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LIU'S COMPLETED BALANCED DEMATEL THEORY BASED ON NORMALIZED INDIRECT RELATION MATRIX WITHOUT SELF-INFLUENCES

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Abstract. *In traditional DEMATEL theory, the indirect influence is always far greater than its responding direct influence that is unbalanced and unfair. To overcome this drawback, the generalized DEMATEL decreased the indirect influence by a shrinkage rate. Liu's balanced DEMATEL theory normalized the indirect relation matrix as that of the direct relation matrix. However, their indirect relation matrix does not exclude the self-influence as that of the direct relation matrix. This is the important cause of imbalance and unfairness. In this paper, we propose an improved theory called Liu's completed balanced DEMATEL theory which not only normalizes the indirect relation matrix but also excludes the self-influence from the indirect relation matrix as that of direct relation matrix. According to Liu's validity index, we can find that the performance of this new method is better than that of both the generalized DEMATEL and the balanced DEMATEL theory, and then, a simple data is also provided in this paper to illustrate the advantages of the proposed theory. For fitting this new theory, a new threshold value of the impact-relations map is also proposed.*
Keywords: Liu's balanced coefficient, Normalized indirect relation matrix, Liu's validity index

1. Introduction.

Decision making trial and evaluation laboratory (DEMATEL) was developed between 1972 to 1979 [1] by Science and Human Affairs Program of the Battelle Memorial Institute of Geneva. It can be used to resolve complex and difficult problems, and it has been widely used as one of the best tools to solve the cause and effect relationship among the evaluation factors [1,2]. However, the indirect relation of a DEMATEL is always far greater than its direct relation, which is unbalanced and unfair [5]. Our previous paper proposed an external shrinkage coefficient to improve it [5]. However, if the indirect relation of a DEMATEL is less than its direct relation, the previous method can do nothing about it. Based on normalizing the indirect relation matrix, the improved method, Liu's balanced DEMATEL [6] was proposed. However, their indirect relation matrix does not exclude the self-influence as that of the direct relation matrix, which is the important cause of imbalance and unfairness. In this paper, for overcoming above-mentioned drawbacks, a completed balanced DEMATEL theory is proposed, in which it not only normalized the indirect relation matrix but also excluded the self-influence from the indirect relation matrix as that of the direct relation matrix.

This paper is organized as follows. Section 2 describes how to construct new DEMATEL based on the traditional DEMATEL: transformed DEMATEL. Section 3 introduces the Liu's validity index of DEMATEL for selecting better DEMATEL. Section 4 introduces the given improved DEMATELs: the generalized and the balanced DEMATELs. Section 5 proposes the new model: the completely balanced DEMATEL. Finally, Section 6 concludes the paper.

2. **The Transformed DEMATEL.** The procedures of the traditional DEMATEL method and the transformed DEMATEL are briefly introduced below [1-6]:

Step 1: Calculate the initial direct relation matrix Q

N experts are asked to evaluate the degree of direct influence between two factors based on pair-wise comparison. The degree to which the expert e perceived factor i effects on factor j is denoted as

$$q_{ij}^{(e)}, \quad e = 1, 2, \ldots, N. \quad q_{ij}^{(e)} \in \{0, 1, 2, 3, 4\}, \quad i, j = 1, 2, \ldots, n \tag{1}$$

For each expert e, an individual direct relation matrix is constructed as

$$Q_e = \left[q_{ij}^{(e)}\right]_{n\times n}, \quad e = 1, 2, \ldots, N, \quad q_{ii}^{(e)} = 0, \quad i = 1, 2, \ldots, n \tag{2}$$

We can obtain their average direct relation matrix, called the initial direct relation matrix Q as follows:

$$Q = [q_{ij}]_{n\times n} = \frac{1}{N}\sum_{e=1}^{N} Q_e, \quad q_{ij} = \frac{1}{N}\sum_{e=1}^{N} q_{ij}^{(e)}, \quad i, j = 1, 2, \ldots, n \tag{3}$$

