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NOVEL THEORETICAL ANALYSES OF TRANSVERSE RESONANCE TECHNIQUE ON LOADED IMPEDANCE WAVEGUIDE

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Abstract. *A novel theoretical analysis of transmission characteristics of waveguide systems by using transverse resonance technique (TRT) is presented. The purpose of this study is to acquire a method which could simply analyze the propagation parameters of the slotted waveguide which is loaded with discrete impedance. The mathematical model is based on the equivalent transverse resonance circuit, in order to define the propagation condition, the fast/slow modes and losses in the fundamental mode. The result shows that the slotted waveguide will lose the fundamental mode if the loaded capacitive impedance is higher than 77pF/m. The different phase constant pattern of slow mode causes the fundamental mode to converge to the unusual direction and the loss increases rapidly. This novel technique has the advantage of simplicity and compares well with results of electromagnetic simulation and measurement.*

Keywords: Transverse resonance technique, Substrate integrated waveguide, Fast mode, Slow mode, Loss

1. **Introduction.** The transverse resonance technique (TRT) has been used to analyze the communication system of the waveguide for many years. It can easily determine the propagation constant by solving the transmission line equivalent circuit, which is based on the cutoff situation of the waveguide.

As the development of the waveguide, this technique is also used to analyze substrate integrated waveguide (SIW) with embedding other components. Lots of the studies based on the full-mode SIW integrated with periodic components are proposed [1] in antennas [2,3], filters [4,5], transverse electromagnetic (TEM) waveguides [6], metamaterial structures [7], miniaturized waveguides and divider [8,9], high quality attenuators [10], half-mode SIW [11], etc. Besides, the mathematical research based on TRT designs a wideband antenna which has an effective half wavelength resonance within a cavity partially loaded with an anisotropic medium [12]. Because of the effective analysis on equivalent circuits, the TRT technique becomes a simpler approach to simulate the characteristics of the guided-wave structures under the resonance condition.

In this paper, a novel theoretical method and mathematics analysis have been demonstrated to model the *TE* (Transverse Electric) mode transmission of a loaded impedance waveguide. It transfers the complex electromagnetic situations to direct mathematical solutions. Section 2 indicates how the loaded impedance affects the TE modes of slotted

SIW under the transverse resonance condition. Section 3 analyzes the fast/slow modes in the fundamental mode. The comparisons between inference and simulation results are presented as well. Section 4 presents the theoretical calculations of loss in fast/slow modes. Finally, Section 5 concludes the paper.

2. **The Analysis of Loaded Impedance Condition.** The slotted SIW has been chosen in this research. There is a slot d along the top layer of the SIW, which can be loaded with impedance Z easily. Figure 1(a) shows the cross section of the slotted SIW model. The a and b dimension has been well designed and RT/duriod 5870 is chosen to be the dielectric material, which has the dielectric constant $\varepsilon_r = 2.33$. Figure 1(b) shows its equivalent transverse resonance circuit mode. Impedance Z occupies the slot d between the long side L_1 and short side L_2, in which $L_1 + d + L_2 = a$.

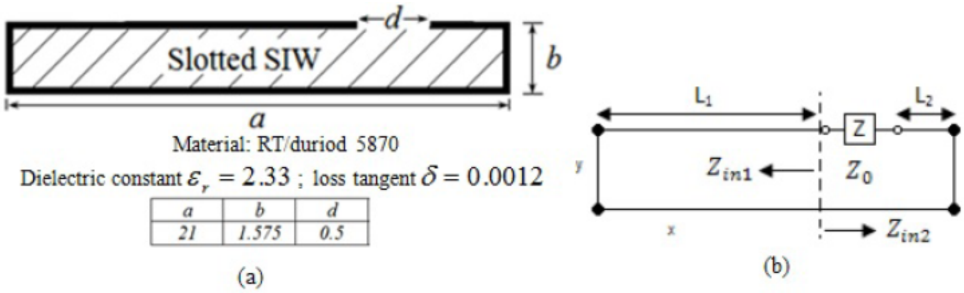

FIGURE 1. (a) Cross section of the slotted SIW (unit: mm); (b) equivalent transverse resonance circuit mode

