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RESEARCH ON THE ROUGH PROGRAMMING METHOD BASED ON RELATIONSHIP EFFECT

Fachao Li and Kai Zhang

School of Economics and Management
Hebei University of Science and Technology
No. 26, Yuxiang Street, Shijiazhuang 050018, P. R. China
lifachao@tsinghua.org.cn; 751799587@qq.com

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Abstract. *Dealing with the relationship among research objects in many decision problems is an important research content in academic field. In this paper, we analyze characteristics and shortages of the existing rough programming methods based on rough decision problem with relationship effect, and then based on basic function of the object and effect function of the relation class, we construct a rough programming model based on relationship effect (denoted by BERC-RPM for short). After that for the problem with positive effect of BERC-RPM, we give the concept of effect value based on upper (lower) approximation, and also give the solving steps of BERC-RPM. Finally we analyze the characteristics of BERC-RPM through a concrete example. All these indicate that BERC-RPM has good interpretability and operability. It has wide application prospects in the fields of resource allocation, complex decision making, artificial intelligence, system optimization and so on.*
Keywords: Rough set, Rough programming, Relationship effect, The effect value based on upper (lower) approximation, Decision

1. **Introduction.** Rough set theory [1], proposed by Pawlak in 1984, the core is to consider the description of concept on the universes by some kind of approximate space. It is an effective tool dealing with imprecise, inconsistent, and incomplete information. In the past decades, many scholars have enriched and perfected the Pawlak rough set theories to make many important research results by combining different theories and application background. In theory, the focus of research is to expand rough set model combining with different backgrounds and theories. For example, Ziarko [2] proposed a variable precision rough set model; as the objects often have multiple ownership, Slowinski and Vanderpooten [3] proposed a rough set model based on similarity relationship; Wang et al. [4] discussed some basic properties of covering information systems and decision systems under homeomorphisms; Zhu [5] studied a variety of different types of rough set models based on extended cover and their relationships; Jensen and Shen [6] proposed the concept of fuzzy rough sets; Dubois [7] proposed the concept of rough fuzzy sets; Liu [8] proposed the concepts of random rough variables and fuzzy rough variables, and analyzed their structural characteristic in theory. In application, the focus of research is to combine with other methods and techniques. By combining with the technology of data mining, taking rough set theory as a tool for describing uncertain information, many scholars constructed feature extraction and attribute reduction methods based on data system. For example, in view of the inconsistency of coverage system, Zhang et al. [9] proposed the attribute reduction method to maintain confidence level based on some typical rules of coverage system; in view of the feature extraction of dynamic data, Shu and Shen [10] proposed a feature selection method for incremental form based on the selection of rough characteristics; aiming at the diagnosis of hepatitis disease, Yilmaz and Murat [11] proposed a new hybrid medical decision support system based on rough

set (RS) and extreme learning machine (ELM); Phophalia et al. [12] proposed an image repairing method based on rough set through roughness is a measure of similarity degree of image as a basic principle. Wang et al. [13] in order to determine the address of the mineral resources, using rough set theory to test and screen index system, established the comprehensive evaluation model based on catastrophe progression, and achieved good results; Wu and Gou [14] put forward an attribute reduction algorithm based on rough set and information entropy.

It is worth noting, there are few researches on theory and method of rough programming. In the production management, resource allocation, personnel assignment, complex system optimization and many other decision-making problems, there often exist different forms of uncertainty. Many scholars study the programming of uncertain environment in different backgrounds and methods, and obtain some important research results. For example, Ebrahim [15] firstly proposed rough programming problem, and gave the concept of global rough optimal solution and local rough optimal solution in rough based on the method of approximate region and distance functions, and studied the rough degree of rough optimal solution; however, this study only considered the answer of roughness, and did not consider the role of equivalence classes as well as the characteristics of decision; Lu and Huang [16] constructed two-stage stochastic programming model of rough interval through using rough interval to describe the most reliable and possible range of complex parameters, but the model only used interval as a description of the rough, did not systematically analyze the characteristics of rough programming; Zhang et al. [17] proposed a data mining method based on multi-objective rough programming for making better use of the reduction of information to classify and predict. However, the researches only use rough set theory as a preprocessing tool for data information, and they did not discuss about the structure of rough programming; Zhou and Li [18] proposed a class of rough programming models and solution method based on synthesis effect, but the model is limited to the synthesis effect, and they did not relate to relationship effect of rough programming.

