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A 3-PRRU TRANSLATIONAL PARALLEL MECHANISM WITH CONSTANT JACOBI MATRIX AND ITS TRANSMISSIBILITY PERFORMANCE ANALYSIS

Yanzhi Zhao[1,2,*], Bowen Liang[1,2], Yachao Cao[1,2], Shuai Li[1,2] and Rui Han[1,2]

[1]Key Laboratory of Parallel Robot and Mechatronic System of Hebei Province
[2]Key Laboratory of Advanced Forging & Stamping Technology and Science
of Ministry of Education of China
Yanshan University
No. 438, Hebei Avenue, Qinhuangdao 066004, P. R. China
*Corresponding author: yzzhao@ysu.edu.cn

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Abstract. *A novel 3-PRRU translational parallel mechanism is proposed, whose Jacobi matrix is constant on condition that the prismatic pairs are actuators. The degrees of freedom (DOFs) of the parallel mechanism are analyzed based on screw theory. A method that can solve Jacobi matrix of the mechanism by the main twist screw (MTS) and transmission wrench screw (TWS) is proposed. It can be seen that curves of the output speed/force of the mechanism are coincident in different poses when the input parameters are definite. Consequently, the conclusion that Jacobi matrix of the 3-PRRU parallel mechanism keeps constant is verified. On the basis of this, the transmissibility performance of the mechanism is further analyzed, obtaining the relationship between transmission power and angle β (an intrinsic parameter of the mechanism) of each limb. The conclusion is: when the input power is definite and invariant, the transmission power of the mechanism decreases with β increasing. The transmissibility performance of the mechanism is optimal as $\beta = 0°$.*
Keywords: Parallel mechanism, Screw theory, Jacobi matrix, The transmissibility performance

1. **Introduction.** Jacobi matrix of the parallel mechanism expresses a mapping between input to output. For most of parallel mechanisms, it varies in different poses. If the matrix can keep constant invariably, namely, the mapping between input to output stays invariant, it will have a vital significance to improve control accuracy for mechanism. Performance evaluation of parallel mechanism is closely related to Jacobi matrix. When Jacobi matrix is constant, the global performance of mechanism is constant. Consequently, constant Jacobi matrix will bring a multitude of convenience to research and control for mechanism.

The translational parallel mechanism [1,2] is widely applied in aviation, medical science, bioengineering and other fields. A 3-DOF translational parallel mechanism – DELTA, invented by Calvel in 1988, has been successfully applied in industrial production. Based on screw theory, S. A. Joshi and L. W. Tsai [3] proposed a method that can solve Jacobi matrix of the limited-DOF parallel mechanism. Y. Zhang et al. [4] invented a new type of 3-CRP translational parallel mechanism and analyzed its motion performance. H. Shen et al. [5] proposed a translational parallel mechanism and did some analyses of its position. However, these mechanisms are without the characteristic that Jacobi matrix is constant. In the field of parallel mechanism transmissibility performance, the dimensional optimization was carried out by X. Liu et al. [6,7] based on the motion/force evaluation index. X. Chen et al. [8] proposed a method that can solve the maximum transmission power of the mechanism in the reachable pose. All literature mentioned provides us some references of the analysis of transmissiblity performance. However, no literature has ever been found which analyzes performance of the mechanism from the point of constant Jacobi matrix by now. In this paper, the power and speed/force transmission performances of a novel 3-PRRU translational parallel mechanism with constant Jacobi matrix are researched.

The reminder of this paper is organized as follows. Section 2 presents a novel 3-PRRU translational parallel mechanism. Then, the DOFs of the mechanism are analyzed based on screw theory. The condition that Jacobi matrix keeps constant is introduced in Section 3. The relationship between the power and β is discussed. The speed/force transmission performances are also researched in Section 4. Conclusions are given in the last section.

