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OPTIMIZATION OF VEHICLE SCHEDULING PROBLEM WITH POTENTIAL DEMAND BASED ON CS-GA

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Abstract. *Considering the purchasing behaviors of customers in the real practice, this paper focuses on the vehicle scheduling problem in which the potential demand is considered. A mathematical model with soft time windows is developed to find the optimal routes of vehicles by minimizing the total cost in the distribution. Considering the proposed model is a typical NP-hard problem, we propose a novel hybrid algorithm CS-GA based on the constraint satisfaction (CS) method and genetic algorithm (GA) to find the optimal solutions, in which the initial solutions are generated by CS and then GA is used to optimize and search in the solution space. Finally, the comparison results illustrate the applicability and effectiveness of the proposed model and algorithm.*
Keywords: Potential demand, Vehicle scheduling, Soft time windows, GA, CS

1. **Introduction.** The classical vehicle routing problem (CVRP) is first introduced by Dantzig and Ramser in their study on the truck dispatching problem [1]. However, there are no limits for the delivery time and the demand of customers is often fixed. Numerous variants of CVRP have been proposed and extensively studied which are based on the complications of real-world problems. Depending on different assumptions and constraints, they can be classified into VRP with time windows (VRPTW) [2,3], in which customers have to be visited within a predefined time interval which can be described by the earliest arrival time and the latest arrival; VRP with multiple depots (VRPMD) [4]; VRP with stochastic demands (VRPSD) [5], which involves demands that are random variables with known distributions; VRP with split delivery, where the demand of a customer can be split and delivered by multiple vehicles [6]; the heterogeneous fleet VRP with several vehicle types [7], etc. These works are more related to the real cases, but the purchasing behaviors of customers may be affected by other customers, i.e., conformity behavior, which could lead to the change of the demand. However, few papers of VRP consider this scenario in the optimization. So, the following model will model VRP with soft time windows and split delivery by considering the conformity behavior.

Due to the fact that the VRP is a typical NP-hard problem, in the literature many heuristics-based studies are proposed for various VRPs. The work of Pang [8] applied the improved route construction heuristics to solve VRPTM, and Wang and Wang [9] raised the novel heuristics based on the traditional two-phase method and the results demonstrated the effectiveness of the proposed heuristics. Most of the recent researches for VRPs paid extensive attention to the development of meta-heuristics. This can be summarized as: genetic algorithms, ant colony optimization (ACO), simulated annealing (SA), variable neighborhood search (VNS), local search algorithm, artificial bee colony (ABC), and particle swarm optimization (PSO). For further details on VRP meta-heuristics, we refer the reader to [10]. Among them, the most commonly used algorithm on VRPS is GA. However, the generation of initial solutions is a key issue in GA. Usually, it is generated randomly, but it may cost more time to be adjusted and modified. Thus, this paper introduces the constraint satisfaction method used to the generation of initial solutions [11]. Followed by it, GA is adopted to optimize the solutions. The research space can be effectively controlled in the proposed algorithm which avoids the time consumption and fails to the local optimum. The outline of the remaining paper is organized as follows: in Section 2 the mathematical model is developed; Section 3 gives the designed algorithm and a numerical example is shown in Section 4; Section 5 draws conclusions this paper.

2. The Mathematical Model of VRP Considering Potential Demand.

2.1. Problem description and assumptions. In this paper, we consider the vehicle routing problem with a central depot "0", and a set of customers $\{1, 2, \ldots, N\}$, which is defined by a directed graph $G = (V, E)$. $V = \{0, 1, 2, \ldots, N\}$ is the node set including 0, N customers and E is the set of arcs (i, j), $i, j \in V$. Arc (i, j) represents the possibility to travel from i to j with an associated distance, duration or cost. The customers are serviced by K homogeneous vehicles with a limited capacity $Q_{\max}$ and the same speed. In the following, some assumptions are given for the modeling.

(1) The order of each customer is allowed to be split, which implies that the customer can be visited more than one by different vehicles.

(2) The duration between the leaving time of a vehicle and the arrival time is defined as a cycle. Since any vehicles can visit customers many times, the route of a vehicle may consist of multiple cycles.

(3) For customers, two kinds of demand are considered, which are subject to the initial demand d_i and the potential demand due to the conformity behavior d_i'. However, it is assumed that the initial demand is given priority to be met.

(4) Soft time windows are imposed on the demand; $[E_i, L_i]$ for the initial demand of customer i and $[E_i', L_i']$ for the potential demand of customer I, in which E_i and E_i' are the earliest arrival time, L_i and L_i' are the latest arrival time that customer i can be serviced by a vehicle. The penalty cost will produce once the time windows are incurred.