Although the above analysis is part of the rough set theory, these reflect current situation basically, the theoretical research on rough set and applicative research on data mining have become more and more mature. However, the research on rough programming is at the starting stage, the main problem is lack of formal description based on structural features. To solve the problem, in this paper, we have the following work: 1) we analyze the essential characteristics of rough programming combining with the reality, and construct the rough programming model based on relationship effect (denoted by BERC-RPM for short); 2) give the concept of effect value based on upper (lower) approximation effect and the steps solving of BERC-RPM; 3) we analyze the characteristics and effectiveness of BERC-RPM through a concrete example. Based on the above work, the structure of this paper is as follows: Section 1 is introduction; Section 2 describes the general form of programming problem and introduces the concepts of rough set; Section 3 constructs rough programming model based on relationship effect (BERC-RPM) and gives formal description; Section 4 analyzes characteristics and solving steps of BERC-RPM combined with a concrete example; Section 5 is the conclusion.

2. **Preliminaries.**

2.1. **The formal description of programming problem.**

The essence of programming problem is to determine the optimal decision scheme under constraint conditions. And its general model is as follows:

$$\begin{cases} \max & f(x), \\ \text{s.t.} & x \in X. \end{cases} \tag{1}$$

Here, X is feasible region, and it expresses a scheme to satisfy the decision requirements on the universe U; $f(x)$ is objective function with quantitative characteristics on U. $f(x)$ is a criterion used to measure the decision scheme is good or not.

According to the characteristics of feasible region and objective function, (1) can be classified into different categories. When X is an exact set on the universe U and $f(x)$ is a real valued function on U, (1) is called certain programming problem; when X or $f(x)$ has certain uncertainty, (1) is called uncertain programming problem; especially, when X or $f(x)$ has randomness (fuzziness) uncertainty, (1) is called a randomness (fuzziness) programming problem.

2.2. **Rough set.** For simplicity, we assume in the following: 1) For the equivalence relation R on universe U (namely $R \subset U \times U$) and meet: i) $(x, x) \in R$ for any $x \in U$; ii) $(x, y) \in R \Leftrightarrow (y, x) \in R$; iii) if $(x, y) \in R$ and $(y, z) \in R$, then $(x, z) \in R$, $[x]_R = \{y | y \in U \text{ and } (x, y) \in R\}$ is the R-equivalence class of x, $U/R = \{[x]_R | x \in U\}$, and called (U, R) is an approximation space; 2) for the finite set of A, $|A|$ represents the number of elements in A.

Definition 2.1. [1] *Let U be a finite universe and R be an equivalence relation on U, $X \subseteq U$,*

$$\underline{R}(X) = \{x | x \in U, \text{ and } [x]_R \subseteq X\}, \ \overline{R}(X) = \{x | x \in U, \text{ and } [x]_R \cap U \neq \varnothing\}. \tag{2}$$

If $\underline{R}(X) = \overline{R}(X)$, then X is called R-definable (or R-exact set); otherwise, X is called R-undefinable (or R-rough set). And $\underline{R}(X)$ is called R-lower approximation set of X, $\overline{R}(X)$ is called R-upper approximation set of X.

