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A COORDINATED MULTI-POINT PRECODING ALGORITHM BASED ON QR-SVD

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Abstract. *This paper proposes a novel linear precoding algorithm suitable for coordinated multi-point (CoMP) transmission system. Firstly, the channel matrix extension effectively excludes the impacts of multi-user and noise interference on the system. Besides, the introduction of low-complexity matrix decomposition method (QR) reduces computational complexity. Finally, by applying singular value decomposition (SVD) to equivalent channel matrix, complete precoding matrix and decoding matrix can be obtained. Therefore, the proposed algorithm can suppress multi-user and noise interference, and can reduce computational complexity. Simulation results show that the proposed algorithm has certain advantages in terms of computational complexity, system capacity and bit error rate (BER), compared with other precoding algorithms.*
Keywords: Coordinated multi-point (CoMP), Precoding algorithm, Matrix decomposition, Computational complexity, Singular value decomposition (SVD)

1. **Introduction.** As a key technology in Long Term Evolution-Advanced (LTE-A) system, coordinated multi-point (CoMP) communication technology can suppress inter-cell interference, increase system throughput, and improve cell-edge users service quality through mutual cooperation among several base stations [1,2].

Coordinated multi-point precoding algorithm is a typical signal preprocessing technology at the transmitting end. When the transmitting end obtains complete channel state information, the signal is preprocessed at the transmitting end. Therefore, the interference caused by transmitting signal going through wireless channel can be eliminated in advance so as to reach the purpose of ensuring communication reliability and improving system performance [3]. As the key technology of LTE-A physical layer, precoding technology can in advance suppress interference between users or data streams at the transmitting end, but there are a variety of defects in commonly used precoding algorithms. In [4], zero-forcing (ZF) precoding algorithm is simple and intuitive, but it does not directly consider the influence of noise, which causes the noise to be amplified. In addition, although ZF can handle its own antenna received signal and can differentiate data flow from multiple transmit antennas at the base station, its interference elimination to other users is not too ideal, and multi-user interference still exists. In [5,6], the main steps of block diagonalization (BD) precoding algorithm is to execute singular value decomposition twice for each user. For multi-user coordinated multi-point communications systems, its computational complexity is very high. In [7], regularized block diagonalization (RBD) precoding algorithm introduces regularization factor, and takes the noise into account, reducing the influence of noise on the system. However, like BD, RBD still has high computational complexity. Through the above analysis, we know that ZF precoding algorithm cannot effectively suppress multi-user and noise interference, and BD and RBD precoding algorithms have extremely high computational complexity. In order to overcome these defects and improve system performance, this paper proposes a novel linear precoding algorithm based on QR-SVD. The proposed precoding algorithm extends the channel matrix to suppress multi-user and noise interference, introduces QR matrix decomposition method to reduce computational complexity, and executes singular value decomposition (SVD) to obtain decoding matrix. According to the above description, we can conclude that the proposed algorithm can reduce computational complexity, improve system capacity, reduce bit error rate, and effectively improve system performance in theory.

The rest of this paper is organized as follows. The system model is given in Section 2. The proposed QR-SVD precoding algorithm is described in detail in Section 3. Section 4 presents computational complexity analysis and simulation. Algorithm performance analysis and simulation are displayed in Section 5. Finally, in Section 6, we summarize our main works.

2. **System Model.** Figure 1 shows multi-user coordinated multi-point (MU-CoMP) communication system in joint processing (JP) mode [8]. First, the transmitted data is precoded. Then the precoded data is forwarded to the user equipment via the base station. And finally the received signal of the user is obtained. We consider an MU-CoMP system with N transmit antennas at the base stations and M_k receive antennas at the user equipment (UE) k, denoted as UE_k. It is assumed that there are K users in the MU-CoMP system. $\mathbf{W} = [\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_K]$ is the transmit precoding matrix, where $\mathbf{W}_k \in^{N\times l_k}$. l_k data streams are transmitted to the user k. The received signal at the kth user is given by:

$$y_k = \mathbf{H}_k \mathbf{W}_k s_k + \mathbf{H}_k \sum_{i\neq k} \mathbf{W}_i s_i + n_k \tag{1}$$

where y_k denotes the kth user's received signal. $\mathbf{H}_k \in^{M_k\times N}$ is the kth user's channel matrix. $s_k \in^{l_k\times 1}$ $(k = 1, 2, \ldots, K)$ is the kth user's transmitted signal. n_k is the kth user's additive Gaussian noise with zero mean and σ_k^2 variance.