2. A Novel 3-PRRU Parallel Mechanism.

2.1. Structure and constraint characteristics. The sketch of 3-PRRU parallel mechanism is illustrated in Figure 1, where axial directions of the three revolute pairs are parallel and the angle between axial direction of the revolute pair and that of prismatic pair refers to β. The prismatic pairs move along the slide rails on the fixed base, and the angles between three slide rails and the fixed base all stand for α. Points A_i, B_i, C_i, D_i ($i = 1, 2, 3$) represent centers of the motion pairs. A fixed coordinate system named $\{O\} : O - XYZ$ is attached to the center point O of the fixed base. Its Z-axis is normal of direction of the fixed base plane, its Y-axis is perpendicular to a line that is formed by the third slide rails of lines projected into the fixed base, and X-axis is determined by the right-hand rule.

A moving coordinate system named $\{P\} : P - xyz$ is attached to the center point P of center of the equilateral triangle. The directions of the z-axis, y-axis, and x-axis are respectively identical with that of the Z-axis, Y-axis and X-axis. Local coordinate systems $\{B_i\} : B_i - x_iy_iz_i$ ($i = 1, 2, 3$) are created through the points B_i ($i = 1, 2, 3$), where the z_i-axis is along the bar, pointing at B_i, the y_i-axis is along the axial direction of the revolute pairs B_i, pointing at O_i and the z_i-axis is determined by the right-hand rule.

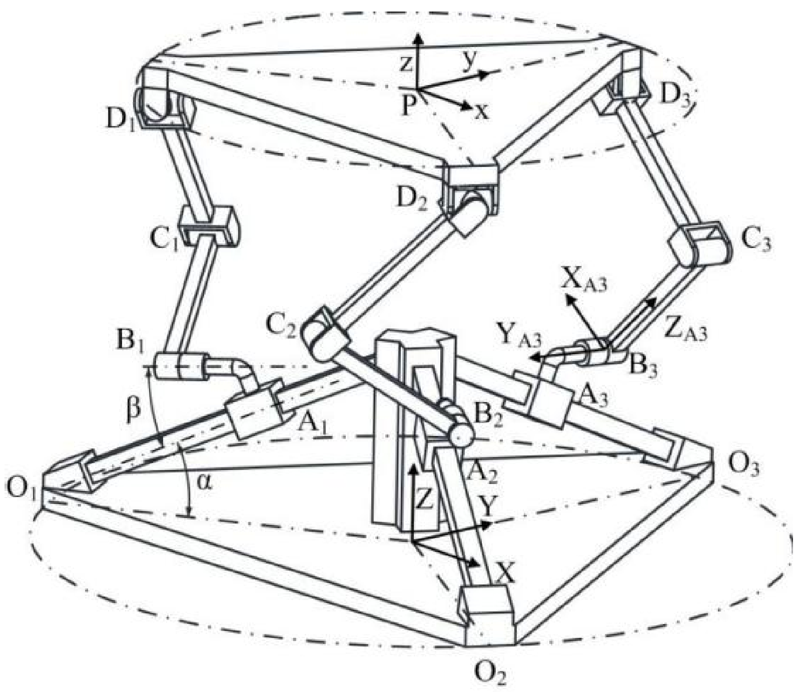

Figure 1. The sketch of 3-PRRU parallel mechanism

2.2. **Analysis of DOF.** The twist screws of the ith limb can be written in the local coordinate system as

$$\begin{cases} \boldsymbol{\$}_{i1} = \begin{pmatrix} 0 & 0 & 0; & 0 & \cos\beta & \sin\beta \end{pmatrix} \\ \boldsymbol{\$}_{i2} = \begin{pmatrix} 0 & 1 & 0; & 0 & 0 & 0 \end{pmatrix} \\ \boldsymbol{\$}_{i3} = \begin{pmatrix} 0 & 1 & 0; & P_{i3} & 0 & R_{i3} \end{pmatrix} \\ \boldsymbol{\$}_{i4} = \begin{pmatrix} 0 & 1 & 0; & P_{i4} & 0 & R_{i4} \end{pmatrix} \\ \boldsymbol{\$}_{i5} = \begin{pmatrix} 1 & 0 & 0; & 0 & Q_{i5} & 0 \end{pmatrix} \end{cases} \tag{1}$$

