada et al. is a type of servomechanism for the periodic reference input. In addition, simple repetitive control systems make transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles. Yamada et al. clarified the parameterization of all stabilizing simple repetitive controllers. Recently, the parameterization of all robust stabilizing simple repetitive controllers for the plant with uncertainty was clarified. However, they did not clarify the parameterization of all robust stabilizing simple repetitive controllers for multiple-input/multiple-output plants. Since many real plants include uncertainty and have multiple-input and multiple-output, this is the important problem to solve. The purpose of this paper is to propose the parameterization of all robust stabilizing simple repetitive controllers for multiple-input/multiple-output plants.*
Keywords: Repetitive control, Uncertainty, Robust stability, Parameterization, Finite numbers of poles, Multiple-input/multiple-output plants

1. **Introduction.** A repetitive control system is a type of servomechanism for periodic reference inputs. That is, the repetitive control system follows the periodic reference input without steady state error, even if a periodic disturbance or an uncertainty exists in the plant [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 15]. Many design methods for repetitive control systems for strictly proper plants have been given [3, 4, 5, 6, 7, 8, 9, 10, 13]. These studies are divided into two types. One uses a low-pass filter [3, 4, 5, 6, 7, 8, 9, 10] and the other uses an attenuator [13]. The latter is difficult to design because it uses a state variable time-delay in the repetitive controller [13]. The former has a simple structure and is easily designed. Therefore, the former type of repetitive control system is called the modified repetitive control system [3, 4, 5, 6, 7, 8, 9, 10]. Recently, Chen and Tomizuka proposed a structural configuration of the internal model in repetitive control, so that designers have more flexibility in the repetitive loop-shaping design, and the amplification of nonrepetitive errors can be reduced [11]. While Li has used a new method affine parameterization perspective instead of an internal model perspective to derive the controller, this method makes the classical prototype repetitive control scheme extended to the angle-domain repetitive disturbance [12].

Using the modified repetitive controllers in [3, 4, 5, 6, 7, 8, 9, 10], even if the plant does not include time-delays, transfer functions from the periodic reference input to the output and from the disturbance to the output have infinite numbers of poles. This makes it difficult to specify the input-output characteristic and the disturbance attenuation characteristic. From the practical point of view, it is desirable that these characteristics should be easy to specify. Therefore, these transfer functions should have finite numbers of poles.

In some cases, the uncertainties in the plant make the control system unstable. When the uncertainties in the plant exist, the stability of the control system is known as robust stability problem [21, 22, 23]. Yamada et al. proposed the parameterization of all robust stabilizing simple repetitive controllers for plants with uncertainties [23]. However, the parameterization in [23] cannot be applied to multiple-input/multiple-output plants, because this parameterization is obtained using the characteristics of single-input/single-output systems. For multiple-input/multiple-output time-delay plants, Sakanushi and Yamada proposed the parameterization of all robust stabilizing simple repetitive controllers [24] using the idea of predictive control. However, the method in [24] cannot apply for non-time-delay plants. Many real plants include multiple-input and multiple-output. In addition, the parameterization is useful to design stabilizing controllers [17, 18, 19, 20]. Therefore, the problem of obtaining the parameterization of all robust stabilizing simple repetitive controllers for multiple-input/multiple-output plants is important.

In this paper, we propose the parameterization of all robust stabilizing simple repetitive controllers for multiple-input/multiple-output plants such that the controller works as a robust stabilizing modified repetitive controller and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles. Obtained simple repetitive controllers make transfer functions from the reference input to the output have finite number of poles, and it is easy to specify the input-output characteristic and the disturbance attenuation characteristic. In addition, even if the plant has uncertainty, this controller guarantees the stability of the control system. Using obtained parameterization, we can easily design stabilizing simple repetitive controllers for multiple-input/multiple-output plants as shown in Section 4.