The input impedance (Z_{in}) of a terminated transmission line with the load end Z_L is: $Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan k_x L}{Z_0 + jZ_L \tan k_x L}$, Z_0 is the impedance characteristics and L is length of the transmission line. Here the wavevector in the waveguide is $k_x = \omega\sqrt{\mu\varepsilon}$. While in the slotted SIW, the load end is the short circuit and $Z_L = 0$. Therefore, $Z_{in} = jZ_0 \tan k_x L$. In Figure 1(b), the resonance condition need satisfy $Z_{in1} = -Z_{in2}$ [13] at the dash line; now in our circuit, the case becomes:

$$jZ_0 \tan k_x L_1 = -(Z + jZ_0 \tan k_x L_2) \quad (1)$$

When considering TE mode problems using the transverse resonance technique the characteristic impedance is given by $Z_0 = E_y/H_z$ [12] and the equivalent voltage (V) and current (I) are set equal to the electric (E_y) and magnetic (H_z) fields respectively. However, because the discrete loaded components will be embedded into the waveguide along the slot, we need to ensure that the equivalent waveguide voltage and current value (and therefore Z_0) are compatible with the voltage and current presented to the embedded impedance Z. If the electric field is independent of y then the voltage at the impedance will be $E_y b$, where b is the height of the waveguide. Hence, above cut-off, we choose $Z_0 = \frac{bV}{I} = \frac{b\omega\mu}{k_x}$, ω is the working frequency and μ is the relative magnetic permeability of the dielectric material.

We consider the loaded impedance Z as discrete capacitance C, which is the capacitance per unit length. Thus, at the resonant point of slotted SIW while setting $\theta = k_x L_1$, $R = L_2/L_1$, $X = -1/\omega^2 \mu b L_1 C$, Equation (1) can be written as:

$$\tan\theta + \tan R\theta = -X\theta \quad (2)$$

For a given frequency ω, Figure 2 shows the plot of Equation (2) with the first solution, the x-axis is the defined θ and the y-axis can be considered as the multiple of C. The variable L_1 is 15mm and L_2 is 5.5mm. As expected, there are a number of solutions to Equation (2). Each solution is corresponding to the order of TE modes. In the graph, we can see that the slope of the $-X\theta$ line will become small if $-X$ is small. The value

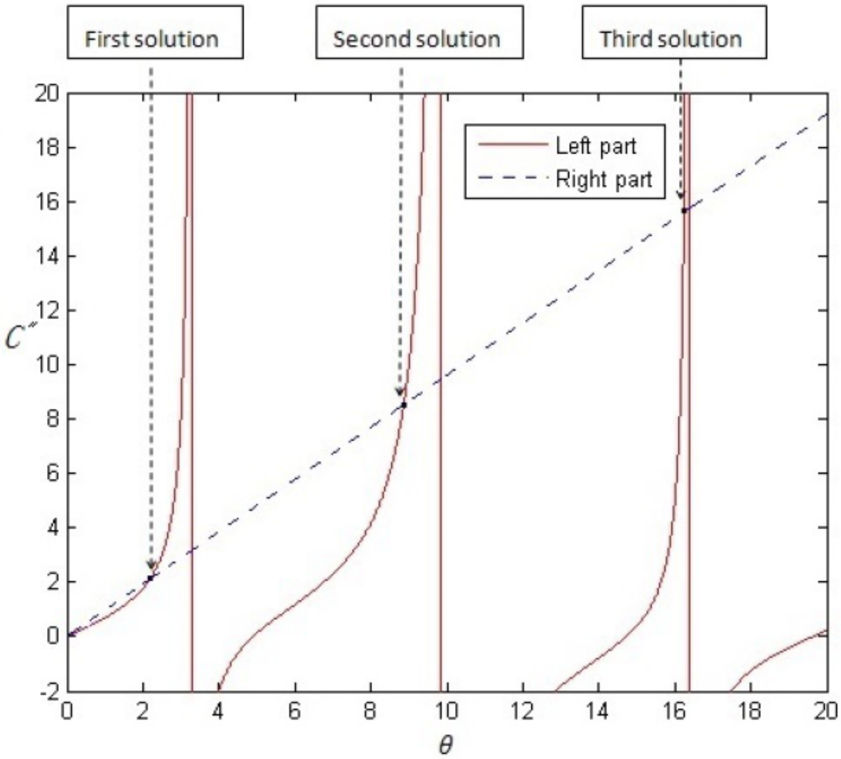

Figure 2. Plot of Equation (2) with the first root

of $-X$ is determined by C. It means if the capacitance we add on the slotted SIW is too large, the fundamental mode will disappear.