Solve reciprocal screw for the twist screws. Then the constraint screws can be obtained as

$$\boldsymbol{\$}_{i1}^{r} = \begin{pmatrix} 0 & 0 & 0; & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

Screw $\boldsymbol{\$}_{i1}^{r}$ is passing through the origin and along the Z_i axis. It is constraint couple that restricts the moving platform to rotate around the Z_i axis. The number of linear independence couples is three. Consequently, the constraint screws restrict the moving platform to rotate around the coordinate axes. That is to say, a 3-PRRU mechanism has three translational DOFs. According to the theory of selecting actuating components of spatial parallel mechanisms [9], it is reasonable that prismatic pairs are chosen as actuators.

3. **Jacobi Matrix.** Jacobi matrix of parallel mechanism denotes a transmissibility relationship between the velocity/force in operation space and joint space. It is a basis of performance analysis and evaluation for parallel mechanism. Based on screw theory, equation to solve the Jacobi matrix [3] of 3-PRRU parallel mechanism can be written as:

$$J = J_m^{-1} J_n \tag{3}$$

$$J_m = \begin{bmatrix} L_{a1} & M_{a1} & N_{a1} \\ L_{a2} & M_{a2} & N_{a2} \\ L_{a3} & M_{a3} & N_{a3} \end{bmatrix} \tag{4}$$

$$J_n = \begin{bmatrix} \boldsymbol{\$}_{b1} \circ \boldsymbol{\$}_{a1} & 0 & 0 \\ 0 & \boldsymbol{\$}_{b2} \circ \boldsymbol{\$}_{a2} & 0 \\ 0 & 0 & \boldsymbol{\$}_{b3} \circ \boldsymbol{\$}_{a3} \end{bmatrix} \tag{5}$$

where (L_{ai}, M_{ai}, N_{ai}) represents axial direction of the TWS of the ith limb. $\boldsymbol{\$}_{bi}$ denotes the TWS of the ith limb. $\boldsymbol{\$}_{ai}$ stands for the MTS of the ith limb.

Choose the prismatic pairs as the actuators. As it is fixed, the TWS can be solved, i.e.,

$$\boldsymbol{\$}_{bi} = \begin{bmatrix} 0 & 1 & 0; & -Q_{i5} & 0 & 0 \end{bmatrix} \tag{6}$$

Axial direction of the TWS is along the positive direction of y_i axis and passes through the point D_i, expressed in the fixed coordinate frame. The angle between the axial direction of the TWS and MTS refers to β as shown in Figure 2. According to Equation (4) and Equation (5), J_m and J_n can be expressed as:

$$J_m = \begin{bmatrix} \frac{\sqrt{3}}{2}A & \frac{1}{2}A & -B \\ -\frac{\sqrt{3}}{2}A & \frac{1}{2}A & -B \\ 0 & -A & -B \end{bmatrix} \tag{7}$$

$$J_n = \begin{bmatrix} AC - BD & 0 & 0 \\ 0 & AC - BD & 0 \\ 0 & 0 & AC - BD \end{bmatrix} \tag{8}$$

where A represents $\cos(\beta - \alpha)$, B denotes $\sin(\beta - \alpha)$, C is referred to as $\cos\alpha$ and D stands for $\sin\alpha$.

Consequently, the Jacobi matrix J will be solved by Equation (3). When we choose the prismatic pairs as actuators, J will be determined by structural parameters: angle α and β, consequently the Jacobi matrix J of 3-PRRU translational parallel mechanism keeps constant invariably whatever the pose is. Assume the structural parameters of mechanism are $\alpha = 15°$, $\beta = 30°$, and the driving velocity equation is

$$\begin{cases} \dot{d}_1 = 30\sin(2t + \pi) \\ \dot{d}_2 = 25\cos(t + \pi/3) \\ \dot{d}_3 = 15\cos(2t + \pi/6) \end{cases} \tag{9}$$

The curve of output velocity of the moving platform is illustrated in Figure 3, where $\dot{P}_{ij}$ ($i = 1, 2, 3$; $j = X, Y, Z$) denotes translational velocity of the moving platform in the ith pose along the direction of axis j.