2. Robust Stabilizing Simple Repetitive Control Systems and Problem Formulation.

Consider the unity feedback control system in

$$\begin{cases} y = G(s)u + d \\ u = C(s)(r - y) \end{cases}, \tag{1}$$

where $G(s) \in R^{m\times p}(s)$ is the plant, $C(s)$ is the controller, $u \in R^p$ is the control input, $d \in R^m$ is the disturbance, $y \in R^m$ is the output and $r \in R^m$ is the periodic reference input with period T satisfying

$$r(t+T) = r(t) \quad (\forall t \geq 0). \tag{2}$$

It is assumed that $m \leq p$. The nominal plant of $G(s)$ is denoted by $G_m(s) \in R^{m\times p}(s)$. Both $G(s)$ and $G_m(s)$ are assumed to have no zero or pole on the imaginary axis. In addition, it is assumed that the number of poles of $G(s)$ in the closed right half plane is equal to that of $G_m(s)$. The relation between the plant $G(s)$ and the nominal plant $G_m(s)$ is written as

$$G(s) = (I + \Delta(s))G_m(s), \tag{3}$$

where $\Delta(s)$ is an uncertainty. The set of $\Delta(s)$ is all rational functions satisfying

$$\bar{\sigma}\left\{\Delta(j\omega)\right\} < |W_T(j\omega)| \quad (\forall \omega \in R_+), \tag{4}$$

where $W_T(s)$ is a stable rational function.

The robust stability condition for the plant $G(s)$ with uncertainty $\Delta(s)$ satisfying (4) is given by

$$\|T(s)W_T(s)\|_\infty < 1, \tag{5}$$

where $T(s)$ is the complementary sensitivity function written by

$$T(s) = (I + G_m(s)C(s))^{-1} G_m(s)C(s). \tag{6}$$

According to [3, 4, 5, 6, 7, 8, 9, 10], in order for the output y to follow the periodic reference input r with period T in (1) with small steady state error, the controller $C(s)$ must have the following structure

$$C(s) = C_1(s) + C_2(s)e^{-sT}\left(I - q(s)e^{-sT}\right)^{-1}, \tag{7}$$

where $q(s) \in R^{m\times m}(s)$ is a low-pass filter satisfying $q(0) = I$ and $\operatorname{rank} q(s) = m$, and $C_1(s) \in R^{p\times m}(s)$ and $C_2(s) \in R^{p\times m}(s)$ satisfy $\operatorname{rank} C_2(s) = m$. In the following, $e^{-sT}(I - q(s)e^{-sT})^{-1}$ defines the internal model for the periodic signal with period T. According to [3, 4, 5, 6, 7, 8, 9, 10], if the low-pass filter $q(s)$ satisfies

$$\bar{\sigma}\left\{I - q(j\omega_i)\right\} \simeq 0 \quad (\forall i = 0, \dots, N_{\max}), \tag{8}$$

where ω_i are frequency components of the periodic reference input r written by

$$\omega_i = \frac{2\pi}{T}i \quad (i = 0, \dots, N_{\max}) \tag{9}$$

and $\omega_{N_{\max}}$ is the maximum frequency component of the periodic reference input r, then the output y in (1) follows the periodic reference input r with small steady state error. The controller written by (7) is called the modified repetitive controller [3, 4, 5, 6, 7, 8, 9, 10].

Using the modified repetitive controller $C(s)$ in (7), transfer functions from the periodic reference input r to the output y and from the disturbance d to the output y in (1) are written as

$$\begin{aligned} y &= (I + G(s)C(s))^{-1}G(s)C(s)r \\ &= (I+\Delta(s))G_m(s)\left\{C_1(s) + (C_2(s) - C_1(s)q(s))e^{-sT}\right\}\left[I + (I+\Delta(s))G_m(s)C_1(s)\right. \\ &\quad \left. -\left[\left\{I + (I+\Delta(s))G_m(s)C_1(s)\right\}q(s) - (I+\Delta(s))G_m(s)C_2(s)\right]e^{-sT}\right]^{-1}r \end{aligned} \tag{10}$$

and

$$\begin{aligned} y &= (I + G(s)C(s))^{-1}d \\ &= \left(I - q(s)e^{-sT}\right)\left[I + (I+\Delta(s))G_m(s)C_1(s)\right. \\ &\quad \left. -\left[\left\{I + (I+\Delta(s))G_m(s)C_1(s)\right\}q(s) - (I+\Delta(s))G_m(s)C_2(s)\right]e^{-sT}\right]^{-1}d, \end{aligned} \tag{11}$$

respectively. Generally, transfer functions from the periodic reference input r to the output y in (10) and from the disturbance d to the output y in (11) have infinite numbers of poles, even if $\Delta(s) = 0$. When transfer functions from the periodic reference input r to the output y and from the disturbance d to the output y have infinite numbers of poles, it is difficult to specify the input-output characteristic and the disturbance attenuation characteristic. From the practical point of view, it is desirable that the input-output characteristic and the disturbance attenuation characteristic are easily specified. In order to specify the input-output characteristic and the disturbance attenuation characteristic easily, transfer functions from the periodic reference input r to the output y and from the disturbance d to the output y are desirable to have finite numbers of poles.