At the fundamental mode, the value of the x-axis (θ) is quite small, and we can approximate $\tan\theta$ as: $\tan\theta \approx \theta$. So the first cross point in the graph only occurs when this condition is satisfied: $\theta(1+R) < -X\theta$. Substituting R and X, we get the condition of $C < \frac{1}{\omega^2 \mu b(L_1+L_2)}$, which means that the fundamental TE mode only occurs under a limited capacitance range. In this case, the slotted SIW has no fundamental mode unless the discrete capacitance C is in the range of 0∼77pF/m, and the working frequency is at 2GHz.

3. **The Phase Dispersion Results of Two Modes.** The loaded impedance leads to two types of propagation which can be categorized as fast and slow modes, where fast and slow are defined with respect to the velocity of propagation in the substrate material.

In fast mode propagation, the field varies sinusoidally across the waveguide cross section, as the case in conventional waveguide. The phase constant β is related to k_x, which is $\beta = \sqrt{\varepsilon_r k_0^2 - k_x^2}$, and k_0 is the wave number of free space.

The slow mode propagation is another class of solution that is unusual. In slow mode propagation, the fields will vary as combinations of exponential functions. Under this condition Equation (2) becomes: $\tan(-j\theta') + \tan(-jR\theta') + jX\theta' = 0$, where we let $\theta' = \alpha_x L_1$, α_x is the attenuation constant of the transverse direction, and the slow mode solution is governed. Thus, impedance Z must be capacitive with $C > \frac{1}{\omega^2 \mu b(L_1+L_2)}$ and the phase constant is given by $\beta = \sqrt{\varepsilon_r k_0^2 + \alpha_x^2}$. Therefore, the transition frequency point f_t between slow and fast is:

$$f_t = \frac{1}{2\pi\sqrt{\mu C b(L_1+L_2)}} \tag{3}$$

Thus, in the fundamental mode, theoretically, the fast mode propagates below f_t and above frequency f_t, the propagation will be in the slow mode region. The impedance Z we used is purely capacitive with value of 60pF/m. The dispersion curves of each TE

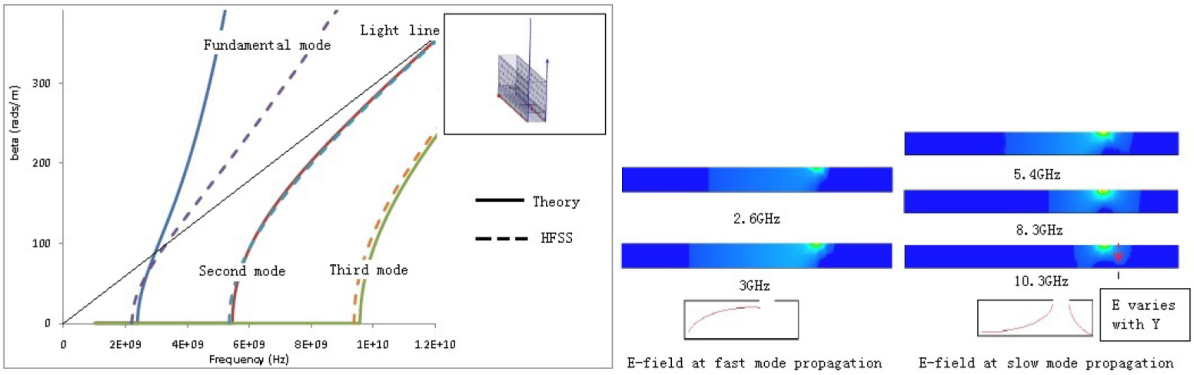

FIGURE 3. The dispersion curves and E-field variations at different frequency points (solid line-theoretical results, dash line-HFSS results)

mode and field variations at different frequency points are shown in Figure 3 by using theoretical analyses and HFSS (High Frequency Structure Simulator). The structure with periodic boundary conditions is shown as well.