Obviously, the elements of $\underline{R}(X)$ surely belong to X according to relationship R, the elements of $U - \underline{R}(X)$ surely do not belong to X and the elements of $\overline{R}(X) - \underline{R}(X)$ possibly belong to X, so X can be approximately described by $(\underline{R}(X), \overline{R}(X))$. For simplicity, $Pos_R(X) = \underline{R}(X)$ is called R-positive region of X, $Neg_R(X) = U - \overline{R}(X)$ is called R-negative region of X, $Bn_R(X) = \overline{R}(X) - \underline{R}(X)$ is called R-boundary region of X, and

$$\alpha_R(X) = \begin{cases} |\underline{R}(X)|/|\overline{R}(X)|, & x \neq \varnothing, \\ 1, & x = \varnothing. \end{cases} \tag{3}$$

is called R-approximate accuracy of X, $\rho_R(X) = 1 - \alpha_R(X)$ is roughness on R of X.

Rough set has many good properties, and the concrete content can refer to [19].

3. **Rough Programming Model Based on Relationship Effect (BERC-RPM).** In this section, we will focus on rough uncertainty to discuss rough programming model based on relationship effect.

3.1. **Description problem.** The rough uncertainty is common in many decision processes. In 2013, Li et al. [20] proposed a rough programming model with a single element which is based on some kind of connection between elements:

$$\begin{cases} \max & f(x), \\ \text{s.t.} & x \in (X, R). \end{cases} \tag{4}$$

Here, R is a relationship on U; X is a subset of universe U (called feasible region); $f(x)$ is a numerical function that depends on both x and $[x]_R$ at the same time, and it is the guideline that describes comprehensively the function of x (called the objective function); $[x]_R = \{y \,|(x, y) \in R\}$ is R relational class of x.

Based on the above description, we can see, the objective function $f(x)$ is still a mapping of $U \to (-\infty, +\infty)$ (can be abstracted as $f(x) = F(x, [x]_R)$), the object of decision is an element in U, so (4) cannot solve the decision-making problem that is a subset of U as decision object. Then, we will combine with a practical problem to analyze the basic features of decision-making problem that makes the subset of U as decision result in rough environment.

Cultivation of innovative team. A company plans to invest a certain amount of research funding for fostering a new technology application innovation team composed of $3 \sim 5$ people to enhance market competitiveness. The formation of any new technology needs a certain amount of time and the interaction among team members, in long-term work in the process, the technical personnel in accordance with the characteristics of technical expertise, interests, preferences have initially formed a five technology application and development team X_i, $i = 1, 2, 3, 4, 5$. According to past data statistics, the ability to work of team X_i and members in nearly 3 years (to score as a unify quantitative platform of all kinds of different performance) are as shown in Table 1 (among them, $F_1(x)$ represents the ability to work of technical personnel x, $F_2(X_i)$ represents the ability to work of team X_i). Try to find the innovation team selection scheme based on determining the best working ability as the principle.

TABLE 1. Statistics for each team and members of the work performance

$F_2(X_i)$	150			165			85		84		112		
X_i	X_1			X_2			X_3		X_4		X_5		
x_k	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}
$F_1(x_k)$	32	40	38	25	48	30	33	38	25	40	34	36	38

As can be seen from Table 1, the cooperation among the team members had a positive effect (that the team's work ability is more than the members' work ability), it shows that the choice to the best work ability of team is not a simple choice to the best work ability of individual, and in the decision-making process, we not only consider the individual performance, but also consider whole performance of the team. There are many decision-making problems with the above characteristics in the portfolio investment, resource allocation, system optimization and other areas. So, it has important application value that constructs a decision object based on a subset of original universe as well as takes account of the individual and group performance of decision-making method.

3.2. **Formal representation of BERC-RPM.** Combined with the discussions of Section 3.1, if we abstract one of the different teams as an approximate space (U, R) formed by a certain relation R on universe U, then the individual performance can be abstracted as a mapping F_1: $U \to (0, +\infty)$, the team performance can be abstracted as a mapping F_2: $U/R \to (0, +\infty)$, then the decision-making problem of Section 3.1 can be abstracted as

$$\begin{cases} \max & z = \Phi(X, R, F_1, F_2), \\ \text{s.t.} & X \in \mathscr{P}(U), \\ & h_i(X) \le 0, \quad i = 1, 2, \cdots, m. \end{cases} \tag{5}$$