Definition 2.1. *(Robust stabilizing simple repetitive controller for multiple-input/multiple-output plants.) We call the controller $C(s)$ a "robust stabilizing simple repetitive controller for multiple-input/multiple-output plants", if following expressions hold true:*

1) *The controller $C(s)$ works as a modified repetitive controller. That is, the controller $C(s)$ is written by (7), where $C_1(s) \in R^{p\times m}(s)$, $C_2(s) \in R^{p\times m}(s)$ satisfy* $\operatorname{rank} C_2(s) = m$ *and $q(s) \in R^{m\times m}(s)$ satisfies $q(0) = I$ and* $\operatorname{rank} q(s) = m$.
2) *When $\Delta(s) = 0$, the controller $C(s)$ makes transfer functions from the periodic reference input r to the output y in (1) and from the disturbance d to the output y in (1) have finite numbers of poles.*
3) *The controller $C(s)$ satisfies the robust stability condition in (5).*

3. **The Parameterization of all Robust Stabilizing Simple Repetitive Controllers for Multiple-Input/Multiple-Output Plants.** In this section, we clarify the parameterization of all robust stabilizing simple repetitive controllers for multiple-input/multiple-output plants defined in Definition 2.1.

In order to obtain the parameterization of all robust stabilizing simple repetitive controllers, we must see that the controller $C(s)$ holds (5). The problem of obtaining the controller $C(s)$, which is not necessarily a simple repetitive controller, satisfying (5) is equivalent to the following H_∞ control problem. In order to obtain the controller $C(s)$ satisfying (5), we consider the control system shown in Figure 1. $P(s)$ is selected such that the transfer function from w to z in Figure 1 is equal to $T(s)W_T(s)$. The state space description of $P(s)$ is, in general,

$$\left\{\begin{array}{l} \dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t) \\ z(t) = C_1 x(t) + D_{12} u(t) \\ y(t) = C_2 x(t) + D_{21} w(t) \end{array}\right. , \tag{12}$$

where $A \in R^{n\times n}$, $B_1 \in R^{n\times r}$, $B_2 \in R^{n\times p}$, $C_1 \in R^{q\times n}$, $C_2 \in R^{m\times n}$, $D_{12} \in R^{q\times p}$, $D_{21} \in R^{m\times r}$, $x(t) \in R^n$, $w(t) \in R^r$, $z(t) \in R^q$, $u(t) \in R^p$ and $y(t) \in R^m$. $P(s)$ is called the generalized plant. $P(s)$ is assumed to satisfy the following assumptions [21].

1) (C_2, A) is detectable, and (A, B_2) is stabilizable.
2) D_{12} has full column rank, and D_{21} has full row rank.
3) rank $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} = n + p \ (\forall\omega \in R_+)$,
   rank $\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} = n + m \ (\forall\omega \in R_+)$.

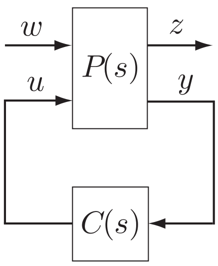

FIGURE 1. Block diagram of H_∞ control problem

Under these assumptions, the parameterization of all robust stabilizing simple repetitive controllers for multiple-input/multiple-output plants is given by the following theorem.

Theorem 3.1. *If simple repetitive controllers satisfying (5) exist, both*

$$X\left(A - B_2 D_{12}^{\dagger} C_1\right) + \left(A - B_2 D_{12}^{\dagger} C_1\right)^T X + X\left\{B_1 B_1^T - B_2\left(D_{12}^T D_{12}\right)^{-1} B_2^T\right\} X + \left(D_{12}^{\perp} C_1\right)^T D_{12}^{\perp} C_1 = 0 \tag{13}$$

