ICIC Express Letters ICIC International ©2017 ISSN 1881-803X
Volume **11**, Number **2**, February **2017** pp. **491–496**

VIBRATIONAL RESONANCE IN A PARALLEL ARRAY OF DYNAMICAL SATURATING NONLINEARITY

Yumei Ma[1], Zhenkuan Pan[1], Fabing Duan[2] and Jinpeng Yu[2]

[1]College of Computer Science & Technology
[2]College of Automation and Electrical Engineering
Qingdao University
No. 308, Ningxia Road, Qingdao 266071, P. R. China
mayumei_qdu@163.com

Received July 2016; accepted October 2016

Abstract. *This paper studies the vibrational resonance in a parallel array of dynamical nonlinearities with saturation and a high-frequency interference signal works as the noise in each nonlinear system. By tuning the high-frequency signal level, the signal-to-noise ratio (SNR) can be enhanced. The numerical results show that the high-frequency interference signal and the dynamical nonlinearity with saturation are positive to weak-periodic signal processing.*

Keywords: High-frequency, Vibrational resonance, Weak signal, Dynamical nonlinearity

1. **Introduction.** Stochastic resonance (SR) is a nonlinear phenomenon where noise plays a constructive role and a weak signal can be enhanced in some certain nonlinear systems [1, 2, 3, 4, 5, 6]. The method of enhancement via noise is still useful for nonlinear signal processing. Then vibrational resonance (VR) [7] is a similar effect to SR where the system is under the action of a two-frequency signal and the internal noise is replaced by a high-frequency interference signal. The VR effect is considered as a new form of SR. Soon afterwards, the research fields of VR are extended to nonlinear circuits, optimal devices, excitable neurons and so on [8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. Then, compared with the SR method, a better result of signal processing can be obtained by VR method in a bistable system [15]. The output signal-to-noise ratio (SNR) was used as a typical quantifier for SR [3, 18, 19], i.e., a non-monotonic function of the background noise intensity. Similarly, the SNR is also employed for studying the VR effects in a nonlinear system.

In this paper, we focus on the enhancement of the SNR, i.e., the high-frequency vibrational resonance effect in an uncoupled parallel array of dynamical nonlinearity with saturation [20, 21] subject to a weak-periodic signal in additive white noise. An internal high-frequency interference sinusoidal signal with different frequencies is as the internal array noise component in each subsystem. A dynamical nonlinearity with saturation can be as a class of potential vibrational resonator or signal processor. A mass of behaviors of the output SNR are observed which is a function of the high-frequency sinusoidal level and the array size. It is shown that the high-frequency vibrational resonance effect appears in an array of dynamical nonlinearity with saturation upon increasing high-frequency sinusoidal level and the array size. It is demonstrated that similar enhanced propagation can be obtained by replacing the array noise with a high-frequency signal.

These results show that a parallel array of dynamical saturating nonlinearities can be practically exploited, and is useful for nonlinear signal processing. In practical processing, sinusoidal interference signal is easier produced and controlled than noise. It is shown that the easily implementable feature of sinusoidal vibrations and the achievable maximum of SNR of an infinite array, indicate a preferable strategy for processing weak signals via the VR mechanism. It is expected the results of VR will be further applicable for telecommunication signal detection, neuroscience, and medicine. For instance, informative signals are usually modulated by high-frequency carriers. Then, in some complicated environments, the high-frequency vibrations, not restricted to sinusoidal waves, might elicit more information about inputs to form an ensemble of suboptimal systems. Also, the tractable method of an infinite array might bring some enlightenment to the high-frequency circuit design.