Step 2: Calculate the direct relation matrix A

$$A = [a_{ij}]_{n\times n} = \lambda^{-1}Q, \quad \lambda = \max_{1\le i,j\le n}\left\{\sum_{j=1}^{n} q_{ij}, \sum_{i=1}^{n} q_{ij}\right\} \tag{4}$$

$$a_{ii} = 0, \quad i = 1, 2, \ldots, n, \quad 0 \le a_{ij} \le 1, \quad i \ne j, \quad i, j = 1, 2, \ldots, n$$

$$0 \le \sum_{i=1}^{n} a_{ij}, \quad \sum_{j=1}^{n} a_{ij} \le 1, \quad i, j = 1, 2, \ldots, n \tag{5}$$

$$\text{Since } 0 \le a_{ij} = \lambda^{-1}q_{ij}, \quad a_{ji} = \lambda^{-1}q_{ji} \le \lambda^{-1}\lambda = 1, \quad i, j = 1, 2, \ldots, n$$

Step 3: Calculate the indirect relation matrix B, the transformed matrix of B and the total transformed relation matrix T:

$$B = [b_{ij}]_{n\times n} = A^2(I - A)^{-1}, \quad B_X = \left[b_{ij}^{(X)}\right]_{n\times n}, \quad T_X = \left[t_{ij}^{(X)}\right]_{n\times n} = A + B_X \tag{6}$$

$$\text{if } B_X = B, \text{ then } T_X = T = [t_{ij}]_{n\times n} = A + B \tag{7}$$

Step 4: Calculate the relation degree and prominence degree of each factor of the transformed DEMATEL.

$$r_i^{(X)} = \sum_{j=1}^{n} t_{ij}^{(X)}, \quad c_i^{(X)} = \sum_{j=1}^{n} t_{ij}^{(X)}, \quad i = 1, 2, \ldots, n \tag{8}$$

The value of $r_i^{(X)}$ indicates the total dispatch both direct and indirect effects, that factor i has on the other factors, and the value of $c_i^{(X)}$ indicates the total receive both direct and indirect effects, that factor i has on the other factors of the transformed DEMATEL.

The relation degree of factor i of the transformed DEMATEL is denoted as

$$x_i^{(X)} = r_i^{(X)} - c_i^{(X)}, \quad i = 1, 2, \ldots, n \tag{9}$$

The prominence degree of factor i of the transformed DEMATEL is denoted as

$$y_i^{(X)} = r_i^{(X)} + c_i^{(X)}, \quad i = 1, 2, \ldots, n \tag{10}$$

Relation prominence matrix of the transformed DEMATEL i is denoted as

$$R(A, B_X) = \left(x_i^{(X)}, y_i^{(X)}\right)_{i=1}^{n} = \left(r_i^{(X)} + c_i^{(X)}, r_i^{(X)} - c_i^{(X)}\right), \quad i = 1, 2, \ldots, n \tag{11}$$

Step 5: Set the threshold value (α_L)

We propose a new threshold value for selecting the significant effects elements in matrix T_X which has no self-influence, and this new threshold is different from that of Yang et al.;

$$\alpha_L = \frac{1}{n(n-1)} \sum_{i=1,}^{n} \sum_{j=1, i\neq j}^{n} t_{ij}^{(X)} \tag{12}$$

Step 6: Build a cause and effect relationship diagram $R_{map}(A, B_X)$

If $t_{ij}^{(X)} > \alpha_L$, or $t_{ji}^{(X)} > \alpha_L$, then factor i is a net dispatch node of factor j, and factor j is a net receive node of factor i, and denoted as

$$\left(x_i^{(X)}, y_i^{(X)}\right) \to \left(x_j^{(X)}, y_j^{(X)}\right), \text{ or } \left(x_i^{(X)}, y_i^{(X)}\right) \leftarrow \left(x_j^{(X)}, y_j^{(X)}\right) \tag{13}$$

The graph of $R_{map}(A, B_X)$ including the net direct edges can present a cause and effect relationship diagram of the transformed DEMATEL.

3. Liu's Validity Index of DEMATEL.

For evaluating the performance of any DEMATEL, the Liu's validity index [5] was defined below.

