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AN OPTIMIZATION INSPECTION POLICY MODEL FOR THREE-STAGE DEGRADATION SYSTEMS WITH HUMAN ERRORS

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Abstract. *This paper proposes an inspection policy for a single-component system based on a three-stage delay time concept, in which the system is periodically inspected and inspections are assumed to be perfect to identify the possible state of the system, i.e., normal, minor defect, severe defect and failed. Repair needs to be done immediately for the defective system regardless of the minor or severe defect. However, the defective system may be renewed after repair with different probabilities when different defects are detected by an inspection. It is because that it needs different abilities for repair labors when different defective states are dealt with. Accordingly, an optimization model is established by minimizing the expected cost per unit time, and we provide a numerical example to demonstrate the proposed model.*

Keywords: Delay time, Preventive maintenance, Inspection, Human errors, Renewal theory

1. **Introduction.** Inspection is one of the commonly preventive maintenance (PM) policies used in industry [1]. Through an inspection policy, the system state could be checked and detected, and then managers make decisions on production planning and equipment management, etc. Therefore, lots of studies on the inspection policy optimization have been extensively developed, including periodic or aperiodic inspection, perfect or imperfect inspection [2]; also, most models are proposed for the multi-state degradation systems [3].

The concept of delay time has been proved to be a supporting theory to establish the relationship between inspections and the removal of defects, which is firstly proposed by Christer [4]. Currently, depending on different state space, there are two kinds of delay time modeling, i.e., a two-stage failure process and a three-stage failure process. Numerous PM or inspection models are proposed based on these two concepts, please refer to the work by Wang [5]. Using the two-stage failure process, the systems' lifetime can be divided into three possible states, i.e., normal, defective and failed, in which the time lapse from the occurrence of the initial defect point to an eventual failure is called as the delay time stage (also named the second stage). Further, Wang et al. proposed the three-stage failure process by dividing the second stage into minor and severe stages, such that it provides more repair decisions depending on different states since it is closer to the real cases [6]. So, a three-stage failure process is used to describe the deterioration process in this study.

Commonly, different repair activities are provided to be chosen and carried out when inspections reveal the system to be in a degraded state; for example, perfect repair which restores the system to a new state, minimal repair bringing the system into the state before repair, and imperfect repair between perfect repair and minimal repair [7]. To the best of our knowledge, few studies of PM models focused on the organizational factors, especially the impact of human behavior on the quality of repair activities [8]. However, the root cause of many accidents has been reported due to human errors. So, this paper considers an inspection policy under the three-stage failure process, in which the failure caused by human errors at the time of repair is incorporated and modeled as the main contribution of this paper.

The rest of the paper is organized as follows. In Section 2, we introduce the modeling assumptions. Then, an inspection policy model is developed in Section 3. A numerical example is given in Section 4. Finally, Section 5 concludes this paper.

2. Problem Description and Notations.

2.1. Modeling assumptions.

The following assumptions are presented and used in this paper for a modeling purpose.

(1) The time horizon is considered as infinite.
(2) A single-component system is considered and according to the three-stage failure process the possible states at the time of an inspection could be normal, minor defect, severe defect, failed, respectively.
(3) These three stages are assumed to be independent of each other.
(4) An inspection scheme is performed with the regular interval and inspections are perfect to fully detect the state of the system.
(5) There is no intervention if it is found to be in the normal state by an inspection.
(6) If the system is revealed to be in the defective state whatever it is minor or severe, repair activities are carried out immediately to reduce the risk of the failure occurrence.
(7) Since human errors resulted from operations or skills may occur randomly in the repair process, repair caused by the identification of any defective state is regarded to bring the system back to the as-good-as-new state with a limited probability. Moreover, it is easier to renew the minor defective system than the severe defective system. Thus, the renewal probability the system is renewed in the minor defective state is higher than that in the severe defective state.
(8) Failure caused by human errors is called as a repair failure, and a failure due to deterioration is a hard failure. Any failure is self-announcing.