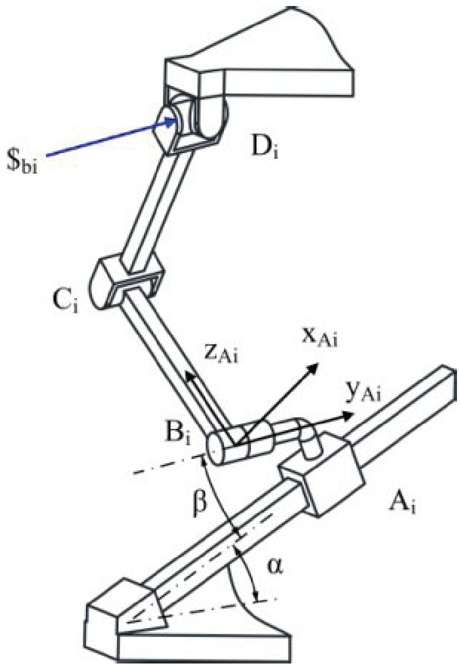

FIGURE 2. The TWS of limb

Similarly, assume the driving force equation of the moving platform is

$$\begin{cases} f_1 = 50\sin(2t + \pi/2) \\ f_2 = 40\sin(t + \pi/3) \\ f_3 = 37\cos(t/2 + \pi/6) \end{cases} \tag{10}$$

When the mechanisms are in different poses, the curve of output force of the moving platform is illustrated in Figure 4, where f_{ij} ($i = 1, 2, 3$; $j = X, Y, Z$) denotes output force of the moving platform in the ith pose along the direction of axis j.

When the input velocity/force of the prismatic pair is defined, the curve of the output velocity/force coincides when the moving platform is along the direction of X, Y, Z axis. Consequently, the conclusion that the Jacobi matrix of the mechanism keeps constant is verified, and the mapping between input and output will not vary when the mechanism is in any poses.

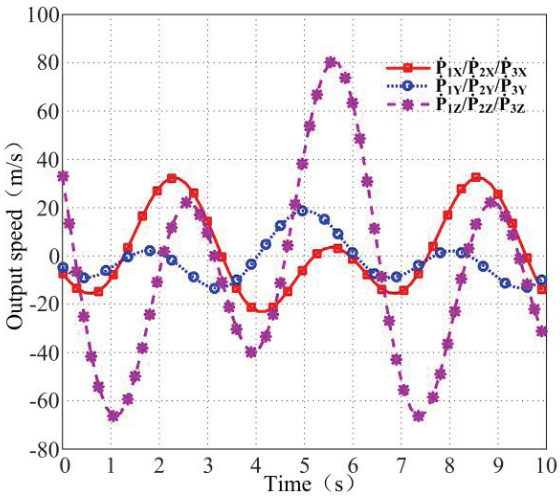

FIGURE 3. The output speed curve of moving platform

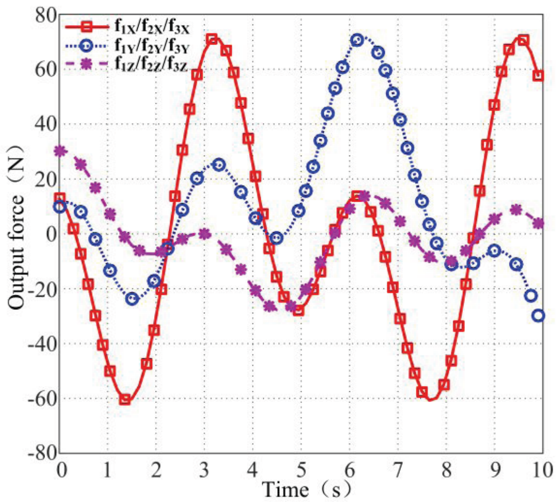

FIGURE 4. The output force curve of moving platform

4. 3-PRRU Transmissibility Performance Analysis.