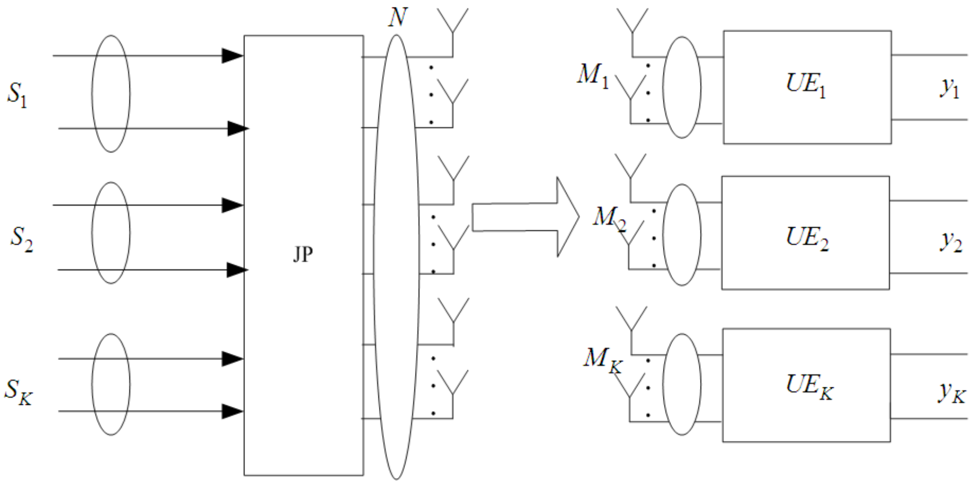

FIGURE 1. MU-CoMP communication system in JP mode

3. **QR-SVD Precoding Algorithm.** Assuming that the kth user's QR-SVD precoding matrix can be expressed as $\mathbf{W}_{QR\text{-}SVD\text{-}k} = \mathbf{W}_k^1 \mathbf{W}_k^2$ and $l_k = M_k$. We can define complement channel matrix as the channel matrix of all users except the kth user's. The complement channel matrix can be expressed as:

$$\overline{\mathbf{H}}_k = \left[\mathbf{H}_1^T, \ldots, \mathbf{H}_{k-1}^T, \mathbf{H}_{k+1}^T, \ldots, \mathbf{H}_K^T\right]^T \in^{\overline{M}_k\times N} \tag{2}$$

where $\overline{M}_k = \sum_{i=1, i \neq k}^{K} M_i$.

QR-SVD precoding algorithm consists of the following steps.

Step 1: Firstly, we extend complement channel matrix to obtain extended channel matrix, and its expression can be expressed as

$$\widetilde{\overline{\mathbf{H}}}_k = \left[\sigma_k \mathbf{I}, \overline{\mathbf{H}}_k\right] \tag{3}$$

where $\mathbf{I}$ is the $\overline{M}_k \times \overline{M}_k$-dimensional unit matrix. Thus, the dimension of $\widetilde{\overline{\mathbf{H}}}_k$ is $\overline{M}_k \times \left(\overline{M}_k + N\right)$. The columns number of $\widetilde{\overline{\mathbf{H}}}_k$ is greater than the rows number, and therefore, it meets the dimension requirements of obtaining its null-space. So this scheme is suitable for MU-CoMP system with users having arbitrary root receive antenna. As can be seen from expression (3), $\widetilde{\overline{\mathbf{H}}}_k$ takes account of both the influence elements of causing multi-user interference $\widetilde{\overline{\mathbf{H}}}_k$, and noise $\sigma_k \mathbf{I}$.