2.2. Notations.

X_i	Random duration for the ith stage of the system, and $i = 1, 2, 3$
$f_{X_1}(x)$	Probability density function (PDF) of X_1
$f_{X_2}(y)$	PDF of X_2
$f_{X_3}(z)$	PDF of X_3
p	Probability that the system is renewed after repair if the minor defective state is revealed by an inspection
q	Probability that the system is renewed due to repairing the severe defect
C_I	Average cost per inspection
C_F	Average cost caused by a failure renewal
C_{RM}	Average cost of an inspection renewal when a minor defect is repaired
C_{RS}	Average cost of an inspection renewal when a severe defect is repaired, and $C_{RS} > C_{RM}$
$E_C(T)$	Expected renewal cycle cost
$E_L(T)$	Expected renewal cycle length
$C(T)$	Expected cost per unit time

3. An Inspection Policy Model for Three-Stage Deterioration Systems.

3.1. Probability of a failure renewal. Depending on assumptions (6)-(8), this model considers two types of failures subject to a hard failure and a repair failure.

(1) The system is renewed due to a failure T_f in $((n-1)T, nT)$, before which any defect is not inspected, as shown in Figure 1.

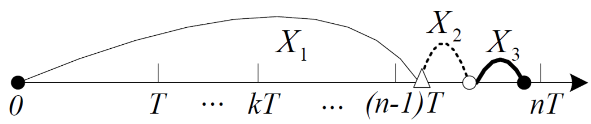

FIGURE 1. The system fails before any defect is found by an inspection, in which a hard failure is denoted by •; and the minor defective stage starts from △ and ends ∘.

Then the corresponding renewal probability can be calculated as

$$\begin{aligned} P_{F1} &= P\left((n-1)T < T_f < nT\right) \\ &= P\left\{(n-1)T < X_1 < nT,\ X_1 + X_2 < nT,\ X_1 + X_2 + X_3 < nT\right\} \\ &= \int_{(n-1)T}^{nT} \int_0^{nT-x} \int_0^{nT-x-y} f_{X_1}(x) f_{X_2}(y) f_{X_3}(z) dz dy dx \end{aligned} \tag{1}$$

(2) As can be seen in Figure 2, the proposed model focuses on two different repair failure renewals, depending to which defect is identified by an inspection. So we have the renewal probability of repair failure is obtained as

$$\begin{aligned} P_{F2} &= P\left(T_f = nT\right) = P_{F2,M} + P_{F2,S} \\ &= P\left\{ \begin{array}{l} (n-1)T < X_1 < nT,\ X_1 + X_2 > nT \\ \text{repair failure occurs with a probability } 1-p \end{array} \right\} \\ &\quad + P\left\{ \begin{array}{l} (n-1)T < X_1 < nT,\ X_1 + X_2 < nT, \\ X_1 + X_2 + X_3 > nT \\ \text{repair failure occurs with a probability } 1-q \end{array} \right\} \\ &= \int_{(n-1)T}^{nT} \int_{nT-x}^{\infty} (1-p) f_{X_1}(x) f_{X_2}(y) dy dx \\ &\quad + \int_{(n-1)T}^{nT} \int_0^{nT-x} \int_{nT-x-y}^{\infty} (1-q) f_{X_1}(x) f_{X_2}(y) f_{X_3}(z) dz dy dx \end{aligned} \tag{2}$$

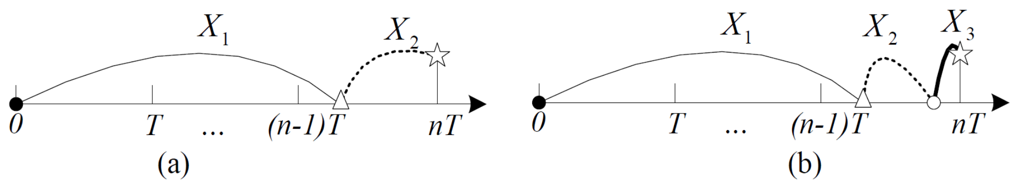

FIGURE 2. Renewal scenarios due to a repair failure when the minor or severe defective state is identified at an inspection nT, denoted by ☆