Among them, 1) $\Phi(X, R, F_1, F_2)$ is the objective function, its value depends on X, (U, R) as well as the performance function F_1, F_2; 2) $h_i(X) \le 0$ represents the requirements of the decision object. If F_2 is understood as the performance description of R relationship class, we understand $\Phi(X, R, F_1, F_2)$ as the relationship effect value of the elements in X, then $F_2([x]_R) \neq \Sigma\{F_1(y)|y \in [x]_R\}$ indicates that the elements in the relationship class have a cooperative effect, this is a basic feature of (5). For simplicity, we called (5) a rough programming model based on relationship effect in the following (denoted by BERC-RPM for short).

Obviously, (5) is still an abstract model, and it can be divided into two basic conditions of the positive effect and negative effect. Then we explore the solution on positive effect (that is $F_2([x]_R) \ge \Sigma\{F_1(y)|y \in [x]_R\}$ for any $x \in U$) of the programming problem (5).

Definition 3.1. *Let U be a finite universe, R be an equivalent relation on U, $F_1 : U \to (0,+\infty)$, $F_2 : U/R \to (0,+\infty)$, $X \subseteq U$. Denote $U/R = \{X_1, X_2, \cdots, X_m\}$,*

$$E([x]_R, R, F_1, F_2) = F_2([x]_R) - \Sigma\{F_1(y)|y \in [x]_R\}, \tag{6}$$

$$W(x, R, F_1, F_2) = E([x]_R, R, F_1, F_2)F_1(x)/\Sigma\{F_1(y)|y \in [x]_R\}, \tag{7}$$

$$\underline{\Phi}(X, R, F_1, F_2) = \Sigma\{F_2(X_i)|X_i \subseteq X \text{ and } X_i \in U/R\} + \Sigma\{F_1(x)|x \in X - \underline{R}(X)\}, \tag{8}$$

$$\begin{aligned}\overline{\Phi}(X, R, F_1, F_2) = {} & \Sigma\{F_2(X_i)|X_i \subseteq X \text{ and } X_i \in U/R\} \\ & + \Sigma\{F_1(x) + W(x, R, F_1, F_2)|x \in X - \underline{R}(X)\}.\end{aligned} \tag{9}$$

Then $E([x]_R, R, F_1, F_2)$ is called the effect value based on $[x]_R$ on (R, F_1, F_2), $W(x, R, F_1, F_2)$ is called the average effect based on x on (R, F_1, F_2), $\underline{\Phi}(X, R, F_1, F_2)$ is called the relationship effect value of lower approximation effect based on X with (R, F_1, F_2), $\overline{\Phi}(X, R, F_1, F_2)$ is called the relationship effect value of upper approximation effect based on X with (R, F_1, F_2).

It is not difficult to see, $\underline{\Phi}(X, R, F_1, F_2)$ represents the minimum effect value of the performance of each element in X, $\overline{\Phi}(X, R, F_1, F_2)$ represents the maximum effect value of the performance of each element in X in mean value. In the decision-making process, it has similar functions compared with the lower and upper approximation sets of Definition 2.1. So, $\underline{\Phi}(X, R, F_1, F_2)$ and $\overline{\Phi}(X, R, F_1, F_2)$ can provide support for the specific solution method of (5).

Theorem 3.1. *Let U be a finite universe, R be an equivalent relation on U, $F_1 : U \to (0,+\infty)$, $F_2 : U/R \to (0,+\infty)$, $X \subseteq Y \subseteq U$. Denote $U/R = \{X_1, X_2, \cdots, X_m\}$. Then:*

1) $\overline{\Phi}(X, R, F_1, F_2) = \underline{\Phi}(X, R, F_1, F_2) + \Sigma\{F_1(x)W(x, R, F_1, F_2)|x \in X - \underline{R}(X)\}$.