Step 2: Then, the Hermitian transpose of extended channel matrix is described by $\widetilde{\overline{\mathbf{H}}}_k^H$. QR decomposition of $\widetilde{\overline{\mathbf{H}}}_k^H$ can be expressed as

$$\widetilde{\overline{\mathbf{H}}}_k^H = \mathbf{Q}_k \mathbf{R}_k \tag{4}$$

where $\mathbf{Q}_k \in^{\left(\overline{M}_k+N\right)\times\left(\overline{M}_k+N\right)}$ is an orthogonal matrix and $\mathbf{R}_k \in^{\left(\overline{M}_k+N\right)\times\overline{M}_k}$ is an upper triangular matrix.

After transforming expression (4), we can obtain expression (5), as shown below.

$$\mathbf{Q}_k^H \widetilde{\overline{\mathbf{H}}}_k^H = \begin{bmatrix} \mathbf{Q}_{k,1}, \mathbf{Q}_{k,2} \\ \mathbf{Q}_{k,3}, \mathbf{Q}_{k,4} \end{bmatrix} \begin{bmatrix} \sigma_k \mathbf{I} \\ \overline{\mathbf{H}}_k^H \end{bmatrix} = \mathbf{R}_k = \begin{bmatrix} \mathbf{R}_{k,1} \\ \mathbf{R}_{k,2} \end{bmatrix} \tag{5}$$

where $\mathbf{Q}_{k,1} \in^{\overline{M}_k \times \overline{M}_k}$, $\mathbf{Q}_{k,2} \in^{\overline{M}_k \times N}$, $\mathbf{Q}_{k,3} \in^{N \times \overline{M}_k}$, $\mathbf{Q}_{k,4} \in^{N \times N}$, $\mathbf{R}_{k,1} \in^{\overline{M}_k \times \overline{M}_k}$, $\mathbf{R}_{k,2} \in^{N \times \overline{M}_k}$.

The following is the proof of $\mathbf{W}_k^1 = \mathbf{Q}_{k,4}$.

From Equation (5), we can deduce $\sigma_k \mathbf{Q}_{k,3} + \mathbf{Q}_{k,4} \overline{\mathbf{H}}_k^H = 0$, that is

$$\sigma_k \mathbf{Q}_{k,3}^H + \overline{\mathbf{H}}_k \mathbf{Q}_{k,4}^H = 0 \tag{6}$$

For unitary matrix, there is

$$\mathbf{Q}_{k,3} \mathbf{Q}_{k,3}^H + \mathbf{Q}_{k,4} \mathbf{Q}_{k,4}^H = \mathbf{I} \tag{7}$$

$$\mathbf{Q}_{k,4} \overline{\mathbf{H}}_k^H \overline{\mathbf{H}}_k \mathbf{Q}_{k,4}^H = \left(\overline{\mathbf{H}}_k \mathbf{Q}_{k,4}^H\right)^H \left(\overline{\mathbf{H}}_k \mathbf{Q}_{k,4}^H\right) = \left(-\sigma_k \mathbf{Q}_{k,3}\right)\left(-\sigma_k \mathbf{Q}_{k,3}^H\right) = \sigma_k^2 \mathbf{Q}_{k,3} \mathbf{Q}_{k,3}^H \tag{8}$$

From Equations (7) and (8), we can get

$$\mathbf{Q}_{k,4} \left(\overline{\mathbf{H}}_k^H \overline{\mathbf{H}}_k + \sigma_k^2 \mathbf{I}\right) \mathbf{Q}_{k,4}^H = \sigma_k^2 \mathbf{I} \tag{9}$$

According to minimum mean square error (MMSE) optimization criterion in [9], we can get the following formula.

$$\mathbf{W}_1 = \min E \left\{ \sum_{k=1}^{K} \left\| \overline{\mathbf{H}}_k \mathbf{W}_{1k} \right\|^2 + \frac{\|n\|^2}{\beta^2} \right\} \tag{10}$$

Referring to the appendix of [9], we can see that $\mathbf{Q}_{k,4}$ is the optimal solution to Formula (10), that is, $\mathbf{W}_k^1 = \mathbf{Q}_{k,4}$.