4.1. **Power transmissibility performance analysis.** The reciprocal product between the TWS and OTS of each limb is defined as the instantaneous transmission power. For a 3-PRRU translational parallel mechanism, the second limb and the third limb are fixed. Assume that v is referred to the input velocity of the actuator, namely the input velocity represents $\dot{d} = [\ v\ \ 0\ \ 0\ ]^T$, and the driving force is denoted by f. Then transmission power of the limb can be expressed as

$$W = \boldsymbol{\$}_{b1} \circ \boldsymbol{\$}_{O1} = \boldsymbol{S}_{b1} \cdot J \cdot \dot{d} = f \cdot v \cdot P \tag{11}$$

where P represents transmission power of the limb under the unit input power, $\boldsymbol{\$}_{O1}$ denotes the twist screw of moving platform, and $\boldsymbol{S}_{b1}$ stands for axial direction vector of the TWS.

From Equation (11), the transmission power of 3-PRRU translational parallel mechanism is determined by the angle β, having nothing to do with the angle α. Assume mechanism's structural parameter α is equal to 15°. Calculate the transmission power of one limb respectively, when β is equal to 0°, 15°, 30°, 45°, 60° and 90°. And the value of transmission power is listed in Table 1. From the table, the transmission power of limb under the unit input power decreases with β increasing. When β is equal to 0°, namely, the axial directions of the TWS and MTS are parallel, the power transmissibility is excellent. When β is equal to 90°, limb cannot transmit power at all.

Now, we will analyze transmission power of the 3-PRRU translational parallel mechanism. Assume the driving velocity equation is as follows:

$$\begin{cases} \dot{d}_1 = 50\cos\theta \\ \dot{d}_2 = 40\sin(\theta+\pi) \\ \dot{d}_3 = 35\cos(2\theta+\pi) + 20\sin(2\theta) \end{cases} \tag{12}$$

TABLE 1. The comparison table of relationship between limb power and β under the unit input

β	0°	15°	30°	45°	60°	90°
Transmission power	1.000	0.956	0.866	0.707	0.500	0.000

The comparisons between the total input power and the total transmission power of all limbs are made as illustrated in Figure 5 to Figure 8, respectively, when β is equal to 0°, 15°, 30° and 60° in different poses. Some values of Figure 6 and Figure 8 are listed in Table 2 and Table 3.

From Figure 5 to Figure 8, when structural parameter α of the mechanism keeps invariant, the transmission power of each limb decreases with β increasing. Comparing Table 2 and Table 3, transmissibility performance with $\beta = 0°$ is much better than that with $\beta = 30°$ in condition that the total input power is identical. Similarly, the transmissibility performance is excellent when β is equal to 0° compared with the transmissibility performance of the any other values of β in $0° \sim 90°$.

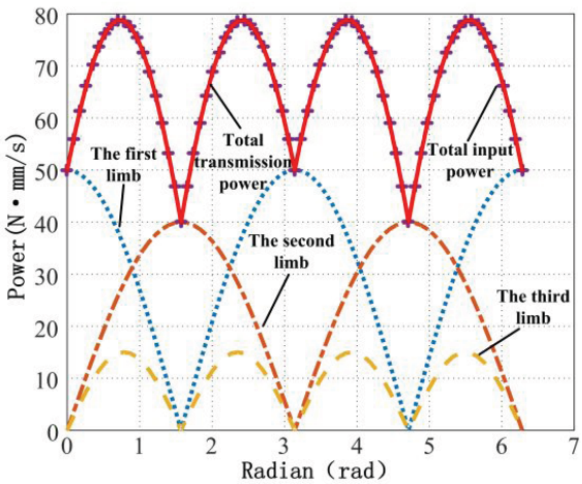

FIGURE 5. Comparison between input power and transmission power of limb with parameter $\beta = 0°$