In the fundamental mode of both results, the fast mode propagates below the light line. It is clear that the E-field keeps the half mode distribution, which is like the sinusoidally function. Above the light line, the slow mode propagation occurs, and the electric fields start to concentrate around the slot like the hyperbolic sine function. The higher order modes compare very well in both methods. The fundamental *TE* mode, however, does not compare very well. In the theoretical analysis, the affection only occurs by the impedance Z between the slot as a capacitance. However, in HFSS, there are other unavoidable effects to the circuit, such as the slot capacitance, and the vertical capacitances of the waveguide. These will slightly change the actual capacitance contribution. That is one reason why the curves in this mode have different slopes.

Another reason is that in the simulation environment, the electric field of slow mode does vary along Y direction in the field distribution. However, in the theory of TRT, we assume that the E-field has no variation with Y, even though, both solution curves in the fundamental mode follow the same shape and demonstrate slow mode propagation.

Using variables given above to calculate the transition point between slow and fast mode is: $f_t \approx 3.2$GHz, which approximately matches with both results. This result shows that the existence of loaded capacitive impedance makes the fundamental *TE* mode only propagate in a limited range (2.4GHz~3.2GHz) and then turn to the slow mode. The different phase constant pattern of slow mode causes the fundamental mode to converge to the unusual direction.

4. **Loss Analysis Results.** The loss is mainly caused by the resistor of impedance Z in the circuit, which affects the attenuation constant. Especially in the slow mode propagation, the loss is much higher than that of fast mode.

We let the component Z include the series resistor r and reactance X. Therefore, written as the impedance form, Z is complex and $Z = r + jX$.

Losses in fast mode

Here we set a function $f(k_x) = \frac{1}{k_x}(\tan k_x L_1 + \tan k_x L_2)$. Substituting $Z_0 = b\omega\mu/k_x$ into Equation (1), we get $f(k_x) = -\frac{z}{jb\omega\mu} = jZ' = j(r' + jX')$, where $Z' = \frac{z}{b\omega\mu}$, $r' = \frac{r}{b\omega\mu}$, $X' = \frac{X}{b\omega\mu}$. Because loaded impedance Z is complex, k_x must also be complex. Then we replace k_x with $k_x - j\alpha_x$. Now, $f(k_x - j\alpha_x) = j(r' + jX') = jr' - X'$. If α_x is small, we can use the Taylor series to expand this function as: $f(k_x - j\alpha_x) \approx f(k_x) - j\alpha_x f'(k_x)$ + higher order terms, where the derivative value $f'(k_x) = -\frac{1}{k_x^2}(\tan k_x L_1 + \tan k_x L_2) + \frac{1}{k_x}(L_1 \sec^2 k_x L_1 + L_2 \sec^2 k_x L_2)$. If we only take the first term as the approximate value of the k_x function, we will get $f(k_x) - j\alpha_x f'(k_x) = jr' - X'$. Hence, equating the real and imaginary parts yields $\alpha_x = -\frac{r'}{f'(k_x)}$. If we set $X'' = \frac{X'}{L_1}$, $r'' = \frac{r'}{L_1}$, $\varphi = \frac{L_2}{L_1}$, the α_x can be expressed as:

$$\alpha_x = \frac{r'' k_x}{(X'' + \sec^2 k_x L_1 + \varphi \sec^2 k_x L_2)} \tag{4}$$

The propagation constant can be expressed as $\gamma = j\beta(k_x)$ when the loss is zero and the phase constant is $\beta(k_x)$, where $\beta(k_x) = \sqrt{\varepsilon_r k_0^2 - k_x^2} = \sqrt{\varepsilon_r} k_0 \left(1 - \frac{k_x^2}{\varepsilon_r k_0^2}\right)^{\frac{1}{2}}$. And its derivative value is $\beta'(k_x) = \frac{\sqrt{\varepsilon_r} k_0}{2} \left(\frac{-2k_x}{\varepsilon_r k_0^2}\right) \left(1 - \frac{k_x^2}{\varepsilon_r k_0^2}\right)^{-\frac{1}{2}} = -\frac{k_x}{\beta(k_x)}$. The phase constant expression with the loss is $\beta(k_x - j\alpha_x) = \beta(k_x) - j\alpha_x \beta'(k_x)$ + higher order terms. It is assumed that the higher order terms are not considered, and the phase expression with the loss will be: $\beta(k_x - j\alpha_x) = \beta(k_x) + j\alpha$. So the attenuation constant of fast mode will be:

$$\alpha = \frac{\alpha_x k_x}{\beta(k_x)} \tag{5}$$

Losses in slow mode

For the slow mode case, a similar analysis yields $k_x = -\frac{r'' \alpha_x}{(X'' + \sec^2 \alpha_x L_1 + \varphi \sec^2 \alpha_x L_2)}$ and the phase constant is $\beta = \sqrt{\varepsilon_r k_0^2 + \alpha_x^2}$. The attenuation constant α of slow mode propagation becomes:

$$\alpha = \frac{\alpha_x k_x}{\sqrt{\varepsilon_r k_0^2 + \alpha_x^2}} \tag{6}$$