Step 3: After Step 2, we can define equivalent channel matrix for each user as $\mathbf{H}_k^{equ} = \mathbf{H}_k \mathbf{Q}_{k,4}$. Therefore, SVD decomposition of $\mathbf{H}_k^{equ}$ can be expressed as

$$\mathbf{H}_k^{equ} = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^H = \mathbf{U}_k \mathbf{\Sigma}_k \left[\mathbf{V}_{k,1}, \mathbf{V}_{k,2}\right]^H \tag{11}$$

where $\mathbf{U}_k$ is an $M_k \times M_k$-dimensional unitary matrix and $\mathbf{\Sigma}_k$ is an $M_k \times N$-dimensional diagonal matrix. The dimension of $\mathbf{V}_k$ is $N \times N$. $\mathbf{V}_{k,1} \in^{N \times M_k}$ is composed of the first

M_k columns of $\mathbf{V}_k$, and is the standard orthogonal basis of $\mathbf{H}_k^{equ}$ row space, so we have $\mathbf{W}_k^2 = \mathbf{V}_{k,1}$.

Step 4: Finally, the kth user's QR-SVD precoding matrix can be expressed as

$$\mathbf{W}_{QR\text{-}SVD\text{-}k} = \mathbf{W}_k^1 \mathbf{W}_k^2 = \mathbf{Q}_{k,4} \mathbf{V}_{k,1}, \quad 1 \le k \le K \tag{12}$$

The decoding matrix for each user at the receiving end is $\mathbf{U}_k^H$. After precoding and channel transmission, the received signal at user k becomes:

$$\begin{aligned} y_k' &= \mathbf{U}_k^H y_k = \mathbf{U}_k^H \left(\mathbf{H}_k \mathbf{W}_k s_k + \mathbf{H}_k \sum_{i \ne k} \mathbf{W}_i s_i + n_k \right) \\ &= \mathbf{\Sigma}_k s_k + \mathbf{U}_k^H \mathbf{H}_k \sum_{i \ne k} \mathbf{W}_i s_i + \mathbf{U}_k^H n_k \end{aligned} \tag{13}$$

The above equation shows that $\mathbf{U}_k^H \mathbf{H}_k \mathbf{W}_k = \mathbf{\Sigma}_k$, where $\mathbf{\Sigma}_k$ is a diagonal matrix. And this shows that user's each stream data is respectively transmitted on a separate sub-channel, which can eliminate the user's internal interference. Besides, the extended channel matrix takes account of both multi-user and noise interference. So QR-SVD precoding scheme can effectively suppress intra-user, inter-user and noise interference at the same time. Thus, QR-SVD precoding scheme can reduce system bit error rate.

4. Computational Complexity Analysis and Simulation.

4.1. Computational complexity analysis. This part is QR-SVD and existing RBD scheme computational complexity analysis. The total number of floating-point operations (flops) is used to measure the precoding algorithm computational complexity. [10,11] gave the number of flops required by real matrix QR and SVD. The number of flops needed for SVD of $m \times n$-dimensional complex matrix is equivalent to that of $2m \times 2n$-dimensional real matrix. Here are the numbers of flops needed to perform various operations on complex matrix.

For an $m \times n$ $(m \ge n)$-dimensional complex matrix, its QR complexity is $16\left(m^2 n - mn^2 + n^3/3\right)$ flops. The complexity of $m \times n$-dimensional complex matrix multiplied by $n \times p$-dimensional complex matrix is $8mnp$. For an $m \times n$ $(m \le n)$-dimensional complex matrix where only $\mathbf{\Sigma}$ and $\mathbf{V}$ are obtained, its SVD complexity is $32\left(nm^2 + 2m^3\right)$ flops. For an $m \times n$ $(m \le n)$-dimensional complex matrix where $\mathbf{U}$, $\mathbf{\Sigma}$ and $\mathbf{V}$ are obtained, its SVD complexity is $8\left(4n^2 m + 8nm^2 + 9m^3\right)$ flops [12,13].