2) $\underline{\Phi}(X, R, F_1, F_2) = \overline{\Phi}(X, R, F_1, F_2) \Leftrightarrow W(X, R, F_1, F_2) = 0$ for any $x \in X - \underline{R}(X)$; specially, when $X = \underline{R}(X)$, $\underline{\Phi}(X, R, F_1, F_2) = \overline{\Phi}(X, R, F_1, F_2)$.

3) $\underline{\Phi}(X, R, F_1, F_2) \le \underline{\Phi}(Y, R, F_1, F_2)$, $\overline{\Phi}(X, R, F_1, F_2) \le \overline{\Phi}(Y, R, F_1, F_2)$.

Theorem 3.1 can be directly proved by Definitions 2.1 and 3.1.

Theorem 3.2. *Let $U = \{x_1, x_2, \cdots, x_n\}$, and R be an equivalent relation on U, $F_1 : U \to (0,+\infty)$, $F_2 : U/R \to (0,+\infty)$, $G(x) = F_1(x) + W(X, R, F_1, F_2)$. If $G(x_1) \ge G(x_2) \ge \cdots \ge G(x_n)$, then*

$$\max\left\{\overline{\Phi}(X, R, F_1, F_2)|X \in \mathscr{P}(U) \text{ and } |X| = s\right\} = \Sigma_{i=1}^{s} G(x_i). \tag{10}$$

Proof: Denote $U/R = \{X_1, X_2, \cdots, X_m\}$, then for $X \in \mathscr{P}(U)$ and $|X| = s$: 1) when $X_k \nsubseteq X$, for any $k \in \{1, 2, \cdots, m\}$ is tenable any time, then $\overline{\Phi}(X, R, F_1, F_2) = \Sigma\{F_2(X_i)|X_i \subseteq X \text{ and } X_i \in U/R\} + \Sigma\{F_1(x) + W(x, R, F_1, F_2)|x \in X - \underline{R}(X)\} = \Sigma\{F_1(x) + W(x, R, F_1, F_2)|x \in X\} = \Sigma\{G(x)|x \in X\} \le \Sigma_{i=1}^{s} G(x_i)$; 2) when $X_k \in U/R$ makes $X_k \subset X$ (might as well assume $X_1 \in U/R$, $X_k \notin U/R$ for any $k \in \{2, 3, \cdots, m\}$ is tenable any time), because $F_2(X_1) = \Sigma\{F_1(x) + W(x, R, F_1, F_2)|x \in X_1\}$ we know $\overline{\Phi}(X, R, F_1, F_2) = F_2(X_1) + \Sigma\{F_1(x) + W(x, R, F_1, F_2)|x \in X - X_1\} = \Sigma\{F_1(x) + W(x, R, F_1, F_2)|x \in X\} = \Sigma\{G(x)|x \in X\} \le \Sigma_{i=1}^{s} G(x_i)$.

4. **Example Analysis.** This part focuses on the cultivation of innovative team in Section 3.1, combining with the discussion of Section 3.2, we will further analyze the characteristics of BERC-RPM and the specific implementation steps of the solution.

Step 1. The specific of BERC-RPM. Using $\Phi(X, R, F_1, F_2)$ on X of the monotonic non-decreasing, we know the innovation team's work ability becomes stronger and stronger with the increase of team size, so, the size of strongest innovation team should be 5 people, from this we know:

$$\begin{cases} \max & z = \Phi(X, R, F_1, F_2), \\ \text{s.t.} & X \in \mathscr{P}(U) \text{ and } |X| = 5. \end{cases} \tag{11}$$

TABLE 2. The effect value of each relationship class and each element

$F_2(X_i)$	X_i	x_k	$F_1(x_k)$	$E([x]_R, R, F_1, F_2)$	$W(x, R, F_1, F_2)$	$G(x)$
150	X_1	x_1	32	40	11.64	43.64
		x_2	40	40	11.55	54.55
		x_3	38	40	13.82	51.82
165	X_2	x_4	25	62	15.05	40.05
		x_5	48	62	28.89	76.89
		x_6	30	62	18.06	48.06
85	X_3	x_7	33	14	6.51	39.51
		x_8	38	14	7.49	45.59
84	X_4	x_9	25	19	7.31	32.31
		x_{10}	40	19	11.69	51.69
112	X_5	x_{11}	34	4	1.26	35.26
		x_{12}	36	4	1.33	37.33
		x_{13}	38	4	1.41	39.41

Step 2. Using (6) and (7) calculate the effect values of each relationship class $E([x]_R, R, F_1, F_2)$ as well as the average effect of each element $W(x, R, F_1, F_2)$ and $G(x) = F_1(x) + W(x, R, F_1, F_2)$, as shown in Table 2.