2.2. Notations.

k	Index of vehicles, $k = 1, 2, \ldots, K$
L_k	The total number of vehicle k's cycle, $l = 1, 2, \ldots, L_k$
p	Penalty cost per time due to waiting for service
q	Penalty cost per time due to delaying service, $p < q$
f_i	Servicing duration at customer i
t_{ij}	Travel time from customer i to customer j with the travel cost per unit time c_{ij}
C	Fixed cost per vehicle
t_{ijl}^k	The arrival time of vehicle k from customer i to customer j in the kth cycle
x_{ijl}^k	Decision variable; it is equal to 1 when vehicle k starts from customer i to customer j in the lth cycle; otherwise, 0
b_{il}^k	Decision variable; the demand of customer i satisfied by vehicle k in the lth cycle.

2.3. **Mathematical model.** The mathematical model is given as follows:

$$\min z = CK + \sum_{k=1}^{K}\sum_{l=1}^{L_k}\sum_{i=0}^{N}\sum_{j=0}^{N} x_{ijl}^{k}\cdot t_{ij}\cdot c_{ij} + \sum_{k=1}^{K}\sum_{l=1}^{L_k}\sum_{i=1}^{N}\sum_{j=1}^{N}\left[p\left(E_j - t_{ijl}^{k}\right)\right.$$
$$\left. +q\left(t_{ijl}^{k} - L_j\right) + p\left(E_j' - t_{ijl}^{k}\right) + q\left(t_{ijl}^{k} - L_j'\right)\right] \quad (1)$$

$$\text{s.t.} \quad \sum_{j=1}^{N} x_{0jl}^{k} = 1, \quad k = \{1,2,\ldots,K\},\ l = \{1,2,\ldots,L_k\} \quad (2)$$

$$\sum_{j=1}^{N} x_{j0l}^{k} = 1, \quad k = \{1,2,\ldots,K\},\ l = \{1,2,\ldots,L_k\} \quad (3)$$

$$\sum_{i=0(i\neq j)}^{N} x_{ijl}^{k} = 1, \quad k = \{1,2,\ldots,K\},\ l = \{1,2,\ldots,L_k\},\ j = \{1,2,\ldots,N\} \quad (4)$$

$$\sum_{i=0}^{N} x_{ihl}^{k} - \sum_{j=0}^{N} x_{hjl}^{k} = 0, \quad k = \{1,2,\ldots,K\},\ l = \{1,2,\ldots,L_k\} \quad (5)$$

$$\sum_{i=0}^{N}\sum_{j=1}^{N} x_{ijl}^{k}\cdot b_{jl}^{k} \le Q_{\max}, \quad k = \{1,2,\ldots,K\},\ l = \{1,2,\ldots,L_k\} \quad (6)$$

$$\sum_{k=1}^{K}\sum_{l=1}^{L_k}\sum_{i=0}^{N} x_{ijl}^{k}\cdot b_{jl}^{k} = d_j + d_j', \quad j = \{1,2,\ldots,N\} \quad (7)$$

$$t_{ijl}^{k} + f_j + t_{jh} \le \sum_{h\in V(h\neq j)}^{N} x_{jhl}^{k}\cdot t_{ihl}^{k}, \quad k = \{1,2,\ldots,K\},\ l = \{1,2,\ldots,L_k\},$$
$$j = \{1,2,\ldots,N\} \quad (8)$$

Objective function, Equation (1), is to minimize the total cost. Equations (2) and (3) indicate that all vehicles start from the central depot and return to the central depot through some customers, respectively. Each vehicle visits only one customer in each cycle, as imposed in Equation (4). The sequence of a route "$i \to h \to j$" for vehicle k in the lth cycle is imposed by Equation (5) and Equation (6) gives the constraint for the vehicle capacity. The demand of customers is met after the distribution process, as shown in Equation (7). The time constraint is controlled by Equation (8) to ensure the time of a vehicle k in the lth cycle reaching to customer j must be not earlier than $t_{ril}^{k} + f_i + t_{ij}$.

3. Design of an Improved Algorithm CS-GA.

3.1. **The framework of the CS-GA.** In this section, we design a hybrid algorithm CS-GA in which an initial population for the vehicle routing problem considering the potential demand is created by the CS method, based on which the genetic algorithm is used to search the optimal solution. The detailed steps are as follows.

Step 1: Initialize the model and algorithm parameters.

Step 2: Generate solutions. Using constraint propagation techniques generates the feasible solution: check whether a variable conflicts to the constraint conditions. If there is no conflict, an assign is carried out for the next variable; otherwise, remove conflict until a feasible solution is obtained.