and

$$Y\left(A - B_1 D_{21}^{\dagger} C_2\right)^T + \left(A - B_1 D_{21}^{\dagger} C_2\right) Y + Y\left\{C_1^T C_1 - C_2^T\left(D_{21} D_{21}^T\right)^{-1} C_2\right\} Y + B_1 D_{21}^{\perp}\left(B_1 D_{21}^{\perp}\right)^T = 0 \tag{14}$$

have solutions $X \geq 0$ and $Y \geq 0$ such that

$$\rho(XY) < 1 \tag{15}$$

and both

$$A - B_2 D_{12}^{\dagger} C_1 + \left\{ B_1 B_1^T - B_2 \left(D_{12}^T D_{12} \right)^{-1} B_2^T \right\} X \tag{16}$$

and

$$A - B_1 D_{21}^{\dagger} C_2 + Y \left\{ C_1^T C_1 - C_2^T \left(D_{21} D_{21}^T \right)^{-1} C_2 \right\} \tag{17}$$

have no eigenvalue in the closed right half plane. Using X *and* Y*, the parameterization of all robust stabilizing simple repetitive controllers satisfying (5) is given by*

$$\begin{aligned} C(s) &= \Big(Z_{11}(s) Q(s) + Z_{12}(s) \Big) \Big(Z_{21}(s) Q(s) + Z_{22}(s) \Big)^{-1} \\ &= \Big(Q(s) \tilde{Z}_{21}(s) + \tilde{Z}_{22}(s) \Big)^{-1} \Big(Q(s) \tilde{Z}_{11}(s) + \tilde{Z}_{12}(s) \Big), \end{aligned} \tag{18}$$

where $Z_{ij}(s)$ $(i = 1, 2; j = 1, 2)$ *and* $\tilde{Z}_{ij}(s)$ $(i = 1, 2; j = 1, 2)$ *are written by*

$$\begin{bmatrix} Z_{11}(s) & Z_{12}(s) \\ Z_{21}(s) & Z_{22}(s) \end{bmatrix} = \begin{bmatrix} C_{12}(s) - C_{11}(s) C_{21}^{-1}(s) C_{22}(s) & C_{11}(s) C_{21}^{-1}(s) \\ -C_{21}^{-1}(s) C_{22}(s) & C_{21}^{-1}(s) \end{bmatrix} \tag{19}$$

and

$$\begin{bmatrix} \tilde{Z}_{11}(s) & \tilde{Z}_{12}(s) \\ \tilde{Z}_{21}(s) & \tilde{Z}_{22}(s) \end{bmatrix} = \begin{bmatrix} C_{21}(s) - C_{22}(s) C_{12}^{-1}(s) C_{11}(s) & C_{12}^{-1}(s) C_{11}(s) \\ -C_{22}(s) C_{12}^{-1}(s) & C_{12}^{-1}(s) \end{bmatrix}, \tag{20}$$

and satisfy

$$\begin{aligned} & \begin{bmatrix} \tilde{Z}_{22}(s) & \tilde{Z}_{12}(s) \\ \tilde{Z}_{21}(s) & \tilde{Z}_{11}(s) \end{bmatrix} \begin{bmatrix} Z_{11}(s) & -Z_{12}(s) \\ -Z_{21}(s) & Z_{22}(s) \end{bmatrix} \\ &= I \\ &= \begin{bmatrix} Z_{11}(s) & -Z_{12}(s) \\ -Z_{21}(s) & Z_{22}(s) \end{bmatrix} \begin{bmatrix} \tilde{Z}_{22}(s) & \tilde{Z}_{12}(s) \\ \tilde{Z}_{21}(s) & \tilde{Z}_{11}(s) \end{bmatrix}, \end{aligned} \tag{21}$$

$C_{ij}(s)$ $(i = 1, 2; j = 1, 2)$ *are given by*

$$\begin{bmatrix} C_{11}(s) & C_{12}(s) \\ C_{21}(s) & C_{22}(s) \end{bmatrix} = \left[\begin{array}{c|cc} A_c & B_{c1} & B_{c2} \\ \hline C_{c1} & D_{c11} & D_{c12} \\ C_{c2} & D_{c21} & D_{c22} \end{array} \right], \tag{22}$$

$$\begin{aligned} A_c = \; & A + B_1 B_1^T X - B_2 \left(D_{12}^{\dagger} C_1 + E_{12}^{-1} B_2^T X \right) \\ & - (I - YX)^{-1} \left(B_1 D_{21}^{\dagger} + Y C_2^T E_{21}^{-1} \right) \left(C_2 + D_{21} B_1^T X \right), \end{aligned}$$