Definition 3.1. *Balanced coefficient, Variation coefficient, Validity index*

If $A = [a_{ij}]_{n\times n}$ is the direct relation matrix of a DEMATEL, $B = [b_{ij}]_{n\times n} = A^2(I-A)^{-1}$, $B_X = \left[b_{ij}^{(X)}\right]_{n\times n}$ is a transformation of B, $\mu_X = \max\limits_{1\le i,j\le n} \left\{\sum_{j=1}^{n} b_{ij}^{(X)}, \sum_{i=1}^{n} b_{ij}^{(X)}\right\}$, $T_X = \left[t_{ij}^{(X)}\right]_{n\times n} = A + B_X$ then

(i) Liu's balanced coefficient of the transformed DEMATEL X, $\beta_L(A, B_X)$ expresses the balance coefficient of B_X on A, the larger, the better, and it is defined as follows:

$$\beta_L(A, B_X) = \frac{2\sqrt{\mu_X}}{1+\mu_X}, \quad 0 \le \beta_X(A, B_X) \le 1 \tag{14}$$

(ii) Liu's variation coefficient of the transformed DEMATEL X, $\sigma_L(A, B_X)$ expresses the variation coefficient of B_X on A, the larger, the better, and it is defined as follows:

$$\sigma_L(A, B_X) = 1 - \frac{1}{1 + 5\sqrt{\sum_{i=1}^{n} \sqrt{\left(x_i^{(X)} - \bar{x}_X\right)^2 + \left(y_i^{(X)} - \bar{y}_X\right)^2}}}, \quad 0 \le \sigma_L(A, B_X) \le 1 \tag{15}$$

(iii) Liu's validity index of the transformed DEMATEL X is defined as

$$V_L(A, B_X) = \frac{\beta_L(A, B_X) + \sigma_L(A, B_X)}{2}, \quad 0 \le V_L(A, B_X) \le 1 \tag{16}$$

Example 3.1. *If the direct relation matrix A is given, then we can obtain the indirect matrix B, the balance coefficient $\beta_L(A, B)$, the variation coefficient $\sigma_L(A, B)$, and the* *validity index $V_L(A,B)$, as follows.*

$$\text{If } A=[a_{ij}]_{4\times4}=\begin{bmatrix}0&0.36&0.32&0.32\\0.32&0&0.34&0.30\\0.34&0.30&0&0.30\\0.28&0.28&0.30&0\end{bmatrix},$$
$$\text{then } B=[b_{ij}]_{4\times4}=A^2(I-A)^{-1}=\begin{bmatrix}3.9448&3.8483&3.9290&3.7995\\3.7483&3.8237&3.7991&3.6907\\3.6807&3.6963&3.8253&3.6331\\3.4499&3.4478&3.4963&3.4508\end{bmatrix} \tag{17}$$

We can obtain

$$\beta_L(A,B)=0.4776,\ \sigma_L(A,B)=0.9015\Rightarrow V_L(A,B)=0.6914 \tag{18}$$

4. **The Given Improved DEMATELs: The Generalized and Balanced DEMATELs.**

4.1. **The generalized DEMATELs with external shrinkage coefficient.** Our previous paper [5] pointed out that the indirect relation of a traditional DEMATEL is always far greater than its direct relation, which is unbalanced and unfair, since it overemphasizes the influence of the indirect relation. For overcoming this drawback, a transformed DEMATEL which is using external shrinkage coefficient of the indirect relation matrix d, was provided to construct a better indirect relation matrix, and a generalized DEMATEL theory as follows:

$$B_d=\left[b_{ij}^{(d)}\right]_{n\times n}=dA^2(I-dA)^{-1},\quad d\in\left[\frac{1}{2},1\right] \tag{19}$$

$$T_d=\left[t_{ij}^{(d)}\right]_{n\times n}=A+B_d=\left[\left(a_{ij}+b_{ij}^{(d)}\right)\right],\quad d\in\left[\frac{1}{2},1\right] \tag{20}$$

$$\beta_L(A,B_X)=\beta_L(A,B_d),\quad \sigma_L(A,B_X)=\sigma_L(A,B_d),\quad V_L(A,B_X)=V_L(A,B_d) \tag{21}$$

Example 4.1. *If A and B are the same as **Example 3.1**, the shrinkage rate is 0.5, and we can obtain transformed indirect relation matrix $B_{0.5}$, the balance coefficient $\beta_L(A,B_{0.5})$, the variation coefficient $\sigma_L(A,B_{0.5})$, and the validity index $V_L(A,B_{o.5})$, as follows.*

Since

$$A=\begin{bmatrix}0&0.36&0.32&0.32\\0.32&0&0.34&0.30\\0.34&0.30&0&0.30\\0.28&0.28&0.30&0\end{bmatrix}\Rightarrow B_{0.5}=0.5A^2(I-0.5A)^{-1}$$
$$=\begin{bmatrix}0.2538&0.1993&0.2144&0.2047\\0.2012&0.2450&0.2002&0.2012\\0.1916&0.2020&0.2450&0.1971\\0.1882&0.1879&0.1872&0.2182\end{bmatrix} \tag{22}$$

we can obtain

$$\beta_L(A,B_{0.5})=0.9977,\quad \sigma_L(A,B_{0.5})=0.7044\Rightarrow V_L(A,B_{0.5})=0.8510 \tag{23}$$

$$V_L(A,B_{0.5})=0.8510>V_L(A,B)=0.6914 \tag{24}$$

This example shows that the generalized DEMATEL is better than the traditional DEMATEL.