Assuming that each user has the same number of receive antennas, denoted as $M_k = M$, $\overline{M} = (K-1)M$. $(6, 2, 3, 2)$ case indicates that the number of transmit antennas is $N = 6$, the system has 2 cells, each user receive antennas number is $M = 3$ and the users number is $K = 2$. In $(6, 2, 3, 2)$ case, Table 1 is RBD precoding algorithm computational complexity and Table 2 is QR-SVD precoding algorithm complexity.

TABLE 1. In $(6, 2, 3, 2)$ case, RBD precoding algorithm computational complexity

Steps	*Operations*	*flops*	$(6, 2, 32)$ *case*
1	$\overline{\mathbf{H}}_k = \overline{\mathbf{U}}_k \overline{\mathbf{\Sigma}}_k \overline{\mathbf{V}}_k^H$	$32K\left(N\overline{M}^2 + 2\overline{M}^3\right)$	*6912*
2	$\mathbf{W}_k^1 = \overline{\mathbf{V}}_k \left(\overline{\mathbf{\Sigma}}_k^T \overline{\mathbf{\Sigma}}_k + \alpha \mathbf{I}_N\right)^{-\frac{1}{2}}$	$K\left(8N^3 + 18N + \overline{M}\right)$	*3678*
3	$\mathbf{H}_k^{equ} = \mathbf{H}_k \mathbf{W}_k^1$	$8KMN^2$	*1728*
4	$\mathbf{H}_k^{equ} = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^H$	$8K\left(4N^2 M + 8NM^2 + 9M^3\right)$	*17712*
			Total 30030

TABLE 2. In $(6, 2, 3, 2)$ case, QR-SVD precoding algorithm computational complexity

Steps	*Operations*	*flops*	$(6, 2, 32)$ *case*
1	$\widetilde{\overline{\mathbf{H}}}_k^H = \mathbf{Q}_k \mathbf{R}_k$	$16K\left(N^2\overline{M} + N\overline{M}^2 + \frac{1}{3}\overline{M}^3\right)$	*5472*
2	$\mathbf{H}_k^{equ} = \mathbf{H}_k \mathbf{Q}_{k,4}$	$8KMN^2$	*1728*
3	$\mathbf{H}_k^{equ} = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^H$	$8K\left(4N^2M + 8NM^2 + 9M^3\right)$	*17712*
			Total 24912

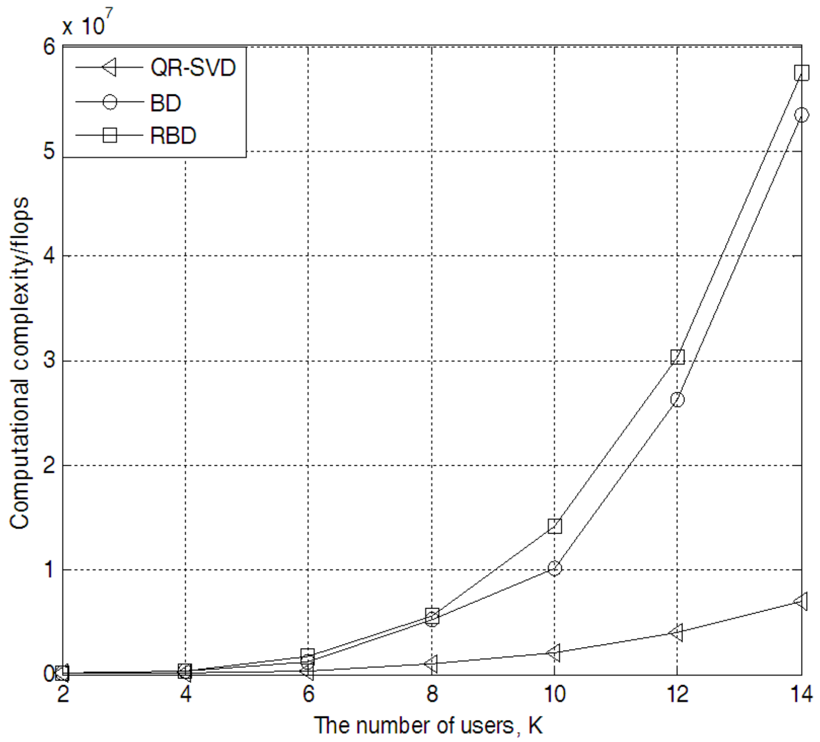

FIGURE 2. Various precoding algorithms computational complexity

As can be seen from Table 1 and Table 2, in $(6, 2, 3, 2)$ case, QR-SVD algorithm computational complexity is far less than RBD algorithm, providing a theoretical basis for practical application.