$$B_{c1} = (I - YX)^{-1} \left(B_1 D_{21}^{\dagger} + Y C_2^T E_{21}^{-1} \right),$$

$$B_{c2} = (I - YX)^{-1} \left(B_2 + Y C_1^T D_{12} \right) E_{12}^{-1/2},$$

$$C_{c1} = -D_{12}^{\dagger} C_1 - E_{12}^{-1} B_2^T X, \; C_{c2} = -E_{21}^{-1/2} \left(C_2 + D_{21} B_1^T X \right),$$

$$D_{c11} = 0, \;\; D_{c12} = E_{12}^{-1/2}, \;\; D_{c21} = E_{21}^{-1/2}, \;\; D_{c22} = 0,$$

$$E_{12} = D_{12}^T D_{12}, \;\; E_{21} = D_{21} D_{21}^T$$

and $Q(s) \in H_\infty^{p \times m}$ *is any function satisfying* $\|Q(s)\|_\infty < 1$ *and written by*

$$Q(s) = \left(Q_{n1}(s) + Q_{n2}(s) e^{-sT} \right) \left(Q_{d1}(s) + Q_{d2}(s) e^{-sT} \right)^{-1}, \tag{23}$$

$$Q_{n2}(s) = G_{2d}(s) \bar{Q}(s) \in RH_\infty^{p \times m} \tag{24}$$

and

$$Q_{d2}(s) = -G_{1d}(s)G_{2n}(s)\bar{Q}(s) \in RH_\infty^{m\times m}. \tag{25}$$

Here, $G_{1n}(s) \in RH_\infty^{m\times m}$, $G_{1d}(s) \in RH_\infty^{m\times m}$, $G_{2n}(s) \in RH_\infty^{m\times p}$ *and* $G_{2d}(s) \in RH_\infty^{p\times p}$ *are coprime factors satisfying*

$$Z_{22}(s) + G_m(s)Z_{12}(s) = G_{1n}(s)G_{1d}^{-1}(s) \tag{26}$$

and

$$G_{1n}^{-1}(s)\left(Z_{21}(s) + G_m(s)Z_{11}(s)\right) = G_{2n}(s)G_{2d}^{-1}(s). \tag{27}$$

$Q_{n1}(s) \in RH_\infty^{p\times m}$, $Q_{d1}(s) \in RH_\infty^{m\times m}$ *and* $\bar{Q}(s) \in RH_\infty^{p\times m}$ *are any functions satisfying*

$$\bar{\sigma}\left\{Z_{22}(0)\left(Q_{d1}(0) + Q_{d2}(0)\right) + Z_{21}(0)\left(Q_{n1}(0) + Q_{n2}(0)\right)\right\} = 0, \tag{28}$$

$$\text{rank }\left(Q_{n2}(s) - Q_{n1}(s)Q_{d1}^{-1}(s)Q_{d2}(s)\right) = m \tag{29}$$

and rank $\bar{Q}(s) = m$.

4. **Numerical Example.** In this section, a numerical example is illustrated to show the effectiveness of the proposed approach.

Consider the problem to design a robust stabilizing modified repetitive controllers for the set of plants $G(s)$ written by (3), where

$$G_m(s) = \begin{bmatrix} \dfrac{s+3}{(s-2)(s+9)} & \dfrac{2}{(s-2)(s+9)} \\ \dfrac{s+3}{(s-2)(s+9)} & \dfrac{s+4}{(s-2)(s+9)} \end{bmatrix} \tag{30}$$

and

$$W_T(s) = \frac{s+400}{550}. \tag{31}$$

The period T of the periodic reference input r is given by $T = 4$[sec].