2. **Theoretical Model.** Consider the observation of a process $k(t) = s(t) + \beta(t)$, where the component $s(t)$ is a weak-periodic sinusoid signal with period T and a maximal amplitude A ($|s(t)| \leq A$). The component $\beta(t)$ is zero-mean additive Gaussian noise, independent of $s(t)$, and has a probability density function (PDF) f_β and variance $\sigma_\beta^2 = \mathrm{E}_\beta[x^2] = \int_{-\infty}^{\infty} x^2 f_\beta(x)dx$. The process $k(t)$ is applied to an uncoupled parallel array of N dynamical nonlinearities g. In these nonlinearities, a high-frequency interference sinusoidal signal as the internal noise has a much higher frequency than input signal $s(t)$ which can be written by Equation (1)

$$\theta_n(t) = A_\theta \sin(2\pi f_n t), \tag{1}$$

where A_θ is the amplitude and f_n is the frequency of the high-frequency sinusoidal signal in the nonlinear array. So the outputs $x_n(t)$ of each nonlinearity can be defined as

$$x_n(t) = g(k(t) + \theta_n(t)), \quad n = 1, 2, \ldots, N, \tag{2}$$

The array outputs $U(t)$ are averaged and the response of the array is written as

$$U(t) = \frac{1}{N}\sum_{n=1}^{N} x_n(t). \tag{3}$$

For a weak-periodic signal $s(t)$ ($A \to 0$), the mean at a fixed time t is $\mathrm{E}[U(t)]$ and the nonstationary expectation is $\mathrm{E}[U^2(t)]$. Based on the former, the variance $\mathrm{var}[U(t)]$ can be obtained. The system performance can be evaluated by the output SNR, defined as the power contained in the output spectral line at fundamental frequency $1/T$ divided by the power contained in the noise background in a small frequency bin ΔB around $1/T$, that is [3]

$$R_{\mathrm{out}} = \frac{|\langle \mathrm{E}[U(t)] \exp[-i2\pi t/T] \rangle|^2}{\langle \mathrm{var}[U(t)] \rangle H(1/T_s)\Delta B}, \tag{4}$$

with the operator $\langle \cdots \rangle = \frac{1}{T}\int_0^T \cdots dt$ indicating a temporal average [3]. Similarly, the input SNR is given by

$$R_{\mathrm{in}} = \frac{A^2/4}{\sigma_\beta^2 \Delta B \Delta t}. \tag{5}$$

As the array size $N \to \infty$, the output SNR of an array of the nonlinearities can be defined as

$$R_{\mathrm{out}}^{\infty} = \frac{|\langle \mathrm{E}[x_n(t)] \exp(-i2\pi t/T) \rangle|^2}{\langle \mathrm{E}[x_n(t)x_m(t)] - \mathrm{E}^2[x_n(t)] \rangle H(1/T_s)\Delta B}, \tag{6}$$

for $n, m = 1, 2, \ldots, N$ and $n \neq m$.

3. **Experiments.** A model with a so-called 'soft' potential is considered as Equation (7).

$$C\dot{x} = -\frac{x}{R} + J\tanh(\nu x) + s(t) + \beta(t) + \theta_n(t), \tag{7}$$

Such a model describes an element of an electronic neural network, where x is the neuron's membrane potential, C is the input capacitance, R is the transmembrance resistance, and J is a self-coupling coefficient. The external noise $\beta(t)$ is the zero-mean generalized Gaussian noise, which is a flexible family containing some common important cases (e.g., Gaussian noise, Laplacian noise). The PDF of the generalized Gaussian noise $\beta(t)$ is written as

$$f_\beta(x) = \frac{c_1}{\sigma_\beta}\exp\left(-c_2\left|\frac{x}{\sigma_\beta}\right|^\alpha\right), \tag{8}$$

where c_1 and c_2 are written by Equation (9) and Equation (10)

$$c_1 = \frac{\alpha}{2}\Gamma^{\frac{1}{2}}\left(3\alpha^{-1}\right)/\Gamma^{\frac{3}{2}}\left(\alpha^{-1}\right), \tag{9}$$

$$c_2 = \left[\Gamma\left(3\alpha^{-1}\right)\Gamma\left(\alpha^{-1}\right)\right]^{\alpha/2}. \tag{10}$$