4.2. **Liu's balanced DEMATEL with normalized indirect relation matrix.** The generalized DEMATEL does not consider that the indirect relation of a traditional DEMATEL may be less than its direct relation, and the improved method, Liu's balanced DEMATEL, is introduced as follows.

If $A = [a_{ij}]_{n\times n}$ is the direct relation matrix, $B = [b_{ij}]_{n\times n} = A^2(I-A)^{-1}$ and

$$\mu = \max_{1\le i,j\le n}\left\{\sum_{j=1}^{n} b_{ij}, \sum_{i=1}^{n} b_{ij}\right\} \tag{25}$$

then normalized indirect relation matrix B_N is defined by

$$B_X = B_N = \left[b_{ij}^{(N)}\right]_{n\times n} = \mu^{-1}B = \left[(\mu^{-1}b_{ij})\right]_{n\times n} \Rightarrow \beta(A, B_N) = 1 \tag{26}$$

$$\beta_L(A, B_X) = \beta_L(A, B_N), \quad \sigma_L(A, B_X) = \sigma_L(A, B_N), \quad V_L(A, B_X) = V_L(A, B_N) \tag{27}$$

Example 4.2. *Let A, B, μ, $\beta(A, B)$ be the same as that of **Example 3.1**, and then we can obtain the normalized indirect relation matrix B_N, the balance coefficient $\beta_L(A, B_N)$, the variation coefficient $\sigma_L(A, B_N)$, and the validity index $V_L(A, B_N)$, as follows.*

$$\begin{aligned} B_N = \mu^{-1}B &= \begin{bmatrix} 0.2541 & 0.2479 & 0.2531 & 0.2448 \\ 0.2415 & 0.2464 & 0.2448 & 0.2378 \\ 0.2371 & 0.2381 & 0.2464 & 0.2341 \\ 0.2223 & 0.2221 & 0.2253 & 0.2223 \end{bmatrix}, \\ T_N = A + B_N &= \begin{bmatrix} 0.2541 & 0.6079 & 0.5731 & 0.5648 \\ 0.5615 & 0.2464 & 0.5848 & 0.5378 \\ 0.5771 & 0.5381 & 0.2464 & 0.5341 \\ 0.5023 & 0.5021 & 0.5253 & 0.2223 \end{bmatrix} \end{aligned} \tag{28}$$

$$\mu_N = \max_{1\le i,j\le n}\left\{\sum_{j=1}^{n} b_{ij}^{(N)}, \sum_{i=1}^{n} b_{ij}^{(N)}\right\} = 1 \Rightarrow \beta(A, B_N) = 1 \tag{29}$$

$$\beta(A, B_N) = 1 > \beta(A, B_{0.5}) = 0.9977 > \beta(A, B) = 0.4769 \tag{30}$$

$$\sigma_L(A, B_N) = 0.7051, \quad \sigma_L(A, B_{0.5}) = 0.7044, \quad \sigma_L(A, B) = 0.9015 \tag{31}$$

$$V_L(A, B_N) = 0.8526 > V_L(A, B_{0.5}) = 0.8510 > V_L(A, B) = 0.6914 \tag{32}$$

This example shows that the balanced DEMATEL is better than the generalized DEMATEL and the traditional DEMATEL.

5. Liu's Completed Balanced DEMATEL Based on Normalized Indirect Relation Matrix without Self-Influences.

Liu's balanced DEMATEL theory normalized the indirect relation matrix as that of the direct relation matrix. However, its indirect relation matrix does not exclude the self-influence as that of the direct relation matrix. That is another kind of imbalance and unfairness, and the improved theory, Liu's completed balanced DEMATEL theory, is introduced as follows.