Using Theorem 3.1, we have the parameterization of all robust stabilizing simple repetitive controllers. We settle the parameters in (23) as $Q_{d1}(s) = I$ and $Q_{n1}(s) = -500I$. In addition, $Q_{n2}(s)$ and $Q_{d2}(s)$ are settled by (24) and (25), where $\bar{Q}(s)$ is given by

$$\bar{Q}(s) = H_o^\dagger(s)\bar{q}_r(s)\left(Q_{d1}(s) - C_{22}(s)Q_{n1}(s)\right) \tag{32}$$

where $H_o \in RH_\infty^{p\times m}$ is an outer function of $H(s)$ written by

$$H(s) = G_{1d}(s)G_{2n}(s) + C_{22}(s)G_{2d}(s) \tag{33}$$

satisfying

$$H(s) = H_i(s)H_o(s), \tag{34}$$

$H_o^\dagger(s)$ is pseudo inverse of $H_o(s)$ satisfying $H_o^\dagger(s)H_o(s) = I$ and $\bar{q}_r(s)$ is a low-pass filter written by

$$\bar{q}_r(s) = \begin{bmatrix} \dfrac{1}{0.01s+1} & 0 \\ 0 & \dfrac{1}{0.01s+1} \end{bmatrix}. \tag{35}$$

Using above-mentioned parameters, we have a robust stabilizing modified repetitive controller. When $\Delta(s)$ is written by

$$\Delta(s) = \begin{bmatrix} \dfrac{s-100}{s+500} & \dfrac{-100}{s+600} \\ \dfrac{-200}{s+500} & \dfrac{s-100}{s+600} \end{bmatrix} \tag{36}$$

and the designed robust stabilizing modified repetitive controller $C(s)$ is used, the tracking error $e(t) = r(t) - y(t)$ in (1) for the periodic reference inputs r

$$r(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} = \begin{bmatrix} \sin\left(\dfrac{\pi t}{2}\right) \\ 2\sin\left(\dfrac{\pi t}{2}\right) \end{bmatrix} \tag{37}$$

is shown in Figure 2. Here, the broken line shows the response of the periodic reference input $r_1(t)$, the dotted line shows that of the periodic reference input $r_2(t)$, the solid line shows that of the error $e_1(t)$ and the dotted and broken line shows that of the error $e_2(t)$. Figure 2 shows that the output y follows the periodic reference input r with small steady state error, even if the plant has uncertainty $\Delta(s)$.

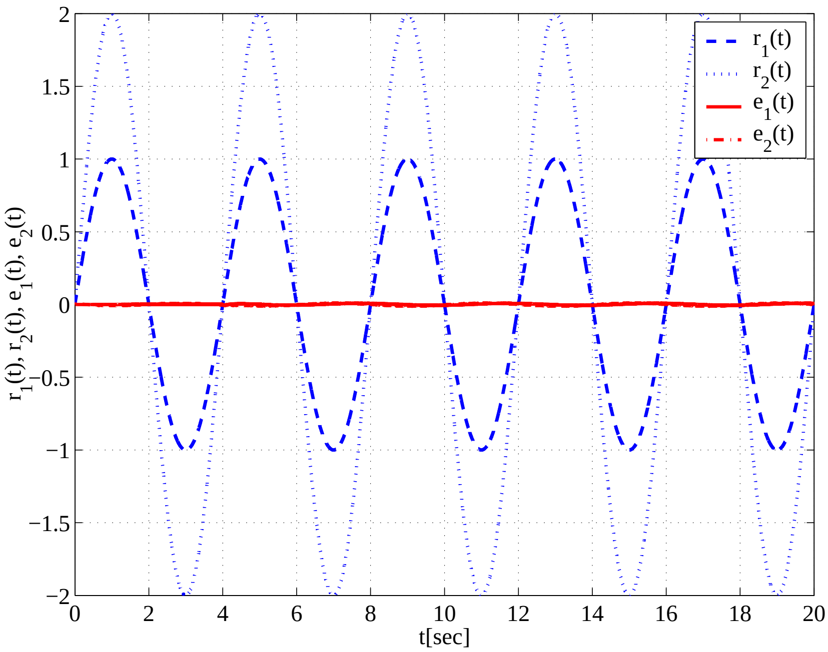

FIGURE 2. Response of the error $e(t)$ for the reference input $r(t)$

In this way, we find that it is easy to design a robust stabilizing simple repetitive controller using Theorem 3.1.

5. **Conclusions.** In this paper, we proposed the parameterization of all robust stabilizing simple repetitive controllers for multiple-input/multiple-output plants such that the controller works as a robust stabilizing modified repetitive controller and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles. Since the robust stabilizing simple repetitive control system has merits, for example, the stability of control system with uncertainty is guaranteed and the robust stabilizing simple repetitive control system can be easily designed, the practical application of the robust stabilizing simple repetitive control is expected.

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