In Figure 1 and Figure 2, the input signal is a weak signal with amplitude $A = 0.2$ and frequency f. The decay parameter of the external noise $\beta(t)$ is set $\alpha = 2$, i.e., Gaussian noise with root-mean-square (RMS) amplitude $\sigma_\beta = 0.3$. The high-frequency sinusoidal interference signal of the array follows $\theta_n(t) = A_\theta \sin[2\pi(20+n)ft]$ and has a much higher frequency $f_n = (20+n)f$, $n = 1, 2, \ldots, N$. Figure 1 and Figure 2 demonstrate the evolution of the output SNR as a function of the amplitude A_θ of the high-frequency interference sinusoidal signal $\theta_n(t)$ and the array size N. From the bottom up, the output SNR of the nonlinear array expressed as Equation (6) is shown for $N = 1, 5, 10, 50$ and ∞ with solid lines. It is shown that the vibrational resonance evolutions of the output SNR are demonstrated upon increasing the array interference level A_θ and the array size N, i.e., the vibrational resonance effect both exist in Figure 1 and Figure 2. It is shown that an uncoupled parallel dynamical array with high-frequency interference signal plays an constructive role for transmitting a weak-periodic signal.

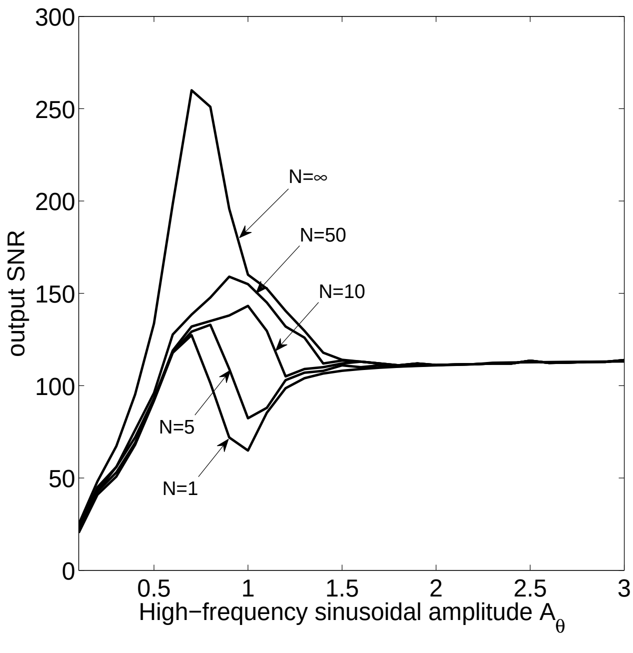

FIGURE 1. Output SNR R_{out} as a function of the amplitude A_θ of the array high-frequency interference sinusoidal signal $\theta_n(t)$ with $J = 0.5$

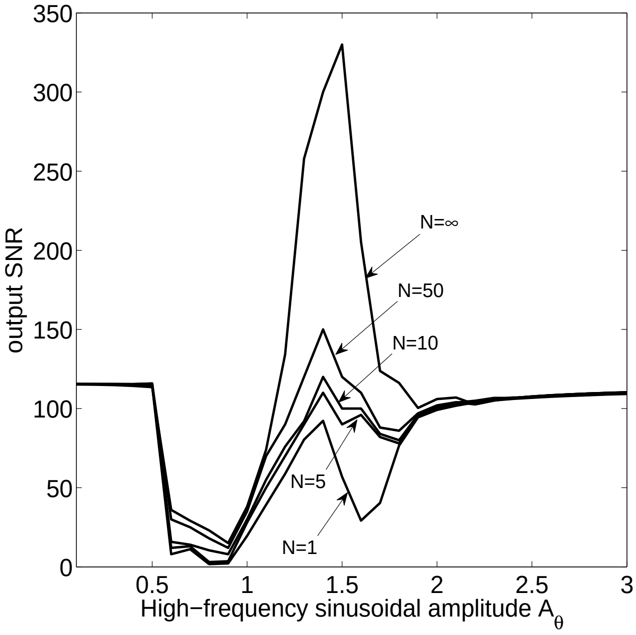

FIGURE 2. Output SNR R_{out} as a function of the amplitude A_θ of the array high-frequency interference sinusoidal signal $\theta_n(t)$ with $J = 1$