3.2. **Probability of an inspection renewal.** If repair can restore the system to the as-good-as-new state once the minor or severe defective state is identified by an inspection nT with the corresponding probabilities p and q, it leads to an inspection renewal. Then the renewal probability can be given as

$$\begin{aligned} P_{PM} &= P\left(T_{PM} = nT\right) = P_{PM,M} + P_{PM,S} \\ &= \int_{(n-1)T}^{nT} \int_{nT-x}^{\infty} p \cdot f_{X_1}(x) f_{X_2}(y) dy dx \\ &\quad + \int_{(n-1)T}^{nT} \int_{0}^{nT-x} \int_{nT-x-y}^{\infty} q \cdot f_{X_1}(x) f_{X_2}(y) f_{X_3}(z) dz dy dx \end{aligned} \tag{3}$$

3.3. **Expected cost per unit time.** Based on all possible renewals, as shown in Equations (1)-(3), the expected renewal cycle cost is firstly calculated by summing up all inspection cost and renewal cost, which is given as follows:

$$\begin{aligned} E_C(T) = \sum_{n=1}^{\infty} &[((n-1)C_I + C_F)P_{F1} + (nC_I + C_F)P_{F2} + (nC_I + C_{RM})P_{PM,M} \\ &+ (nC_I + C_{RS})P_{PM,S}] \end{aligned} \tag{4}$$

Equations (1) and (2) give the probabilities of a hard and repair failure, respectively, from which the corresponding expected lengths can be derived. Initially, the PDF of a hard failure in $((n-1)T + z,\ (n-1)T + z + dz)$ $(z \in (0, T))$ is given by

$$\begin{aligned} p_{F1} &= P\left((n-1)T < T_f < (n-1)T + z\right)/dz \\ &= P\left\{ \begin{array}{l} (n-1)T < X_1 < (n-1)T + z,\ X_1 + X_2 < (n-1)T + z, \\ X_1 + X_2 + X_3 < (n-1)T + z \end{array} \right\} \Big/ dz \\ &= \int_{(n-1)T}^{(n-1)T+z} \int_{0}^{(n-1)T+z-x} \left[f_{X_1}(x) f_{X_2}(y) f_{X_3}\left((n-1)T + z - x - y\right)\right] dy dx \end{aligned} \tag{5}$$

Using Equation (5) and considering that the system may be renewed due to a repair failure and an inspection renewal at an inspection nT, the expected renewal cycle length is given by

$$E_L(T) = \sum_{n=1}^{\infty} \left[\int_0^T \left((n-1)T + z\right) \cdot p_{F1} dz + nT \cdot \left(P_{F2} + P_{PM}\right) \right] \tag{6}$$

By using the renewal award theorem, the expected cost per unit time of the proposed inspection policy is given by

$$C(T) = \frac{\text{The expected renewal cycle cost}}{\text{The expected renewal cycle length}} = \frac{E_C(T)}{E_L(T)} \tag{7}$$

We aim to find the optimal inspection interval T^* by minimizing the expected cost per unit time Equation (7).

4. **A Numerical Example.** In this example, the lifetime distribution of the system in the ith stage is assumed to follow the Weibull distribution with scale parameter α_i and shape paramter β_i, which has been widely used in the previous delay-time-modeling literature. Table 1 gives the distribution parameters of three stages and some parameters mentioned in the proposed model (7) are also given.

Based on the parameters in Table 1 and the optimization model (7), the results of the cost model are shown in Figure 3 with the inspection interval T from 5 to 30. It is obvious that the optimal solution is $T^* = 13$ with the minimal expected cost per unit time 29.9024. It is obvious that the expected cost per unit time decreases firstly and then increases with the increase of the inspection interval T, which is what we expected.