Step 3. Use (8) and (9), combined with appropriate methods or algorithms to determine $\overline{\Phi}(X, R, F_1, F_2)$, or $\underline{\Phi}(X, R, F_1, F_2)$, or $\omega_1\overline{\Phi}(X, R, F_1, F_2) + \omega_2\underline{\Phi}(X, R, F_1, F_2)$ (ω_1, ω_2 is nonnegative, and $\omega_1 + \omega_2 = 1$) as the decision results X^* based on the optimization criteria, as shown in Table 3 (the solving method is shown in Remark 4.1 $\sim$ Remark 4.3).

TABLE 3. Satisfactory decision results based on effect value of lower (upper) approximation effect

objective function	$\overline{\Phi}(X, R, F_1, F_2)$	$\underline{\Phi}(X, R, F_1, F_2)$	$\omega_1\overline{\Phi}(X, R, F_1, F_2) + \omega_2\underline{\Phi}(X, R, F_1, F_2)$		
			$\omega_1 = 0.7$, $\omega_2 = 0.3$	$\omega_1 = \omega_2 = 0.5$	$\omega_1 = 0.3$, $\omega_2 = 0.7$
decision result X^*	x_2, x_3, x_5, x_6, x_{10}	x_4, x_5, x_6, x_7, x_8	x_1, x_2, x_3, x_5, x_{10}	x_1, x_2, x_3, x_5, x_{10}	x_2, x_3, x_5, x_6, x_{10}
effect value X^*	283.01	250.00	266.41	258.29	252.87

Remark 4.1. *With $\overline{\Phi}(X, R, F_1, F_2)$ as the optimization criterion, by using Theorem 3.2, the optimal decision result is $X^* = \{x_2, x_3, x_5, x_6, x_{10}\}$, and the solving method is general, namely when $U = \{x_1, x_2, \cdots, x_n\}$, $X^* = \{x_{i1}, x_{i2}, \cdots, x_{is}\}$ meet $\overline{\Phi}(X^*, R, F_1, F_2) = \max\{\overline{\Phi}(X, R, F_1, F_2) | X \in \mathscr{P}(U)$ and $|X| = s\}$. Among them $G(x) = F_1(x) + W(x, R, F_1, F_2)$, $G(x_{i1}) \geq G(x_{i2}) \geq \cdots \geq G(x_{in})$.*

Remark 4.2. *With $\underline{\Phi}(X, R, F_1, F_2)$ as the optimization criterion, then there is no formal method for determining the optimal decision, and by enumeration method to determine the optimal decision-making is a hard problem of NP (with the increase of $|U|$, the computational complexity increases exponentially). However, we can determine the optimal decision scheme according to the following procedure ($U = \{x_1, x_2, \cdots, x_n\}$, $U/R = \{X_1, X_2, \cdots, X_m\}$), $M(Y) = \Sigma_{x \in Y} F_1(x)$.*

1) In accordance with the decreasing rank, the $F_1(x_1), F_1(x_2), \cdots, F_1(x_n)$ is sorted to $F_1(x_{i1}) \geq F_1(x_{i2}) \geq \cdots \geq F_1(x_{in})$, make $X^* = \{x_{i1}, x_{i2}, \cdots, x_{is}\}$.