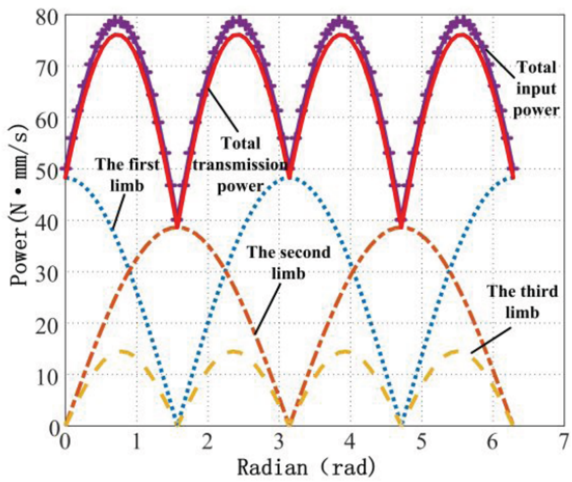

FIGURE 6. Comparison between input power and transmission power of limb with parameter $\beta = 15°$

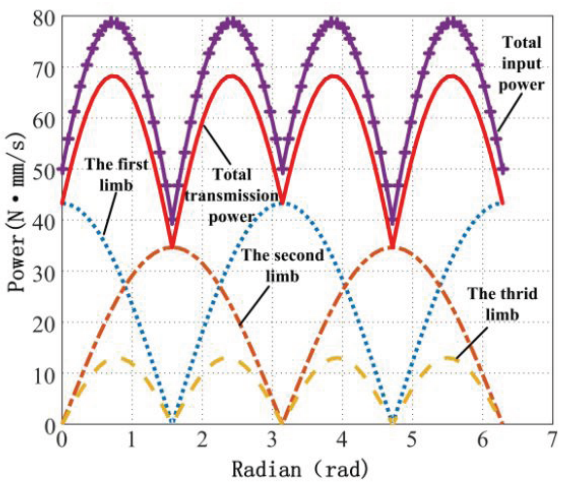

FIGURE 7. Comparison between input power and transmission power of limb with parameter $\beta = 30°$

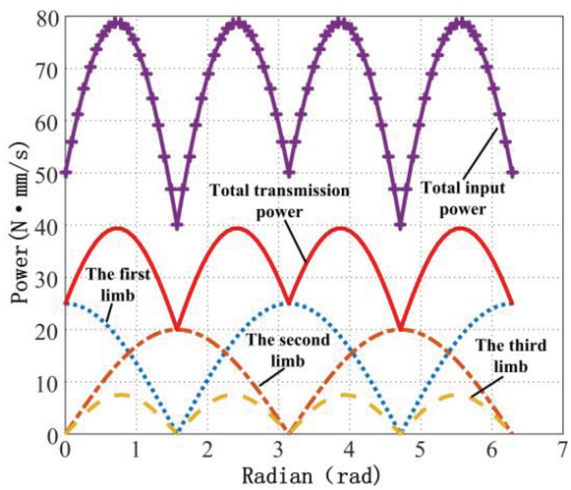

FIGURE 8. Comparison between input power and transmission power of limb with parameter $\beta = 60°$

Table 2. Comparison between input power and transmission power with parameter $\beta = 0°$

	Limb 1 (N·mm/s)	Limb 2 (N·mm/s)	Limb 3 (N·mm/s)	Total input (N·mm/s)	Total transmission (N·mm/s)
$\theta = 0°$	50.0000	0.0000	0.0000	50.0000	50.0000
$\theta = 30°$	43.3013	20.0000	12.9904	76.2917	76.2917
$\theta = 60°$	25.0000	34.6410	12.9904	72.6314	72.6314
$\theta = 90°$	0.0000	40.0000	0.0000	40.0000	40.0000
$\theta = 120°$	25.0000	34.6410	12.9904	72.6314	72.6314

Table 3. Comparison between input power and transmission power with parameter $\beta = 30°$

	Limb 1 (N·mm/s)	Limb 2 (N·mm/s)	Limb 3 (N·mm/s)	Total input (N·mm/s)	Total transmission (N·mm/s)
$\theta = 0°$	43.3013	0.0000	0.0000	50.0000	43.3013
$\theta = 30°$	37.5000	17.3205	11.2500	76.2917	66.0705
$\theta = 60°$	21.6506	30.0000	11.2500	72.6314	62.9006
$\theta = 90°$	0.0000	34.6410	0.0000	40.0000	34.6410
$\theta = 120°$	21.6506	30.0000	11.2500	72.6314	62.9006