4.2. **Computational complexity simulation.** This part is QR-SVD, BD and RBD precoding algorithm computational complexity simulation result. Figure 2 shows various precoding algorithms complexity varying with the number of users. From Figure 2, with M and N fixed ($M = 3$, $N = 6$), BD and RBD precoding algorithms computational complexity grows relatively faster than QR-SVD algorithms with the increase of K. The reason is that the first SVD operation of BD and RBD precoding algorithms is implemented K times on $\overline{\mathbf{H}}_k$ with dimension $\overline{M}_k \times N$.

The proposed QR-SVD precoding algorithm shows the lowest computational complexity in Figure 2. The reason is that we use a less complex QR decomposition to replace the first SVD operation in BD and RBD precoding algorithms.

5. Algorithm Performance Analysis and Simulation.

5.1. System capacity analysis and simulation. This part is QR-SVD, ZF, BD and RBD precoding algorithm system capacity analysis and simulation. System capacity is calculated using the following formula.

$$C = \sum_{k=1}^{K} \log_2 \left(1 + \frac{\|\mathbf{H}_k \mathbf{W}_k\|^2}{\sum_{i \neq k} \|\mathbf{H}_k \mathbf{W}_i\|^2 + \sigma_k^2} \right) \quad \text{(bps/HZ)} \tag{14}$$

The simulation is carried out in MATLAB 7.0 simulation environment. Assuming that cell number of multi-user CoMP model is 2, per base station transmit antennas number is 3, so the total number of transmit antennas at the base stations is 6. User number is 2, each user's receive antennas number is 3, so the total number of receive antennas at the user equipments is 6. Modulation mode adopts quadrature phase shift keying (QPSK). The channel model employs flat Rayleigh channel, and Rayleigh distribution is a zero mean and σ_k^2 variance stationary narrowband Gauss process. Simulation parameters are shown in Table 3 [14].

Figure 3 shows various precoding algorithms system capacity performance. RBD precoding algorithm illustrates almost the same system capacity performance as BD precoding algorithm. However, the system capacity of ZF precoding algorithm is slightly inferior

TABLE 3. Simulation parameters

Simulation parameters	*Values*
Total number of transmit antennas	*6*
Cell number	*2*
Each user receive antennas number	*3*
Users number	*2*
Modulation mode	*QPSK*
Channel mode	*Flat Rayleigh*