Figure 4 shows the theoretical attenuation plots of Equations (5) and (6). The series resistance is 0.01125Ω·m. The solid lines are the fast modes, and the dash line is the slow mode. It is clear that the attenuation constant increased rapidly in slow mode range above the transition frequency point. It causes a dip between the first and second modes. At 3GHz, the fundamental mode has about 5dBs/m loss; the second mode has about 3dBs/m loss at 8GHz.

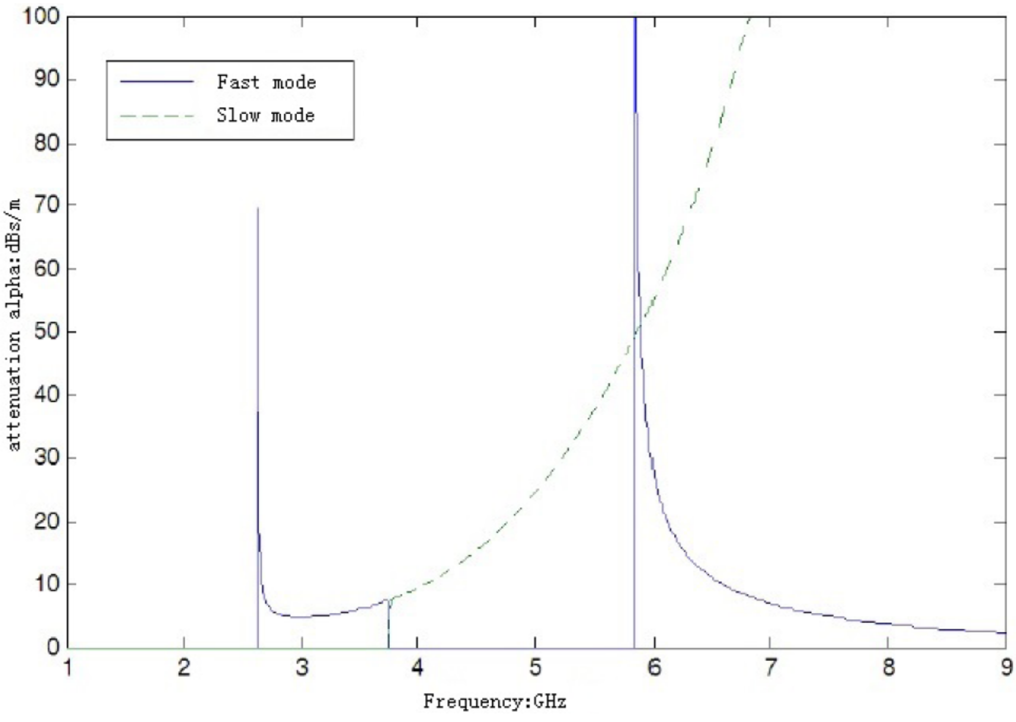

Figure 4. The theoretical attenuation constants (series resistance $r = 0.01125\Omega\cdot$m)

5. **Conclusions.** In this paper, based on the transverse resonance technique (TRT), we introduced a novel mathematics method to model the slotted substrate integrated waveguide with loaded components. For capacitive loaded circuit, the fundamental *TE* mode may disappear while the capacitance per unit is too large. Furthermore, the governing equations and the transition point between two types of propagations – the fast and slow modes in the loaded waveguide are discussed. Both theoretical and simulation results match well. The different phase constant pattern causes the fundamental mode to converge to the unusual direction and the loss increases significantly in the slow mode that leads to a frequency dip during the transmission.

This novel method can define the properties of the signal propagation theoretically, and give a promising technique for analyzing other communication systems. The next research plan is to utilize this method to design a flexible guide-wave structure which is loaded with adjustable impedances.

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