TABLE 1. The values of the parameters

i	α_i	β_i	p	q	C_I	C_F	C_{RM}	C_{RS}
1	0.04	1.15						
2	0.09	1.27	0.85	0.70	50	2500	500	800
3	0.17	1.31						

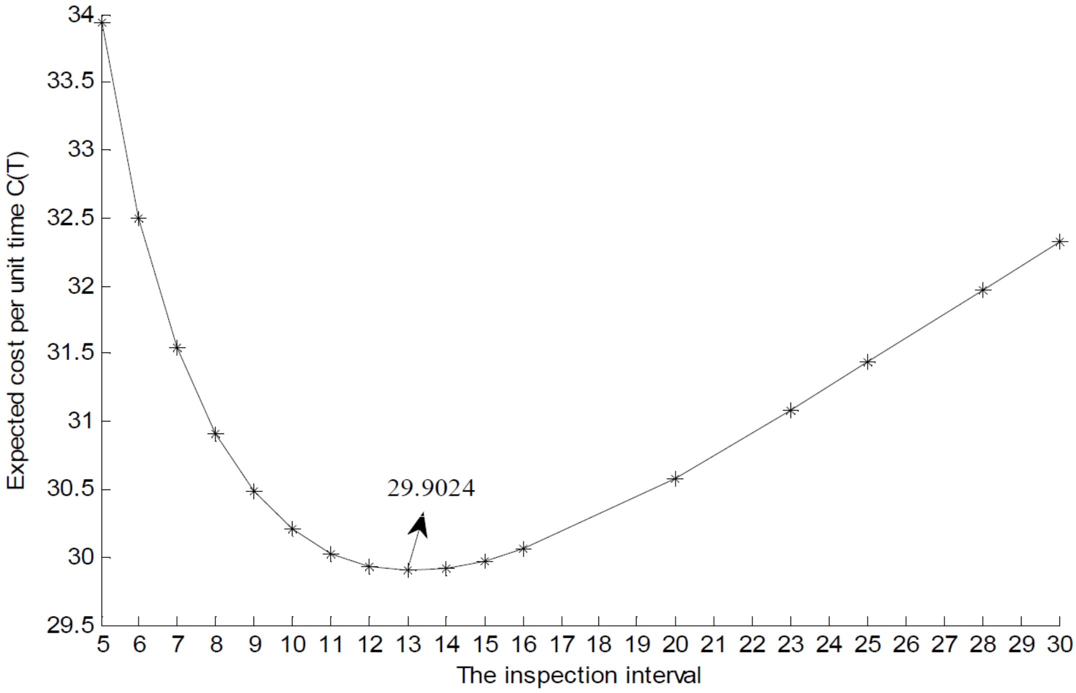

FIGURE 3. The simulation results

TABLE 2. Sensitivity analysis

$q = 0.70$			$p = 0.85$		
p	T^*	$C(T^*)$	q	T^*	$C(T^*)$
0.75	17	33.1278	0.40	12	32.2425
0.80	15	31.5691	0.50	12	31.4730
0.85	13	29.9024	0.60	13	30.6900
0.90	12	28.1404	0.70	13	29.2024
0.95	11	25.5520	0.80	13	29.1149

It can be explained that when the system is frequently checked, i.e., a relatively smaller inspection interval, more inspection cost will be required; reversely, a renewal caused by a failure is performed for the system with a higher cost.

Table 2 shows the sensitivity analysis of different values for p and q. It is noted that the minimal expected cost per unit time decreases with the increase of these two parameters, which is because that a higher renewal probability in the minor or severe defective state costs less than a failure renewal. However, it is noted that the trend of the optimal inspection interval T^* is different based on different values of p and q. When q is fixed, i.e., it is assumed to be $q = 0.70$, the optimal inspection interval decreases as the value of p increases. It is consistent with the practical case in the industry. It is clear that from Table 2 the optimal inspection interval T^* increases with the increase of q. Such a trend is probably because the system is repaired and renewed when it is revealed to be in the minor defective state. Thus, the parameter q works marginally on the optimal inspection interval.

5. **Conclusions.** This paper models a periodic inspection policy for single-component systems subject to the three-stage failure process. The proposed model considered the organization risk due to human errors when repair is carried out for the defective system, which is more realistic than the previous works on inspection and PM models where human errors are neglected. Further work should be done to consider imperfect inspection and maintenance.

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