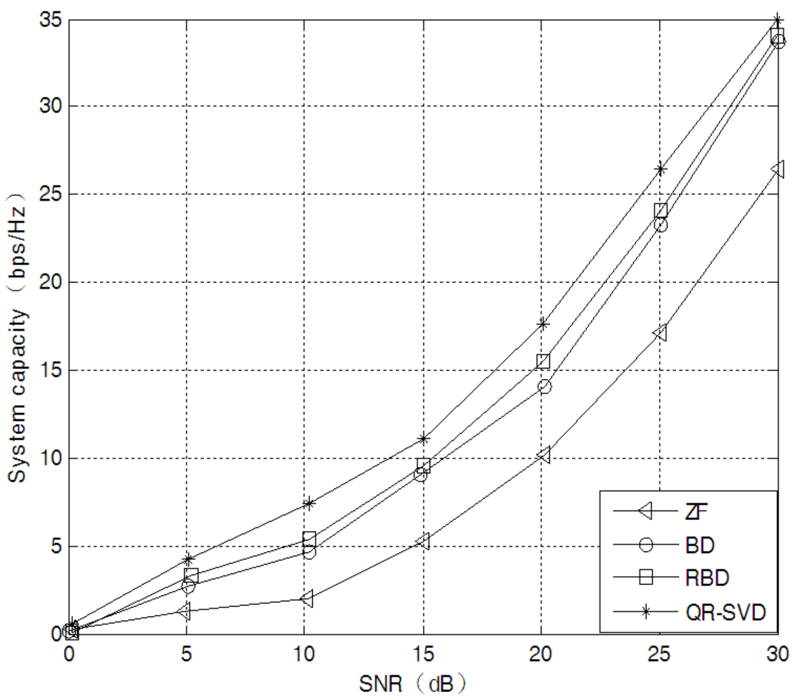

FIGURE 3. Various algorithms system capacity

to the other precoding algorithms. The proposed QR-SVD precoding algorithm shows the best system capacity performance. The reason is that QR-SVD precoding algorithm can suppress intra-user, inter-user and noise interference at the same time.

5.2. **Bit error rate analysis and simulation.** This part is QR-SVD, ZF, BD and RBD precoding algorithm bit error rate (BER) analysis and simulation. Bit error rate is calculated using the following formula.

$$BER = 1 - \left[1 - \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)\right] \tag{15}$$

where E_b is the average bit energy, and N_0 is the noise power spectral density.

The bit error rate has the same simulation settings as system capacity. Figure 4 illustrates various precoding algorithms bit error rate performance. The proposed QR-SVD precoding algorithm shows the best BER performance in the range of 0∼30dB. The reason is that QR-SVD precoding algorithm can suppress intra-user, inter-user and noise interference at the same time. However, ZF and BD precoding algorithms cannot eliminate noise interference. And RBD precoding algorithm cannot eliminate intra-user interference.

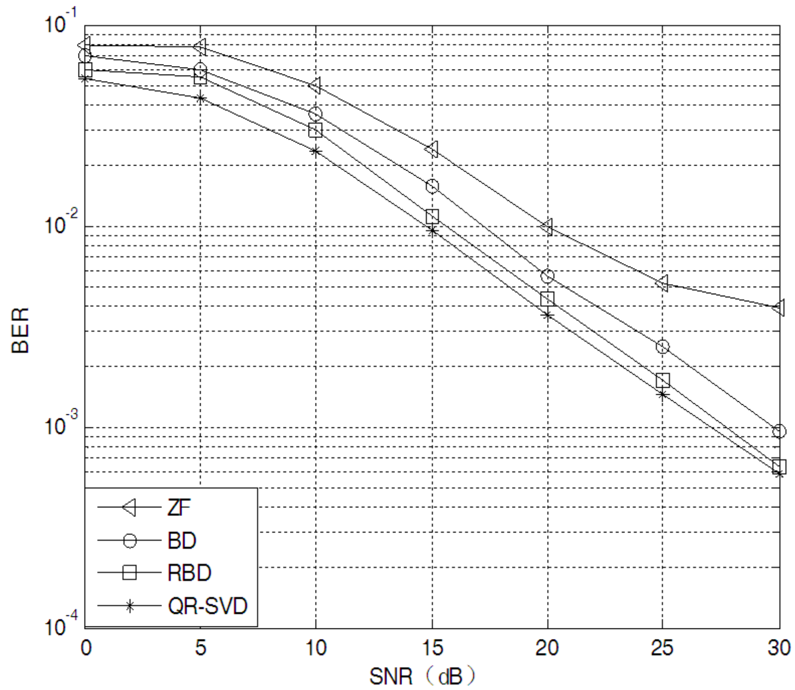

Figure 4. Various algorithms BER

6. **Conclusion.** This paper proposes a new coordinated multi-point precoding algorithm based on QR-SVD. The proposed algorithm extends channel matrix to suppress multi-user and noise interference, then introduces matrix decomposition method QR, requiring a much lower computational complexity, and finally applies SVD to equivalent channel matrix to obtaining complete precoding matrix and decoding matrix. The proposed algorithm has certain advantages over other algorithms in terms of computational complexity, system capacity and bit error rate.

In LTE-A system, there are many CoMP precoding algorithms. This paper just describes a part of linear precoding algorithms in CoMP application. Future research could focus on better linear and nonlinear precoding algorithms, obtaining more outstanding performance and providing more solid theoretical basis for practical applications.

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