Step 3: Check that whether the number of feasible solutions meets the population size N, if it is met, turn to GA for the optimization; otherwise, continue to generate feasible solutions.

Step 4: Evaluate the fitness of each individual in the population.

Step 5: (Find the optimal solution) Perform the genetic operators until the iteration termination condition is met, and ouput the results.

3.2. **Solution presentation.** Integer representation is used in this paper. However, since a route consists of multiple cycles, rather than delivery only once, we defined an index ki where k and i are the indices of vehicle and node respectively. The generalizable chromosome representation is then described in Figure 1.

3.3. **Fitness evaluation.** Using the objective function shown in Equation (1), the fitness is defined as the reciprocal of z, i.e., $F = M - z$, in which M is infinite positive integer. Thus, the fitness value of each individual from the initial population and its offspring can be calculated to scale the performance of each individual.

3.4. **Genetic operators.** (1) Selection operator. The Roulette Wheel selection is used to select fitter individuals. It implies that the probability of individuals with lower cost to be selected for the next generation is higher. (2) Crossover operator. We propose a novel crossover operator, as shown in Figure 2. (3) Mutation operator. 2-change mutation technique is performed in this paper and the operator process is shown in Figure 3.

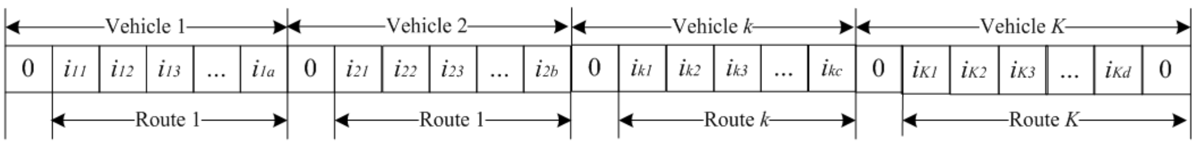

FIGURE 1. The chromosome structure

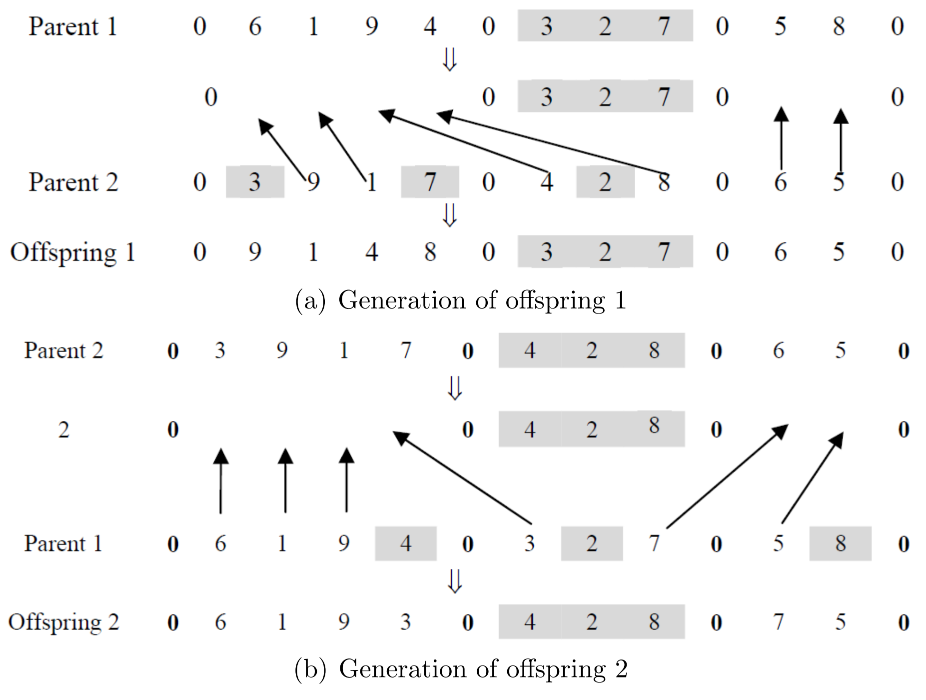

FIGURE 2. Crossover process

FIGURE 3. The mutation operator

3.5. **Algorithm parameters.** The algorithm parameters can affect directly the performance of the algorithm, which involve the crossover probability P_c, the mutation probability P_m, and the population size N. Thus, a self-adaptive adjust strategy is proposed as shown in Equation (9).

$$P_c = \begin{cases} k_1(f_0 - f')/\left(f_0 - \overline{f}\right) & f' \geq \overline{f} \\ k_2 & f' < \overline{f} \end{cases}, \quad P_m = \begin{cases} k_3(f_0 - f)/\left(f_0 - \overline{f}\right) & f \geq \overline{f} \\ k_4 & f < \overline{f} \end{cases} \tag{9}$$

where k_1, k_2, k_3, k_4 are constants in the interval $(0, 1]$, f' is the individual with the larger fitness, $\overline{f}$ denotes the average fitness in the population, and f is used to represent the fitness value of the mutation individual. Commonly, k_1 and k_3 are set to 0.75-0.95 and 0.005-0.01; the values of k_2 and k_4 are usually set to the relatively great values to obtain the fitter individuals.