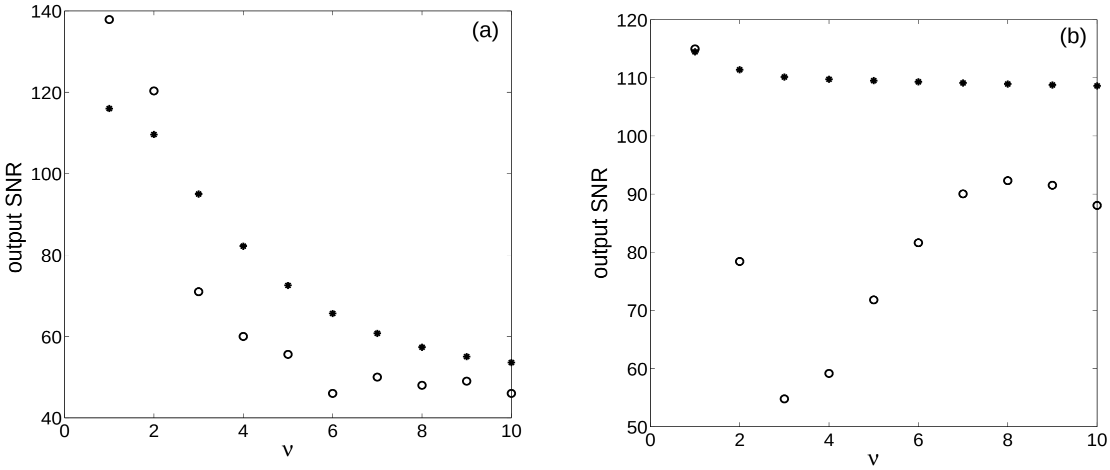

FIGURE 3. Output SNR R_{out} as a function of the slope ν of the dynamical nonlinearity of Equation (6) with array size $N \to \infty$

There are some differences between Figure 1 and Figure 2. The high-frequency vibrational resonance effect is obviously in Figure 1, but the output SNR is bigger in Figure 2 when the array size $N \to \infty$. What is more, when $J = 0.5$ the maximum of the output SNR can be obtained nearby the sinusoidal signal amplitude $A_\theta = 1$. However, with $J = 1$ the peak of the output SNR is acquired around the sinusoidal signal amplitude $A_\theta = 1.5$. The high-frequency interference sinusoidal signal of the dynamical nonlinear array can induce the vibrational resonance effect for transmitting weak-periodic signal upon increasing the high-frequency interference signal level and the array size.

It is shown that the output SNR is plotted as a function of the slope ν of the dynamical nonlinearity of Equation (7) as the array size $N \to \infty$ in Figure 3. The slope of Equation (7) takes $C = 1$, $R = 1$. The amplitude A_θ of the array high-frequency interference sinusoidal signal $\theta_n(t)$ is set $A_\theta = 1$ in Figure 3(a) and $A_\theta = 1.5$ in Figure 3(b) respectively.

Aterisks correspond to the output SNR with $J = 0.5$, and circles represent the output SNR with $J = 1$. It is demonstrated the results as the array size $N \to \infty$ accord to the output SNR of Figure 1 and Figure 2.

4. **Conclusion.** In this paper, the high-frequency vibrational resonance effect of an uncoupled parallel array of dynamical nonlinearities with saturation is studied. The output SNR can be enhanced by an uncoupled parallel array of dynamical nonlinearities. When the parameters of the dynamical nonlinearities with saturation vary, different VR effects can be obtained. We only study the dynamical nonlinearities with saturation. There are other interesting nonlinearities such as static nonlinearities, which may be of interest for further studies of weak signal processing in the context of VR. Sinusoidal interference signal is easier obtained and controlled than noise in practical situations and VR is meaningful for practical signal processing.

Acknowledgment. This work is partially supported by the National Natural Science Foundation of China (61501276, 61573204), the China Postdoctoral Science Foundation (2016M592139), and the Qingdao Postdoctoral Application Research Project (2015120).