2) To do the following adjustments for X^* from 1 to m: i) when $X_k \cap X^* = \varnothing$, X^* unchanged; ii) when $|X_k| > |X^*|$, X^* unchanged; iii) when $X_k \cap X^* \neq \varnothing$, $|X_k| < |X^*|$, and $F_2(X_k) + M(Y_k) \leq M(X^*)$, X^* unchanged; iv) when $X_k \cap X^* \neq \varnothing$, $|X_k| < |X^*|$ and $F_2(X_k) + M(Y_k) > M(X^*)$, adjust the X^* to $X_k \cup Y_k$ (here: $X^* - X_k \triangleq \{y_1, y_2, \cdots, y_{s-|X \cap X_k|}\}$ and $F_1(y_1) \geq F_1(y_2) \geq \cdots \geq F_1(y_{s-|X_k|})$, $Y_k = \{y_1, y_2, \cdots, y_{s-|X_k|}\}$).

Remark 4.3. *With* $\omega_1\overline{\Phi}(X, R, F_1, F_2) + \omega_2\underline{\Phi}(X, R, F_1, F_2)$ *as the optimization criterion, the solution features are similar to* $\underline{\Phi}(X, R, F_1, F_2)$*, and we use* $F_1(x) + \omega_2 W(x, R, F_1, F_2)$ *to replace* $F_1(x)$ *of Remark 4.2 as the solution method.*

Remark 4.4. *With* $\omega_1\overline{\Phi}(X, R, F_1, F_2) + \omega_2\underline{\Phi}(X, R, F_1, F_2)$ *as the optimization criterion, the selection rules for parameter* ω_1*,* ω_2 *are as follows: If tending to* $\overline{\Phi}(X, R, F_1, F_2)$*, then select* $\omega_1 > \omega_2$*; If tending to* $\underline{\Phi}(X, R, F_1, F_2)$*, then select* $\omega_1 < \omega_2$*; If giving consideration to* $\overline{\Phi}(X, R, F_1, F_2)$ *and* $\underline{\Phi}(X, R, F_1, F_2)$*, then select* $\omega_1 = \omega_2$*.*

Synthesizing the above discussions, we see that from Table 3: 1) the decision result changes with the optimization criterion, and the difference is obvious (for example, the decision result is $\{x_2, x_3, x_5, x_6, x_{10}\}$ when the $\overline{\Phi}(X, R, F_1, F_2)$ is the optimization criterion; the decision result is $\{x_4, x_5, x_6, x_7, x_8\}$ when the $\underline{\Phi}(X, R, F_1, F_2)$ is the optimization criterion); 2) the decision of $\overline{\Phi}(X, R, F_1, F_2)$ as optimization criterion is a kind of optimistic decision-making; the decision of $\underline{\Phi}(X, R, F_1, F_2)$ as optimization criterion is a kind of conservative decision-making; the decision of $\omega_1\overline{\Phi}(X, R, F_1, F_2) + \omega_2\underline{\Phi}(X, R, F_1, F_2)$ as optimization criterion is a kind of compatible decision-making, which can be reflected by the different values of ω_1 and ω_2.

5. **Conclusion.** In this paper, based on the analysis of characteristics and shortages of the current rough programming method, we mainly do the following work: 1) based on basic function of the object and effect function of the relation class, we propose the rough programming model based on relationship effect (BERC-RPM); 2) aiming at solving the problem with positive effect of BERC-RPM, give the concept of effect value based on upper (lower) approximation, discuss the property of effect value based on upper (lower) approximation, and give the steps of solving BERC-RPM by using effect value based on upper (lower) approximation; 3) we analyze the characteristics of BERC-RPM through a concrete example. Finally, theoretical analysis and practical applications show that BERC-RPM not only contains existing method, but also has good interpretability and operability. So our discussions can enrich the existing theories and methods to a certain degree, have wide application prospect in many fields such as distribution of resources, complex decision, artificial intelligence, and optimization of systems. However, it is worth noting that the relationship effect often has negative effect in practical problems, the solving method constructed in this paper is not suitable for this kind of situation, so, how to construct a general solution of BERC-RPM, will be carried out as further work.

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