4. **Numerical Example.** The model parameters are shown in Table 1 and other parameters are set to $K = 6$, $Q_{\max} = 65$, $C = 1000$, $p = 2.5$, $q = 4$. The corresponding algorithm parameters are $N = 10$, $T = 500$, $k_1 = 0.85$, $k_2 = 0.90$, $k_3 = 0.01$, $k_4 = 0.95$.

Based on the proposed algorithm CS-GA, the route results are obtained as shown in Table 2 and the comparison results of the proposed CS-GA with GA are given in Table 3. From Table 1, the optimal route for each vehicle can be determined and the unloading quantity at each customer is given. There are five experiments given in Table 3, in which it is obvious that the results obtained by the proposed algorithm are better and

Table 1. The demand information

Customer i		$[E_i, L_i]$	d'_i	$[E'_i, L'_i]$	Customer i		$[E_i, L_i]$	d'_i	$[E'_i, L'_i]$
1	34	$[20, 50]$	15	$[85, 150]$	9	100	$[50, 100]$	0	–
2	50	$[35, 80]$	10	$[175, 200]$	10	35	$[40, 80]$	10	$[75, 110]$
3	10	$[53, 115]$	40	$[90, 110]$	11	47	$[130, 170]$	10	$[140, 165]$
4	75	$[20, 45]$	5	$[85, 130]$	12	30	$[150, 200]$	20	$[200, 230]$
5	55	$[60, 90]$	10	$[105, 120]$	13	0	–	40	$[70, 130]$
6	20	$[120, 150]$	35	$[130, 170]$	14	45	$[100, 140]$	25	$[110, 160]$
7	0	–	60	$[65, 100]$	15	80	$[135, 170]$	0	–
8	55	$[80, 130]$	15	$[90, 150]$					

Table 2. The optimal routes

Vehicle	Optimal routes	Demand
1	$0 \to 1 \to 3 \to 4 \to 9 \to 0' \to 11 \to 15 \to 0$	$0 \to d34 \to d10 \to d10 \to d11 \to 0' \to d47 \to d15 \to 0$
2	$0 \to 4 \to 0' \to 9 \to 6 \to 0' \to 3 \to 14 \to 0$	$0 \to d65 \to 0' \to d44 \to d20 \to 0' \to d'40 \to d'25 \to 0$
3	$0 \to 2 \to 9 \to 0' \to 8 \to 5 \to 0' \to 5 \to 0$	$0 \to d50 \to d15 \to 0' \to d55 \to d'10 \to 0' \to d'60 \to 0$
4	$0 \to 10 \to 9 \to 0' \to 15 \to 0' \to 8 \to 12 \to 0$	$0 \to d35 \to d30 \to 0' \to d65 \to 0' \to d'15 \to d'20 \to 0$
5	$0 \to 7 \to 4 \to 0' \to 6 \to 12 \to 0$	$0 \to d'60 \to d'5 \to 0' \to d'35 \to d30 \to 0$
6	$0 \to 13 \to 1 \to 10 \to 0' \to 14 \to 2 \to 11 \to 0$	$0 \to d'40 \to d'15 \to d'10 \to 0' \to d45 \to d'10 \to d'10 \to 0$

Table 3. The comparison results

No.	GA		CS-GA	
	Z ($*\ 10^4$)	Time (s)	z	Time (s)
1	1.87	365	1.45	201
2	2.19	452	1.34	248
3	1.64	584	1.29	189
4	1.74	617	1.47	239
5	1.92	514	1.51	274

the convergence speed is accelerated by the use of the constraint satisfaction method. It can be explained that the infeasible solution is removed by the constraint satisfaction technique and the convergence speed is relatively faster than GA.

5. **Conclusions.** This paper focused on the vehicle routing problem with the potential demand and soft time windows, and then a mathematical model is established to find the optimal routes of vehicles by minimizing the total cost. In order to solve the proposed model, an improved algorithm CS-GA is designed based on the constraint satisfaction method and genetic algorithm. The results of the proposed algorithm compared to GA illustrated the effectiveness and the applicability of the proposed algorithm in the numerical example. Future research will focus on different types of vehicles.

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