REFERENCES

[1] R. Benzi, G. Parisi, A. Sutera and A. Vulpiani, Stochastic resonance in climate changes, *Tellus*, vol.34, pp.10-16, 1981.
[2] L. Gammaitoni, P. Hänggi, P. Jung and F. Marchesoni, Stochastic resonance, *Reviews of Modern Physics*, vol.70, no.1, pp.233-287, 1998.
[3] F. Chapeau-Blondeau and X. Godivier, Theory of stochastic resonance in signal transmission by static nonlinear systems, *Physical Review E*, vol.55, no.2, pp.1478-1495, 1997.
[4] B. Chen, Y. Zhu, J. Hu and M. Zhang, Weak signal detection: Condition for noise induced enhancement, *Digital Signal Processing*, vol.23, pp.1585-1591, 2013.
[5] H. Droogendijk, M. J. de Boer, R. G. P. Sanders and G. J. M. Krijnen, Stochastic resonance in a voltage-controlled micromechanical slider, *Microelectromechanical Systems*, vol.24, no.3, pp.651-660, 2015.
[6] T. Djurhuus and V. Krozer, A generalized model of noise driven circuits with application to stochastic resonance, *IEEE Trans. Circuits and Systems I: Regular Papers*, vol.62, no.8, pp.1981-1990, 2015.
[7] P. Landa and P. V. McClintock, Vibrational resonance, *Journal of Physics A: Mathematical and General*, vol.33, no.45, 2002.
[8] E. Ullner, A. Zaikin, J. Garcia-Ojalvo, R. Bascones and J. Kurths, Vibrational resonance and vibrational propagation in excitable systems, *Physics Letters A*, vol.312, no.5, pp.348-354, 2003.
[9] A. Zaikin, L. Lopez, J. Baltanás, J. Kurths and M. Sanjuán, Vibrational resonance, *Physics Review E*, vol.66, no.1, 2002.
[10] V. Chizhevsky and G. Giacomelli, Experimental and theoretical study of vibrational resonance in a bistable system with asymmetry, *Physics Review E*, vol.73, no.2, 2006.
[11] V. Gandhimathi, S. Kurths and M. Sanjuán, Vibrational and stochastic resonances in two coupled overdamped anharmonic oscillators, *Physics Letters A*, vol.360, no.2, pp.279-286, 2006.
[12] C. Yao and M. Zhan, Signal transmission by vibrational resonance in one-way coupled bistable systems, *Physics Review E*, vol.81, no.6, 2010.
[13] V. Chizhevsky and G. Giacomelli, Improvement of signal-to-noise ratio in a bistable optical system: Comparison between vibrational and stochastic resonance, *Physics Review E*, vol.71, no.1, 2005.
[14] M. Bordet and S. Morfu, Experimental and numerical enhancement of vibrational resonance in neural circuits, *Electronics Letters*, vol.48, no.15, 2012.
[15] P. R. VenKatesh and A. VenKatesan, Vibrational resonance and implementation of dynamic logic gate in a piecewise-linear Murali-Lakshmanan-Chua circuit, *Commun. Nonlinear. Sci. Numer. Simulat.*, vol.39, pp.271-282, 2016.
[16] Y. Ren and F. Duan, Theoretical and experimental implementation of vibrational resonance in an array of hard limiters, *Physica A*, vol.456, pp.319-326, 2016.
[17] K. B. Alici, Hybridization of Fano and vibrational resonances in surface-enhanced infrared absorption spectroscopy of streptavidin monolayers on metamaterial substrates, *IEEE Trans. Nanotechnology*, vol.13, no.2, 2016.
[18] P. Jung and P. Hänggi, Amplification of small signals via stochastic resonance, *Physical Review A*, vol.44, pp.8032-8042, 1991.

[19] F. Duan, F. Chapeau-Blondeau and D. Abbott, Noise-enhanced SNR gain in parallel array of bistable oscillators, *Electronics Letters*, vol.42, pp.1008-1009, 2006.

[20] A. R. Bulsara, J. F. Lindner, V. In, A. Kho, S. Baglio, V. Sacco, B. Ando, P. Longhini, A. Palacios and W.-J. Rappel, Coupling-induced cooperative behaviour in dynamic ferromagnetic cores in the presence of a noise floor, *Physics Letters A*, vol.353, pp.4-10, 2005.

[21] A. R. Bulsara, V. In, A. Kho, P. Longhini, A. Palacios, W.-J. Rappel, J. Acebron, S. Baglio and B. Ando, Emergent oscillations in unidirectionally coupled overdamped bistable systems, *Physical Review E*, vol.70, 2004.