If $A = [a_{ij}]_{n\times n}$ is the direct relation matrix, $B = [b_{ij}]_{n\times n} = A^2(I-A)^{-1}$ and

$$B_E = \left[b_{ij}^{(E)}\right]_{4\times 4} = B - diagB, \quad \mu_E = \max_{1\le i,j\le n}\left\{\sum_{j=1}^{n} b_{ij}^{(E)}, \sum_{i=1}^{n} b_{ij}^{(E)}\right\} \tag{33}$$

then completed balanced indirect relation matrix B_C is defined by

$$B_C = \left[b_{ij}^{C}\right] = \mu_E^{-1}\left[b_{ij}^{(E)}\right]_{n\times n} \Rightarrow \beta(A, B_C) = 1 \tag{34}$$

$$\beta_L(A, B_X) = \beta_L(A, B_C), \quad \sigma_L(A, B_X) = \sigma_L(A, B_C), \quad V_L(A, B_X) = V_L(A, B_C) \tag{35}$$

Example 5.1. *Let A, B be the same as that of **Example 3.1**, and then we can obtain the indirect relation matrix without self-influence B_E, the completed normalized indirect relation matrix B_C, the balance coefficient $\beta_L(A, B_C)$, the variation coefficient $\sigma_L(A, B_C)$, and the validity index $V_L(A, B_C)$, as follows.*

Let

$$B_E = B - diagB = \begin{bmatrix} 0 & 3.8483 & 3.9290 & 3.7995 \\ 3.7483 & 0 & 3.7991 & 3.6907 \\ 3.6807 & 3.6963 & 0 & 3.6331 \\ 3.4499 & 3.4478 & 3.4963 & 0 \end{bmatrix}, \tag{36}$$

$$\mu_E = \max_{1 \le i,j \le n} \left\{ \sum_{j=1}^{n} b_{ij}^{(E)}, \sum_{i=1}^{n} b_{ij}^{(E)} \right\} = 11.5768$$

then

$$B_C = \left[b_{ij}^C\right] = \mu_E^{-1} \left[b_{ij}^{(E)}\right]_{4\times4} = \begin{bmatrix} 0 & 0.3324 & 0.3394 & 0.3282 \\ 0.3238 & 0 & 0.3282 & 0.3188 \\ 0.3179 & 0.3193 & 0 & 0.3138 \\ 0.2980 & 0.2978 & 0.3020 & 0 \end{bmatrix} \tag{37}$$

$$T_c = \begin{bmatrix} 0 & 0.6924 & 0.6594 & 0.6482 \\ 0.6438 & 0 & 0.6682 & 0.6188 \\ 0.6579 & 0.6193 & 0 & 0.6138 \\ 0.5780 & 0.5778 & 0.6020 & 0 \end{bmatrix},$$
$$R(A, B_c) = \left(x_i^{(c)}, y_i^{(c)}\right)_{i=1}^{n} = \begin{pmatrix} 0.1203 & 3.8797 \\ 0.0413 & 3.8203 \\ -0.0386 & 3.8206 \\ -0.1230 & 3.6386 \end{pmatrix} \tag{38}$$

$$\beta(A, B_C) = \beta(A, B_N) = 1 > \beta(A, B_{0.5}) = 0.9977 > \beta(A, B) = 0.4769 \tag{39}$$

$$\sigma_L(A, B_C) = 0.8023, \quad \sigma_L(A, B_N) = 0.7051, \quad \sigma_L(A, B_{0.5}) = 0.7044, \quad \sigma_L(A, B) = 0.9015 \tag{40}$$

$$V_L(A, B_N) = 0.9012 > V_L(A, B_N) = 0.8526 > V_L(A, B_{0.5}) = 0.8510 > V_L(A, B) = 0.6914 \tag{41}$$

This example shows that Liu's completed balanced DEMATEL is better than Liu's balanced DEMATEL, the generalized DEMATEL and the traditional DEMATEL.

6. **Conclusion.** In this paper, we propose a further improved theory called Liu's completed balanced DEMATEL theory which is based on normalized indirect relation matrix without self-influence as the direct relation matrix which has been done can obtain a perfect balanced DEMATEL. According to Liu's validity index, for comparing all of above-mentioned DEMATEL theories, a simple data is also provided, and the results show that the valid performance of this new method is better than before. To fit this new theory, a new threshold value of the impact-relations map is also proposed.

In the future, we will develop the integrated model of this new DEMATEL and Liu's ordering theory [7].

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