4.2. **Analysis of speed and force transmission performance.** Assume mechanism's structural parameter α is equal to 15° and the driving velocity of each limb is shown as:

$$\begin{cases} \dot{d}_1 = 20\sin(t+\pi) \\ \dot{d}_2 = 30\cos(t+\pi) \\ \dot{d}_3 = 50\cos(t+\pi/2) \end{cases} \tag{13}$$

According to Equation (3) to Equation (5), Jacobi matrixes of the mechanism in both cases that β is equal to 0° and 30° are solved. Then the curve of mechanism's output speed with time is shown as Figure 9. Similarly, assume the driving force f_i equation is expressed as

$$\begin{cases} f_1 = 20\sin(t+\pi/2) \\ f_2 = 40\cos(t+\pi) \\ f_3 = 60\sin(t+\pi/2) \end{cases} \tag{14}$$

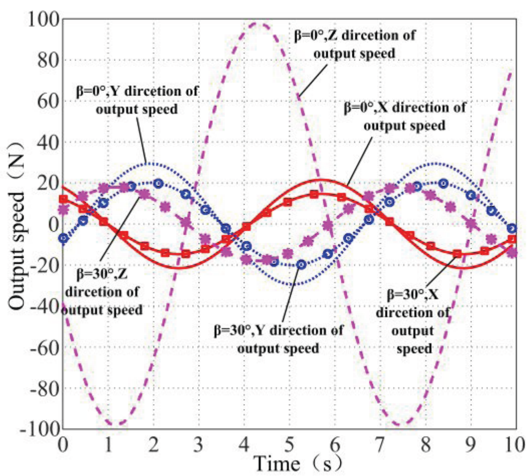

Figure 9. Output speed curve of moving platform

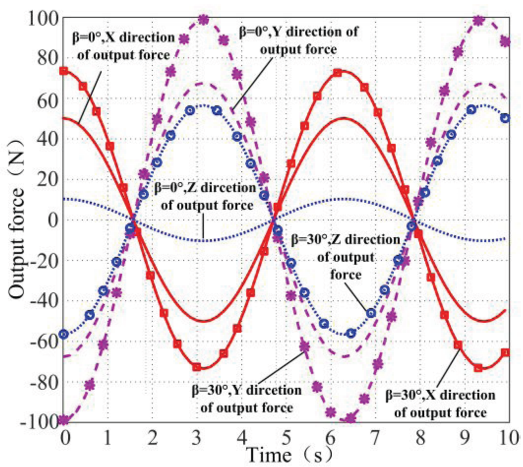

Figure 10. Output speed curve of moving platform

When β is equal to $0°$ and $30°$, the curve of moving platform's output velocity with time is illustrated as Figure 10.

From Figure 9 and Figure 10, when the input velocity is given, the curves of output velocity/force in the direction of axis X, Y, Z are not identical in two cases of $\beta = 0°$ and $\beta = 30°$. Obviously, the output velocity/force of $\beta = 0°$ is greater than that of $\beta = 30°$ at the same time. Similarly, comparisons between the output velocity/force of $\beta = 0°$ and that of other values of β are made. The conclusion is that the output velocity/force is maximum as $\beta = 0°$, namely, the transmissibility performance is excellent.

5. **Conclusions.** In this paper, a novel 3-PRRU translational parallel mechanism is proposed. The DOFs of the parallel mechanism are analyzed based on screw theory. The conclusion that Jacobi matrix of the 3-PRRU parallel mechanism keeps constant is verified when the prismatic pairs are chosen as actuators. On the basis of this, the transmissibility performance of the mechanism is further analyzed. The last but not the least, the criterion of the optimal transmissibility performance of the mechanism is given. It is expected to be a theoretical basis for 3-PRRU translational parallel mechanism to further research. At the same time, this paper is to provide guidance for the optimization of the performance of the mechanism.

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